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TRAVELING ELECTROMAGNETIC WAVES ON LINEAR PERIODIC ARRAYS OF SMALL LOSSLESS PENETRABLE SPHERES[1]

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1. Introduction

Electromagnetic waves on infinite linear periodic arrays of lossless penetrable spheres can be conveniently analyzed using the source scattering-matrix framework [1] and vector spherical wave functions. Our investigation of these arrays is motivated by the theoretical demonstration that a doubly negative (DNG) medium can be formed by embedding an array of spherical particles in a background matrix [2]. In this paper we apply the spherical-wave source scattering-matrix approach to obtain an implicit transcendental equation for the propagation constants of the traveling waves that can exist on infinite linear periodic arrays of small penetrable (magnetodielectric) spheres. The spheres are assumed lossless and small enough so that only the electric and magnetic dipole vector spherical waves are significant, but no other restrictive approximations are made. The paper focuses on determining the $kd-\beta d$ diagram for the traveling waves that can be supported. Backward waves and waves with low group velocity are shown to be supportable in narrow wavebands by arrays composed of spheres with appropriately chosen permittivity and permeability. Interestingly, for certain spheres and separations it is possible to have two different traveling waves supported by the array.

2. Analyis

We consider a linear periodic array of lossless penetrable spheres with radius a, and relative permittivity and permeability $\epsilon_{\rm r}$ and $\mu_{\rm r}$. The separation between adjacent sphere centers is d. The array axis is taken to be the z axis of a cartesian coordinate with the sphere centers at $x=0, y=0, z=nd, n=0,\pm1,\pm2,\cdots$. Harmonic time dependence $\exp(-{\rm i}\omega t), \omega>0$, is assumed with the free space wave number $k=\omega/c$, c the vacuum speed of light.

We assume that a traveling wave $\exp({\rm i}\beta z)$ with real $\beta>0$, propagating in the z direction, can be supported by the array and obtain an equation that relates β to $k, d, a, \epsilon_{\rm r}$ and $\mu_{\rm r}$. In broad outline we proceed as follows. The EM field incident on each sphere of the array gives rise to a field scattered from that sphere with the scattered field related to the incident field by the Mie coefficients. The scattered field is in turn incident on all the other spheres of the array. The desired equation for β is then obtained by equating the field incident on a sphere of the array with the sum of the fields scattered from all the other spheres of the array. Accordingly the first step in the derivation of the equation for β is to obtain an expression for the field scattered from a sphere with center $x=0, y=0, z=nd$, given the field incident on that sphere. To do this we employ a local spherical polar coordinate system with origin at the sphere's center and whose corresponding cartesian axes are parallel to those of the global coordinate system. The EM field incoming on the sphere can in general be expanded as a weighted sum of divergenceless vector spherical wave functions $\mathbf{M}_{lm}^{(1)}(\mathbf{r})$ and $\mathbf{N}_{lm}^{(1)}(\mathbf{r})$, $l=1,2,\cdots,\infty, m=-l,\cdots,l$, [3],[4] whose radial dependence is given by the spherical Bessel function $j_l(kr)$. The EM field scattered from the sphere can be expressed as a weighted sum of vector spherical wave functions $\mathbf{M}_{lm}^{(2)}(\mathbf{r})$ and $\mathbf{N}_{lm}^{(2)}(\mathbf{r})$ with radial dependence given by the spherical Hankel function $h_l^{(1)}(kr)$. We assume that the spheres are small enough that only the electric and magnetic dipole functions, $l=1$, need be included, and also assume that the incident EM field at the center of the sphere is polarized with the electric field in the x direction and the magnetic field in the y direction. Then the incoming EM field can be represented as

$$\mathbf{E}_n^0(\mathbf{r}) = a_{1,-}^n \mathbf{N}_{1,-}^{(1)}(\mathbf{r}) + a_{1,+}^n \mathbf{M}_{1,+}^{(1)}(\mathbf{r}) \tag{1a}$$

$$\mathbf{H}_n^0(\mathbf{r}) = -{\rm i}Y_0 \left[a_{1,-}^n \mathbf{M}_{1,-}^{(1)}(\mathbf{r}) + a_{1,+}^n \mathbf{N}_{1,+}^{(1)}(\mathbf{r})\right] \tag{1b}$$

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and the scattered EM field as

$$\mathbf{E}_n^{sc}(\mathbf{r}) = b_{1,-}^n \mathbf{N}_{1,-}^{(2)}(\mathbf{r}) + b_{1,+}^n \mathbf{M}_{1,+}^{(2)}(\mathbf{r}) \tag{2a}$$

$$\mathbf{H}_n^{sc}(\mathbf{r}) = -\mathrm{i}Y_0 \left[b_{1,-}^n \mathbf{M}_{1,-}^{(2)}(\mathbf{r}) + b_{1,+}^n \mathbf{N}_{1,+}^{(2)}(\mathbf{r}) \right] \tag{2b}$$

where Y_0 is the free space admittance,

$$\mathbf{N}_{1,\pm}^{(i)}(\mathbf{r}) = \mathbf{N}_{1,-}^{(i)}(\mathbf{r}) \pm \mathbf{N}_{1,+}^{(i)}(\mathbf{r}), \quad \mathbf{M}_{1,\pm}^{(i)}(\mathbf{r}) = \mathbf{M}_{1,-}^{(i)}(\mathbf{r}) \pm \mathbf{M}_{1,+}^{(i)}(\mathbf{r}) \tag{3}$$

and the coefficients $b_{1,-}^n, b_{1,+}^n$ of the scattered waves are related to the coefficients $a_{1,-}^n, a_{1,+}^n$ by

$$b_{1,-}^n = b_1^{sc} a_{1,-}^n, \quad b_{1,+}^n = a_1^{sc} a_{1,+}^n \tag{4}$$

where b_1^{sc} and a_1^{sc} are the magnetic and electric dipole Mie scattering coefficients, respectively [5]. The fields $[\mathbf{E}, \mathbf{H}] = [\mathbf{N}_{1,-}^{(2)}(\mathbf{r}), -\mathrm{i}Y_0\mathbf{M}_{1,-}^{(2)}(\mathbf{r})]$ and $[\mathbf{M}_{1,+}^{(2)}(\mathbf{r}), -\mathrm{i}Y_0\mathbf{N}_{1,+}^{(2)}(\mathbf{r})]$ are proportional to the fields radiated by an infinitessimal x directed electric dipole and y directed magnetic dipole, respectively.

It is convenient to write the scattering equations (4) in the normalized form

$$b_-^n = S_- E_{nx}^0, \quad b_+^n = S_+ H_{ny}^0 / Y_0 \tag{5}$$

where b_-^n and b_+^n are the respective coefficients of $\exp(\mathrm{i}kr)/(kr)$ in the transverse components of the scattered electric fields $b_{1,-}^n \mathbf{N}_{1,-}^{(2)}(\mathbf{r})$ and $b_{1,+}^n \mathbf{M}_{1,+}^{(2)}(\mathbf{r})$, S_- and S_+ equal $-\mathrm{i}(3/2)b_1^{sc}$ and $-\mathrm{i}(3/2)a_1^{sc}$, respectively, and $E_{nx}^0 \hat{\mathbf{x}}$ and $H_{ny}^0 \hat{\mathbf{y}}$ are the electric and magnetic incoming fields at the center of the nth sphere. Making use of explicit expressions for the vector spherical wave functions [4], (5) implies that the scattered electric and magnetic fields on the array axis corresponding to an incident electric field equal to $E_{nx}^0 \hat{\mathbf{x}}$ at the nth sphere's center are

$$\mathbf{E}_n^{sc}(z) = b_-^n \frac{\mathrm{e}^{\mathrm{i}k|z-nd|}}{k|z-nd|} \left(1 + \frac{\mathrm{i}}{k|z-nd|} - \frac{1}{(k|z-nd|)^2} \right) \hat{\mathbf{x}} \tag{6a}$$

$$\mathbf{H}_n^{sc}(z) = \pm Y_0 b_-^n \frac{\mathrm{e}^{\mathrm{i}k|z-nd|}}{k|z-nd|} \left(1 + \frac{\mathrm{i}}{k|z-nd|} \right) \hat{\mathbf{y}}, z \gtrless nd \tag{6b}$$

and that the scattered fields on the array axis corresponding to an incident magnetic field equal to $H_{ny}^0 \hat{\mathbf{y}}$ at the nth sphere's center are

$$\mathbf{E}_n^{sc}(z) = \pm b_+^n \frac{\mathrm{e}^{\mathrm{i}k|z-nd|}}{k|z-nd|} \left(1 + \frac{\mathrm{i}}{k|z-nd|} \right) \hat{\mathbf{x}}, z \gtrless nd \tag{7a}$$

$$\mathbf{H}_n^{sc}(z) = Y_0 b_+^n \frac{\mathrm{e}^{\mathrm{i}k|z-nd|}}{k|z-nd|} \left(1 + \frac{\mathrm{i}}{k|z-nd|} - \frac{1}{(k|z-nd|)^2} \right) \hat{\mathbf{y}}. \tag{7b}$$

Note that even though there is no cross-coupling of the electric and magnetic dipole modes in the scattering equations (4), the two dipole modes are coupled through the scattered fields since the EM field scattered by each sphere in response to either an x-directed electric field or a y-directed magnetic field incident on the sphere has both an x-directed electric field and a y-directed magnetic field on the array axis.

Assuming that the array supports a traveling wave $\exp(\mathrm{i}\beta z)$ with a real propagation constant β to be determined, the coefficients b_-^n and b_+^n are equal to $b_-^0 \exp(\mathrm{i}\beta nd)$ and $b_+^0 \exp(\mathrm{i}\beta nd)$, respectively. Since the x-directed electric field $E_{nx}^0 \hat{\mathbf{x}}$ and y-directed magnetic field $H_{ny}^0 \hat{\mathbf{y}}$ incident on the nth sphere at the sphere's center equal the sum of the on-axis x-directed electric field and y-directed magnetic field, respectively, scattered from all the other spheres in the array, we can use (6) and (7) to obtain the pair of on-axis scattering equations

$$(kd)^3 = S_- \left(\Sigma_1 + p\Sigma_2 \right), \quad (kd)^3 = S_+ \left(\Sigma_1 + \frac{1}{p}\Sigma_2 \right). \tag{8}$$

where $p = b_+^0/b_-^0$, and closed forms are available for the two summations

$$\Sigma_1 = \sum_{m=1}^{\infty} \left(\frac{\mathrm{e}^{\mathrm{i}(k+\beta)dm} + \mathrm{e}^{-\mathrm{i}(\beta-k)dm}}{m} \right) \left((kd)^2 + \frac{\mathrm{i}kd}{m} - \frac{1}{m^2} \right) \tag{9a}$$

and

$$\Sigma_2 = kd \sum_{m=1}^{\infty} \left(\frac{\mathrm{e}^{\mathrm{i}(k+\beta)dm} - \mathrm{e}^{-\mathrm{i}(\beta-k)dm}}{m} \right) \left(kd + \frac{\mathrm{i}}{m} \right). \tag{9b}$$

We can then solve for p in each of the two equations (8) and equate the resulting expressions, thereby obtaining

$$\frac{(kd)^3 - S_-\Sigma_1}{S_-\Sigma_2} = \frac{S_+\Sigma_2}{(kd)^3 - S_+\Sigma_1} \tag{10}$$

The two sides of (10) are real quantities and hence (10) can be easily solved numerically for βd given kd.

4. RESULTS

Computer calculations have been performed to illustrate the theory of traveling waves on linear periodic arrays of small lossless spheres that we have presented. In Fig. 1 we show some $kd-\beta d$ diagrams for a linear periodic array of spheres with $\epsilon_\mathrm{r} = \mu_\mathrm{r} = 10$ for ka equal to several different resonances of the Mie scattering coefficients. The smallest value of kd in these diagrams must be greater than $2ka$, otherwise the spheres will overlap. We observe that the curves end when $\beta d = kd$, that is, when the traveling wave propagation constant β equals the free-space wave number k. The importance of having ka equal to, or close to, a resonance of the Mie coefficients is intuitively clear since a traveling wave with β greater than k cannot be excited without a high degree of scattering coupling between the spheres composing the array. In Fig. 2 we show the $kd-\beta d$ diagram for an array of spheres with $\epsilon_\mathrm{r} = 10$, $\mu_\mathrm{r} = 1$, and $ka = 1.1$. Note that for $2.337 < kd < 2.346$ there are two values of βd corresponding to each value of kd meaning that the array can support two different traveling waves.

As an example of calculations with varied frequency, in Fig. 3 we show the $kd-\beta d$ diagram for a linear array of spheres with $\epsilon_\mathrm{r} = 10$, $\mu_\mathrm{r} = 10$, and $a/d = 0.45$. (The dotted line shows $\beta d = kd$.) We note that there are four sections of the diagram, each section corresponding to a narrow window of ka in the vicinity of one of the resonances of the Mie scattering coefficients at $ka = 0.405, 0.693, 0.988$, and 1.299. Each section begins and ends when $\beta d = kd$. Each of the four sections consists of two branches, the lower branch with βd increasing from kd to π as kd increases, and the upper branch with βd decreasing from π to kd as kd continues to increase. The phase velocity of the traveling wave is positive on both branches, while the group velocity $(\mathrm{d}k/\mathrm{d}\beta)c$ of the traveling wave is positive on the lower branch and negative on the upper branch. Hence the traveling waves corresponding to the upper branches of the four sections of the $kd-\beta d$ diagram are "backward" traveling waves. The group velocity of the traveling wave in the lowest branch of Fig. 3 is very small in the interval between $ka = .3925$ and $.3928$ as shown in Fig. 4.

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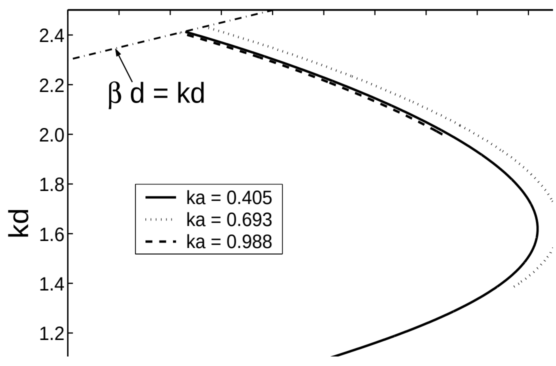


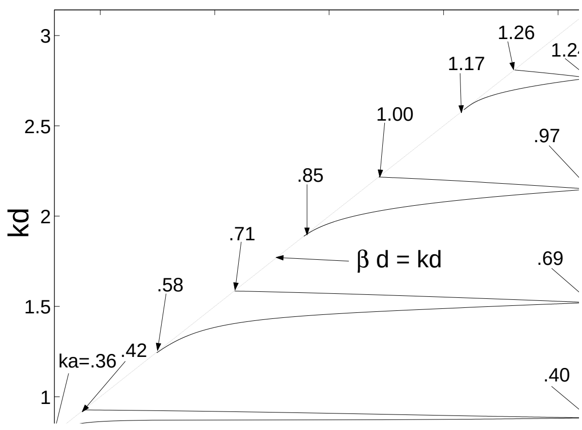
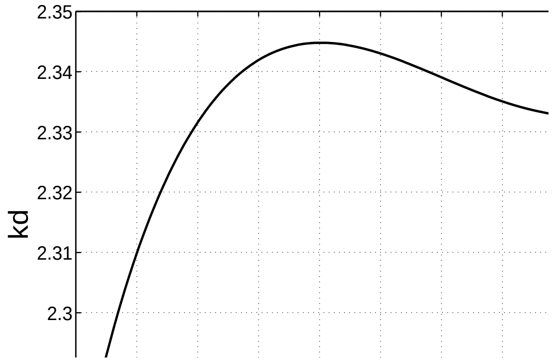


Figure 1: $kd - \beta d$ diagrams for traveling waves on array of spheres, $\epsilon_{\mathrm{r}} = 10$, $\mu_{\mathrm{r}} = 10$.

Figure 3: $kd - \beta d$ diagram for traveling wave on array of spheres, $\epsilon_{\mathrm{r}} = 10$, $\mu_{\mathrm{r}} = 10$, $a/d = 0.45$.



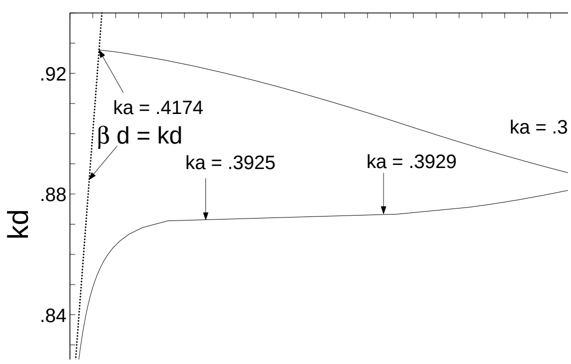


Figure 2: $kd - \beta d$ diagram for traveling wave on array of spheres, $\epsilon_{\mathrm{r}} = 10$, $\mu_{\mathrm{r}} = 1$, $ka = 1.1$.

Figure 4: Lowest branch of $kd - \beta d$ diagram of Fig. 3.