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NUMERICAL ANALYSIS OF A BINARY THIN METALLIC GRATING WITH SUBWAVELENGTH STRUCTURES

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1 Introduction

It is well known that a dielectric grating with very small periodicity can be analyzed by approximation of equivalent uniform permittivity. The approximation is often used for the design of periodic wedge-type absorber at radio wavelengths. In optics, the form birefringence caused by subwavelength gratings works as a diffractive optical device. Recently, it has been known that a binary relief grating with subwavelength structures can be used as more efficient diffractive optical device and the grating was analyzed [1, 2].

A metallic grating with small periodicity must be approximated by an equivalent sheet of surface resistance. However, uniform approximation of equivalent resistance has not been reported. Authors formulated the uniform approximation and showed the validity [3]. In addition, a binary resistive grating with subwavelength structures may be an efficient polarizer in radio wavelength. In this paper, it is numerically demonstrated that a binary extremely thin metallic gratings having surface resistance with subwavelength structures shows anisotropic characteristics of surface resistance. The derivation of uniform approximation of surface resistance is described. Then, the matrix eigenvalue calculations and the spatial harmonics expansions of flux densities are applied to analyze a binary thin metallic grating with subwavelength structures and anisotropic resistive gratings. The flux densities expansion is shown to be successful in the analysis of subwavelength gratings.

2 Description of the Problem

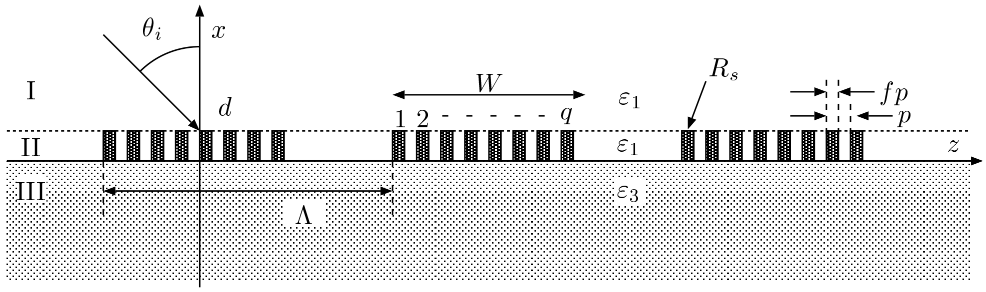

Fig. 1 A binary thin metallic grating with subwavelength structures.

The geometry of the problem is shown in Fig.1 where a one-dimensional binary thin metallic grating with periodicity Λ and width W is placed at $x = 0$. q is the number of subwavelength periodicity p. The width of subwavelength grating having surface resistance of R_s are given by fp. Let us consider scattering problem by a linearly polarized electromagnetic wave at an incident angle θ_i. Regions I and III, which have relative permittivity ε_1 and ε_3, are lossless materials. Since this paper treats the situation where thickness d is very thin and conductivity σ is very large, the materials of a thin metallic grating can be approximated by surface resistance $R_s = 1/(\sigma d)$ [4]. Complex relative permittivity ε_2 of region II can be written by using the surface resistance R_s as follows:

$$\varepsilon_2 = \varepsilon_2' - i\frac{\sigma}{\omega\varepsilon_0} = \varepsilon_1 - i\frac{1}{(R_s/Z_0)\ k_0 d} \tag{1}$$

where the real part of ε_2 is assumed to be $\varepsilon_2' = \varepsilon_1$ throughout the paper, since the grating in the region $x \geq 0$ is surrounded by region I.

In the following formulations, the space variables (x, y, z) are normalized by the wave number in vacuum $k_0 = \omega\sqrt{\varepsilon_0\mu_0}$. Using the normalized space variables, Maxwell's equations can be rewritten in dimensionless form as

$$\overline{\text{curl}}\sqrt{Y_0}\boldsymbol{E} = -i\mu(z)\sqrt{Z_0}\boldsymbol{H}, \quad \overline{\text{curl}}\sqrt{Z_0}\boldsymbol{H} = i\varepsilon(z)\sqrt{Y_0}\boldsymbol{E} \tag{2}$$

where $Z_0 = 1/Y_0 = \sqrt{\mu_0/\varepsilon_0}$ and $\overline{\text{curl}}$ is the rotation of the normalized space variables.

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3 Uniform Approximation of Equivalent Surface Resistance

When the periodicity is very small compared with incident wavelength, a metallic grating having surface resistance $R_s(z)$ as a function of the position parameter z can be expressed by uniform approximation of anisotropic surface resistance $(R_{sy},\ R_{sz})$. The y component of electric field E_y is uniform in the case of TE incidence and the z component of surface current J_z is uniform in the case of TM incidence. Therefore, the average current and electric field can be written as

$$\frac{E_y}{R_{sy}} = \left(\frac{1}{p}\int_0^p \frac{1}{R_s(z)}\,dz\right) E_y, \quad R_{sz}J_z = \left(\frac{1}{p}\int_0^p R_s(z)\,dz\right) J_z\ . \tag{3}$$

$R_s(z)$ is considered as $R_s(z) = R_{s1}$ $(0 \le z < fp)$, R_{s2} $(fp < z \le p)$. Substituting into Eqs. (3), uniform approximation of surface resistance can be obtained as

$$\frac{1}{R_{sy}} = \frac{1}{p}\int_0^p \frac{1}{R_s(z)}\,dz = \frac{f}{R_{s1}} + \frac{1-f}{R_{s2}}, \quad R_{sz} = \frac{1}{p}\int_0^p R_s(z)\,dz = fR_{s1} + (1-f)\,R_{s2}\ . \tag{4}$$

As shown in Fig. 1, the subwavelength structure is surrounded by the region I. From R_s corresponding to R_{s1} and $R_{s2} \to \infty$ in Eqs. (4), we obtain that $R_{sy} = R_s/f$ and $R_{sz} \to \infty$. It is seen that in the TE case, the analytical model is equivalent to a thin metallic grating having surface resistance R_{sy} with no subwavelength structures. In the TM case, the analytical model is equivalent to a nonperiodic multilayer.

4 Analysis of Method for a Thin Metallic Grating

4.1 Electromagnetic Fields in the Grating

Since the structure of grating layer is periodic, the relative permittivity $\varepsilon(z)$ and its inverse $1/\varepsilon(z)$, and the relative permeability $\mu(z)$ and its inverse $1/\mu(z)$ are expanded as a Fourier series of N_f terms with Fourier coefficients $\widetilde{\varepsilon}_m$, $(1/\tilde{\varepsilon})_m$, $\widetilde{\mu}_m$ and $(1/\tilde{\mu})_m$, respectively :

$$\varepsilon(z) = \sum_{m=-N_f}^{N_f} \widetilde{\varepsilon}_m \exp\{\, im(\lambda/\Lambda)\ z\}\,, \quad \frac{1}{\varepsilon(z)} = \sum_{m=-N_f}^{N_f} \left(\frac{1}{\tilde{\varepsilon}}\right)_m \exp\{\, im(\lambda/\Lambda)\ z\} \tag{5}$$

$$\mu(z) = \sum_{m=-N_f}^{N_f} \widetilde{\mu}_m \exp\{\, im(\lambda/\Lambda)\ z\}\,, \quad \frac{1}{\mu(z)} = \sum_{m=-N_f}^{N_f} \left(\frac{1}{\tilde{\mu}}\right)_m \exp\{\, im(\lambda/\Lambda)\ z\}\ . \tag{6}$$

The x and y components of electromagnetic fields $\sqrt{Y_0}E_\ell$ and $\sqrt{Z_0}H_\ell$ $(\ell = x, y)$ are continuous. The z components of electric flux density $\sqrt{Y_0}D_z = \varepsilon(z)\sqrt{Y_0}E_z$ and magnetic flux density $\sqrt{Z_0}B_z = \mu(z)\sqrt{Z_0}H_z$ are continuous against along the z axis. $\sqrt{Y_0}E_\ell$, $\sqrt{Z_0}H_\ell$, $\sqrt{Y_0}D_z$ and $\sqrt{Z_0}B_z$ are expressed in terms of spatial harmonics with expansion coefficients $e_{\ell m}(x)$, $h_{\ell m}(x)$, $d_{zm}(x)$ and $h_{zm}(x)$:

$$\sqrt{Y_0}E_\ell(x,z) = \sum_{m=-M}^{M} e_{\ell m}(x)\exp(-is_m z), \quad \sqrt{Y_0}D_z(x,z) = \sum_{m=-M}^{M} d_{zm}(x)\exp(-is_m z) \tag{7}$$

$$\sqrt{Z_0}H_\ell(x,z) = \sum_{m=-M}^{M} h_{\ell m}(x)\exp(-is_m z), \quad \sqrt{Z_0}B_z(x,z) = \sum_{m=-M}^{M} b_{zm}(x)\exp(-is_m z) \tag{8}$$

where s_m and s_0 are expressed in terms of the periodicity Λ, wavelength λ, relative permittivity ε_1 and incident angle θ_i by $s_m = s_0 + ms = \sqrt{\varepsilon_1\mu_1}\sin\theta_i + m\,(\lambda/\Lambda)$. The quantities $\boldsymbol{e}_\ell(x)$, $\boldsymbol{h}_\ell(x)$, $\boldsymbol{d}_z(x)$ and $\boldsymbol{b}_z(x)$ are defined by the (2M+1)-dimensional column vectors of the expansion coefficients. Substituting Eqs. (5) $\sim$ (8) into Maxwell's equations (2), the differential equation for the y and z field components can be derived as

$$\boldsymbol{F}(x)/dx = i\ [C]\,\boldsymbol{F}(x) \tag{9}$$

TE-wave:
$$\boldsymbol{F}(x) = \begin{bmatrix} \boldsymbol{e}_y(x) \\ \boldsymbol{b}_z(x) \end{bmatrix}, \quad [C] = \begin{bmatrix} [0] & -[1] \\ [1/\mu]^{-1}\,(-[\varepsilon] + [s][\mu]^{-1}[s]) & [0] \end{bmatrix} \tag{10}$$

TM-wave:
$$\boldsymbol{F}(x) = \begin{bmatrix} \boldsymbol{d}_z(x) \\ \boldsymbol{h}_y(x) \end{bmatrix}, \quad [C] = \begin{bmatrix} [0] & [1/\varepsilon]^{-1}\,([\mu] - [s][\varepsilon]^{-1}[s]) \\ [1] & [0] \end{bmatrix} \tag{11}$$

where $[C]$ consists of $m \times n$ submatrices: $[\varepsilon] = [\widetilde{\varepsilon}_{n-m}]$, $[1/\varepsilon] = [(1/\widetilde{\varepsilon})_{n-m}]$, $[\mu] = [\widetilde{\mu}_{n-m}]$, $[1/\mu] = [(1/\widetilde{\mu})_{n-m}]$, $[s] = [s_m\delta_{mn}]$, $[1] = [\delta_{mn}]$, $[0]$ is the zero matrix, $[\varepsilon]^{-1}$, $[1/\varepsilon]^{-1}$, $[\mu]$ and

$[1/\mu]^{-1}$ are the inverse matrices of $[\varepsilon]$, $[1/\varepsilon]$, $[\mu]$ and $[1/\mu]$, respectively and δ_{mn} is the Kronecker delta. The solutions of the differential equation (9) can be obtained by matrix eigenvalue calculations of the constant matrix $[C]$. By using a $2(2M+1)$-dimensional column vector $\boldsymbol{a}(x)$ and transforming $\boldsymbol{F}(x) = [T]\boldsymbol{a}(x)$, Eq. (9) can be transformed into

$$d\boldsymbol{a}(x)/dx = i[T]^{-1}[C][T]\boldsymbol{a}(x) = i[\kappa]\boldsymbol{a}(x) \tag{12}$$

where the matrix $[\kappa]$ is a diagonal matrix expressed in terms of the eigenvalue κ_m of the matrix $[C]$ and $[T]$ is a diagonalization matrix for the coefficient matrix $[C]$ and consists of eigenvectors corresponding to κ_m. The eigenvalues κ_m can be assigned to $(2M+1)$ terms of $\kappa_m^{\pm}$ for propagation in the $\pm x$ directions. $\boldsymbol{a}(x)$ can also be partitioned into complex amplitudes $\boldsymbol{a}^{\pm}(x)$ in the $\pm x$ direction corresponding to κ_m. The solution of Eq. (9) is given by

$$\boldsymbol{F}(x) = [T]\begin{bmatrix}\boldsymbol{a}^{+}(x)\\ \boldsymbol{a}^{-}(x)\end{bmatrix} = [T]\begin{bmatrix}[U(\kappa^{+}, x-x_0)] & [0]\\ [0] & [U(\kappa^{-}, x-x_0)]\end{bmatrix}\begin{bmatrix}\boldsymbol{a}^{+}(x_0)\\ \boldsymbol{a}^{-}(x_0)\end{bmatrix} \tag{13}$$

where $[U(\kappa^{\pm}, x-x_0)] = [\delta_{mn}\exp\{i\kappa_m^{\pm}(x-x_0)\}]$ and x_0 is a standard phase position.

4.2 Electromagnetic Fields in Uniform Regions

Since the electromagnetic fields in uniform region are continuous, the fields can be expanded in terms of spatial harmonics. From Maxwell's equations (2), the differential equations can be expressed in terms of the coefficient matrix $[C]$ as follows:

TE-wave:
$$\frac{d}{dx}\begin{bmatrix}\boldsymbol{e}_y(x)\\ \boldsymbol{h}_z(x)\end{bmatrix} = i\,[C]\begin{bmatrix}\boldsymbol{e}_y(x)\\ \boldsymbol{h}_z(x)\end{bmatrix}, \quad [C] = \begin{bmatrix}[0] & -[\mu]\\ -[\varepsilon]+[s][\mu]^{-1}[s] & [0]\end{bmatrix} \tag{14}$$

TM-wave:
$$\frac{d}{dx}\begin{bmatrix}\boldsymbol{e}_z(x)\\ \boldsymbol{h}_y(x)\end{bmatrix} = i\,[C]\begin{bmatrix}\boldsymbol{e}_z(x)\\ \boldsymbol{h}_y(x)\end{bmatrix}, \quad [C] = \begin{bmatrix}[0] & [\mu]-[s][\varepsilon]^{-1}[s]\\ [\varepsilon] & [0]\end{bmatrix}. \tag{15}$$

Uniform regions I and III with no periodicity are described by the relative permittivity $\varepsilon(z) = \varepsilon$ and the relative permeability $\mu(z) = \mu$. The submatrices can be given by $[\varepsilon] = \varepsilon[1]$ and $[\mu] = \mu[1]$. Therefore, the coefficient matrix $[C]$ in Eqs. (14) and (15) consist of diagonal submatrices. The eigenvalue κ_m of 2×2 matrix $[C_m]$ corresponding to the m-th mode and the diagonalization matrix $[T]$ can be obtained analytically as $\kappa_m^{\pm} = \mp\xi_m = \mp\sqrt{\varepsilon\mu - s_m^2}$ and

TE-wave: TM-wave:

$$[T] = \begin{bmatrix}[\delta_{mn}\sqrt{\mu}] &]\delta_{mn}\sqrt{\mu}]\\ [\delta_{mn}\xi_m/\sqrt{\mu}] & -[\delta_{mn}\xi_m/\sqrt{\mu}]\end{bmatrix} \qquad [T] = \begin{bmatrix}[\delta_{mn}\xi_m/\sqrt{\varepsilon}] & [\delta_{mn}\xi_m/\sqrt{\varepsilon}]\\ -[\delta_{mn}\sqrt{\varepsilon}] & [\delta_{mn}\sqrt{\varepsilon}]\end{bmatrix}. \tag{16}$$

4.3 Boundary Conditions

The continuity of the fields $\boldsymbol{e}_\ell$ and $\boldsymbol{h}_\ell$ $(\ell = y, z)$ at each boundary requires that

For $x = d$:
$$[T_1]\begin{bmatrix}\boldsymbol{a}_1^{+}(d)\\ \boldsymbol{a}_1^{-}(d)\end{bmatrix} = [M_2][T_2]\begin{bmatrix}[U(\kappa_2^{+}, d)] & [0]\\ [0] & [1]\end{bmatrix}\begin{bmatrix}\boldsymbol{a}_2^{+}(0)\\ \boldsymbol{a}_2^{-}(d)\end{bmatrix} \tag{17}$$

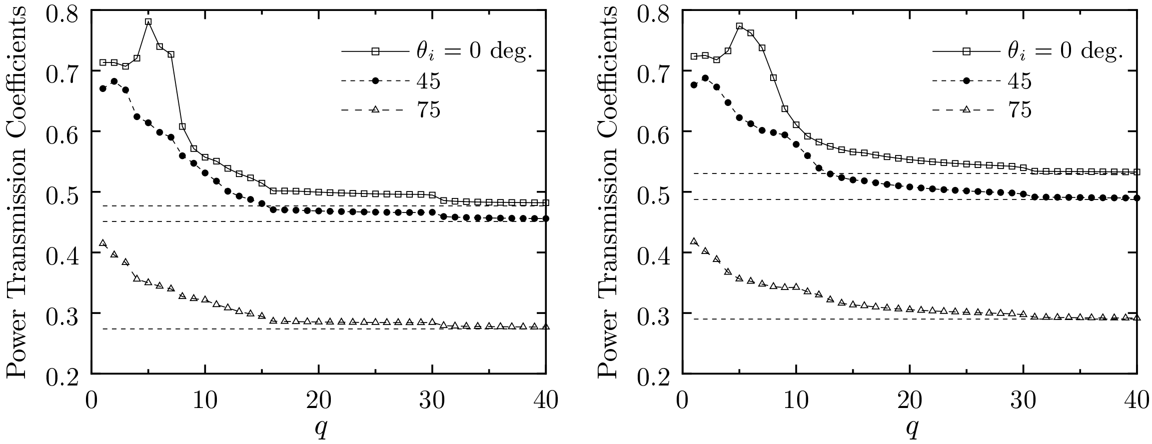
For $x = 0$:
$$[M_2][T_2]\begin{bmatrix}[1] & [0]\\ [0] & \left[U(\kappa_2^{+}, -d)\right]\end{bmatrix}\begin{bmatrix}\boldsymbol{a}_2^{+}(0)\\ \boldsymbol{a}_2^{-}(d)\end{bmatrix} = [T_3]\begin{bmatrix}\boldsymbol{a}_3^{+}(0)\\ \boldsymbol{a}_3^{-}(0)\end{bmatrix} \tag{18}$$

where $[M]$ is transformation matrix is defined by $\boldsymbol{e}_z = [1/\varepsilon]\,\boldsymbol{d}_z$ and $\boldsymbol{h}_z = [1/\mu]\,\boldsymbol{b}_z$ in the grating region. $\boldsymbol{a}_1^{-}(x_1)$ and $\boldsymbol{a}_L^{+}(x_{L-1})$ are the incident amplitude in region I and the radiation condition in region III, respectively, and are given by $\boldsymbol{a}_1^{-} = [\,0\cdots 1\cdots 0\,]^t$, $\boldsymbol{a}_L^{+} = [\,0\cdots 0\cdots 0\,]^t$. Unknowns in Eqs. (17) and (18) are $\boldsymbol{a}_1^{+}(d)$ and $\boldsymbol{a}_3^{-}(0)$. Diffraction efficiency η_m^r of the reflected wave and η_m^t of the transmitted wave are obtained.

5 Numerical Results

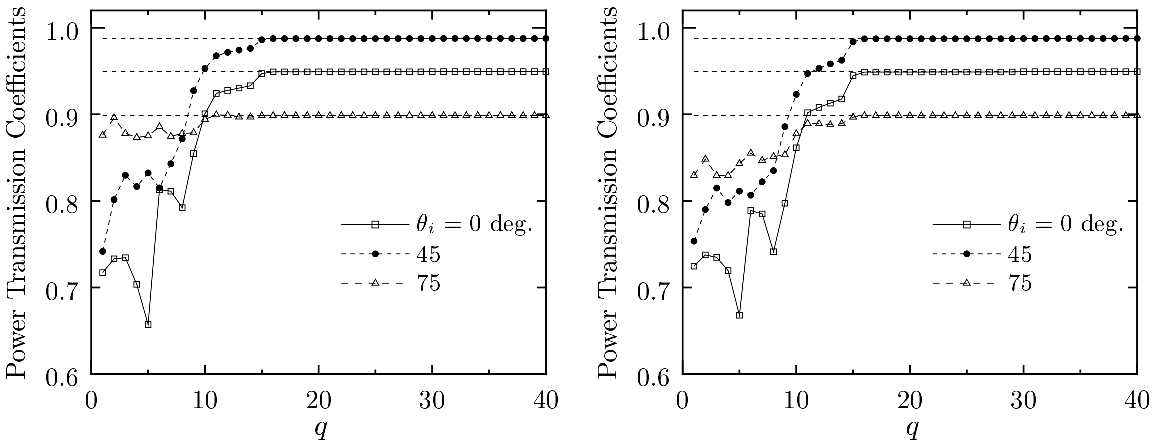
In this paper, we show that a binary thin metallic grating with subwavelength structures has anisotropic characteristics of surface resistance. It is numerically demonstrated that in the case of TE incidence, a binary grating with subwavelength structures having surface resistance of R_s is equivalent to a grating having R_s/f, and in the case of TM incidence, a binary grating is equivalent to nonperiodic structure which consists of regions I and III.

Permeability is assumed to be μ_0 throughout. Parameter values chosen are $\varepsilon_1 = 1$, $\varepsilon_3 = 2.5$, $2M+1 = 121$, $\Lambda/\lambda = 10$, $W/\Lambda = 0.5$ and $d/\lambda = 0.001$. Figures 2 and 3 show power transmission coefficients $\sum_{m=-M}^{m=M}\eta_m^t$ against the number q of subwavelength periodicity. Figures 2 and 3 are



(a) $R_s/Z_0 = 0.01$ (b) $R_s/Z_0 = 0.1$

Fig. 2 Power transmission coefficients against the number q of subwavelength periodicity for various incident angles θ_i of TE wave.



(a) $R_s/Z_0 = 0.01$ (b) $R_s/Z_0 = 0.1$

Fig. 3 Power transmission coefficients against the number q of subwavelength periodicity for various incident angles θ_i of TM wave.

TE and TM cases, respectively. In the TE case, as the number of subwavelength periodicity increases, the solutions of subwavelength gratings approach those of thin metallic gratings with no subwavelength structures drawn by the dotted lines. At $p/\lambda \leq 0.16$, the solutions agree well where R_s and θ_i change. In the TM case, as the number increases, the solutions of the subwavelength gratings approach those of nonperiodic structure drawn by the dotted lines. At $p/\lambda \leq 0.31$, the solutions agree well.

6 Conclusions

We have demonstrated numerically that a binary thin metallic grating with subwavelength structures has anisotropic characteristics of surface resistance. Therefore, it is possible that the grating is used as a polarization-selective elements in radio wavelength. In the future, we will investigate subwavelength structures that consist of two metallic gratings arranged by turns.

References

[1] M. Komatsu, H. Wakabayashi, J. Yamakita and Y. Cho, "Analysis of characteristics of subwavelength gratings by the matrix eigenvalue method", The Proc. of 2001 International Laser, Lightwave and Microwave Conference, pp. 41–43, 2001.

[2] M. Fujimoto, Y. Okuno and T. Matsuda, "Numerical evaluation of polarization-selective elements with subwavelength gratings", Tech. on Electromagnetic Theory, IEE Japan, EMT-98-35, pp. 67–72, 1998.

[3] K. Sumitani, H. Wakabayashi and H. Inai, "A study of uniform approximation of equivalent impedance sheet for parallel metallic gratings", IEEJ Trans, EIS, 123, 11, pp. 1930–1935, 2003.

[4] H. Wakabayashi, J. Yamakita, K. Matsumoto and M. Asai, "vailability of resistive boundary conditions for plane gratings", Trans. IEICE (C-I), J80-C-I, 9, pp. 387–396, 1997.