2C2-5

Capacity of MIMO System Using Polarimetric Antenna Elements

#Dao Manh Tuan [1], Park Seong-Ook [2]
[1] Electrical Engineering Department, KAIST
7117-N27-KAIST, tuandm@kaist.ac.kr
[2] Electrical Engineering Department, KAIST
N5-KAIST, sopark@ee.kaist.ac.kr

1. Introduction

Exploitation of polarization for MIMO system is a promising approach to increase multiple parallel channels, particularly for space-limited mobile and base-stations. For this purpose, the use of polarimetric antenna elements has been studied by both theoretical studies and experiment [1]-[4]. The studies, however, present different conclusion on the number of possible independent parallel channels from polarization diversity. In this paper, we study the use of polarimetric antennas under different scattering environments by using a three dimensional (3D) polarized channel modelling in which the Fisher-Bingham (FB5) distribution was used to model direction of scatterers in 3D space. We found that due to a significant correlation between electric and magnetic dipoles, only four independent channels are obtained in fully scattering condition and that in order to achieve full capacity, a certain separation between them is required. Our results can be used for optimisation of array antenna design using polarimetric antenna elements.

2. Point source

Consider characteristics of a point source, which refers to polarimetric antenna elements composed of three orthogonal electric dipoles and three orthogonal magnetic dipoles that are all collocated. We first describe the radiation pattern of each dipole. For convenience, the six co-located orthogonal dipoles in the spherical coordinate are shown in Fig. 1. We assume that all the electric dipoles are ideal dipoles with central feed point at (0, 0, 0). From the field theory, the radiated far field pattern can be derived using a constant excitation current [6]. For electric dipoles, marked as (1), (2) and (3), which are z axis oriented, x axis oriented, and y axis oriented dipoles, respectively, in Fig. 1, the $\theta-$ and $\phi-$far-field components at the propagation direction (ϕ, θ) are given by

$$\begin{bmatrix} G_\theta^{(1)} & G_\theta^{(2)} & G_\theta^{(3)} \\ G_\phi^{(1)} & G_\phi^{(2)} & G_\phi^{(3)} \end{bmatrix} = \begin{bmatrix} \sin\theta & -cos\theta cos\phi & -cos\theta sin\phi \\ 0 & sin\phi & cos\phi \end{bmatrix}. \qquad (1)$$

From the duality theorem, the far field components of the magnetic dipoles can be given by

$$\begin{bmatrix} G_\theta^{(4)} & G_\theta^{(5)} & G_\theta^{(6)} \\ G_\phi^{(4)} & G_\phi^{(5)} & G_\phi^{(6)} \end{bmatrix} = \begin{bmatrix} G_\phi^{(1)} & G_\phi^{(2)} & G_\phi^{(3)} \\ -G_\theta^{(1)} & -G_\theta^{(2)} & -G_\theta^{(3)} \end{bmatrix}. \qquad (2)$$

Observe that the electric and magnetic dipoles oriented along z axis (dipole (1) and (4)) have only vertical and horizontal component, respectively, while the others have both components.

3. Channel Modeling

In order to study performance of MIMO system using the polarimetric antennas, we extend the 2D model in [5] for 3D channel model as shown in Fig. 2.

Consider a MIMO system with N_t transmit antennas and N_r receive antennas under the non-line-of-sight (NLOS) and flat-fading environment, the channel coefficient between receiver u and transmitter s can be expressed as

$$h_{u,s}(t) = \sum_{l=1}^{L} A_l \sqrt{P_A(\Omega_l^{Tx}, \Omega_l^{Rx})}$$
$$\times \begin{bmatrix} G_s^{Tx,(v)}(\Omega_l^{Tx}) \\ G_s^{Tx,(h)}(\Omega_l^{Tx}) \end{bmatrix}^T \begin{bmatrix} \exp\left(j\Phi_l^{(v,v)}\right) & \sqrt{\varsigma_l^h}\sqrt{\chi_l}\exp\left(j\Phi_l^{(v,h)}\right) \\ \sqrt{\varsigma_l^v}\exp\left(j\Phi_l^{(h,v)}\right) & \sqrt{\chi_l}\exp\left(j\Phi_l^{(h,h)}\right) \end{bmatrix}$$
$$\times \begin{bmatrix} G_u^{Rx,(v)}(\Omega_l^{Rx}) \\ G_u^{Rx,(h)}(\Omega_l^{Rx}) \end{bmatrix} \exp\left(\vec{k}_l^{Tx} \cdot \vec{r}_s^{Tx}\right) \exp\left(\vec{k}_l^{Rx} \cdot \vec{r}_u^{Rx}\right) \exp\left(\vec{k}_l^{Rx} \cdot \vec{v}t\right) \quad (3)$$

The channel coefficient is the summation of a large number of sinusoidal wave components which can be grouped as single cluster or multiple clusters with power angular profile $P_A(\Omega_l^{Tx}, \Omega_l^{Rx})$. A cluster here is defined as a group of scatterers located within an isolated solid angle. Each cluster of scatterers has an FB5 distribution in the azimuth and elevation domain. The FB5 distribution, which has been used and proved to be suitable for 3D scattering model [7], is characterized by five parameters: 1) the concentration parameter κ; 2) the ellipticity parameter β; 3) the mean direction vector $\mathbf{\Gamma}$; 4) the major axis direction $\boldsymbol{\nu}_1$; and 5) the major axis direction $\boldsymbol{\nu}_2$. The depolarization is characterized by the parameters ς_l^v, ς_l^h, and χ_l which are the inverse of XPD_v(cross-polarization discrimination), XPD_h and CPR (co-polar ratio), respectively, and have the log-normal distribution. Other parameters are denoted in Fig. 2.

4. Simulation Result and Discussion

The point source, described in Section 2, is now used for both transmitter and receiver. This means that we have a 6×6 MIMO system. Performance of MIMO system strongly depends on power angular profile which varies with propagation environment. For convenience, we use the result of the angular of arrival (AoA) profile which was obtained in measurement campaign in outdoor environment [7]. We assume that the angular of departure (AoD) profile specified by (κ =100, $\beta = 48$). The specified setting gives the azimuth spread of 20^0 and 123^0 at transmitter and receiver, respectively. We set equal power for all wave components. By this way, we easily generate the channel realizations following the specified angular distributions. Fig. 3 shows the reconstruction of the pdf for the mixture of ten components which were obtained by generating 1000 waves (L = 1000) following the specified parameters in TABLE I in [7]. To compare the performance of MIMO system under different propagation conditions, we denote A as the propagation environment with the described power angular profile (PAP), and B as the propagation environment with uniform PAP at both transmitter and receiver, which was generated by setting the concentration parameter $\kappa = 0$.

We generated 10,000 samples of channel realizations. The entries of channel matrix **H** are normalized with respect the average of the vertical-to-vertical path gain of h_{11}. That is, $H = H/ E\{|h_{11}|^2\}$, where $E\{\cdot\}$ denotes the expectation. The mean gain value of the six eigenchannels of HH^H for different scattering conditions is plotted in Fig. 4. As can be seen, the eigenvalue of B is significantly higher than that of A. It slightly increases as the mean of XPD reduces and gets maximum as XPD = 0 and B scattering condition. This is an expected result because the condition of XPD = 0 and B provides richest multipath environment. It is found that, though there are no eigenchannels equal to zero, or rank (**H**) = 6, the 5th and 6th eigenchannels are negligible, much lower than the ideal i.i.d case. This result is different from that in [2], which shows that if full azimuth angular spread is considered, all eigenvalues tend toward the same value, indicating full six independent channels.

The eigenvalues are used to calculate capacity as follow, assuming that the transmitter has no knowledge of the channel:

$$C = \sum_{i=1}^{M} \log_2\left(1 + \lambda_i SNR/N_t\right) \quad (4)$$

where M is the number of eigenvalues (M = 6 in this case). It is noted that if known knowledge of the channel, the water filling algorithm can be used to achieve maximum capacity [8].

The result is shown in Fig. 5 in which the capacity of the submatrices 3×3 MIMO systems using orthogonal electric and magnetic dipoles, respectively, and capacity of the SISO system, are added for comparison. It is expected that behavior of capacity is similar to that of the eigenvalues, increasing with more angular spread (B) and less XPD. Most noteworthy is that the capacity of submatrices 3×3 MIMO systems approach the 3 × 3 ideal case (cyan curve), agreeing with the measurement in [1], while that of the full scattering 6 × 6 case (green curve) is much lower than its ideal case (red curve marked with upward-pointing triangle). At SNR of 20 dB, capacity of the 6 × 6 case is 23.5 bps/Hz and that of SISO is 6 bps/Hz, meaning that maximum of approximately 4 equivalent independent channels is achievable with the use of polarimetric antenna elements.

We have seen that the 3 × 3 electric and magnetic configurations are able to provide full capacity while the co-collated combination of them, the full 6 × 6 one has the limitation. Therefore, it is reasonable to speculate that there is some correlation between channel paths originating from the electric and magnetic dipoles.

Denote R as the normalized correlation matrix of the 6 × 6 MIMO system,

$$\mathrm{R} = \left[\mathrm{R}_{i,j}\right]_{i,j=\overline{1,6}} \tag{5}$$

with the component $\mathrm{R}_{i,j}$ is expressed as

$$\mathrm{R}_{i,j} = \frac{\sum_{k=1}^{6} \mathrm{E}\{h_{i,k}h_{jk}^{*}\}}{\sqrt{\left(\sum_{k=1}^{6} \mathrm{E}\left\{\left|h_{i,k}\right|^{2}\right\}\right)\left(\sum_{k=1}^{6} \mathrm{E}\left\{\left|h_{j,k}\right|^{2}\right\}\right)}} \tag{6}$$

where the $h_{i,j}$ denotes the components of the channel matrix **H**. The correlation component $\mathrm{R}_{i,j}$ represents the correlation between two paths originating the same transmitter antenna to the two receiver antennas *i* and *j*, respectively. The correlation matrix R can be decomposed as

$$\mathrm{R} = \begin{bmatrix} \mathrm{R}_1 & \mathrm{R}_2 \\ \mathrm{R}_3 & \mathrm{R}_4 \end{bmatrix} \tag{7}$$

where R_i is the sub3 × 3 correlation matrix. Considering the numbered order of antennas in the Tx array and Rx array, we can see that R_1 and R_4 show the correlation between the electric dipoles, and between the magnetic dipoles, respectively, and that R_2 and R_3 ($\mathrm{R}_2 = \mathrm{R}_3{}^{*}$) indicate the correlation between electric and magnetic dipoles.

For the case B and XPD = 0, or fully-scattered condition, we obtain the following correlation matrix:

$$\mathrm{R} = \begin{bmatrix} 1.0000 & 0.1627 & 0.2515 & 0.0160 & 0.7539 & 0.6381 \\ 0.1627 & 1.0000 & 0.5186 & 0.7817 & 0.3120 & 0.6160 \\ 0.2515 & 0.5186 & 1.0000 & 0.6785 & 0.6152 & 0.3999 \\ 0.0160 & 0.7817 & 0.6785 & 1.0000 & 0.2018 & 0.2828 \\ 0.7539 & 0.3120 & 0.6152 & 0.2018 & 1.0000 & 0.5291 \\ 0.6381 & 0.6160 & 0.3999 & 0.2828 & 0.5291 & 1.0000 \end{bmatrix} \tag{8}$$

Now by comparing (7) and (8), we see that, except of the unity in the matrix diagonal, most entries R_1 and of R_4 are less than 0.5 while those of R_2 and of R_3 are higher than 0.5, meaning that there is a significant correlation between electric and magnetic dipoles. As a result, only 4 independent channels are achievable. In order to achieve full capacity, therefore, it is necessary to separate the three electric dipoles and three magnetic dipoles. Fig. 6 shows the capacity for different separation. It is found that full capacity can be reached with the separation of one wave length. This result provides a compromise between capacity and spacing for optimisation of antenna design using polarimetric antenna elements.

Acknowledgement

This work was supported by the National Research Foundation of Korea(NRF) grant funded by the Korea government (MEST, No. 20120005270) and by the KCC(Korea Communications Commission), Korea, under the R&D program supervised by the KCA(Korea Communications Agency,KCA-2012-1297204002-120010200)

References

[1] M. R. Andrews, P. P. Mitra, and R. deCarvalho, "Tripling the capacity of wireless communications using electromagnetic polarization," Nature, vol. 409, pp. 316–318, Jan. 2001.

[2] T. Svantesson, M. A. Jensen, and J. W. Wallace, "Analysis of electromagnetic field polarizations in multiantenna systems," IEEE Trans. Wireless Commun., vol. 3, no. 2, pp. 641–646, Mar. 2004.

[3] M. Gustafsson and S. Nordebo, “Characterization of MIMO antenna using spherical vector waves,” IEEE Trans. Antennas Propagat., vol. 54, no. 9, pp. 2679–2682, Sept. 2006.
[4] T. L. Marzetta, “Fundamental limitations on the capacity of wireless links that use polarimetric antenna arrays,” in Proc. IEEE ISIT, July 2002, p. 51.
[5] H. Xu, D. Chizhik, H. Huang, and R. Valenzula, “A generalized spaced-time multiple-input multiple-ouput (MIMO) channel model,” IEEE Trans. Wireless Commun., vol. 3, no. 3, pp. 966–975, May 2004.
[6] C. A. Balanis, Antenna theory, New York: Wiley, 1997.
[7] Mammasis and R. W. Stewart, “The Fisher-Bingham spatial correlation model for multielement antenna systems,” IEEE Trans. Veh. Technol., vol. 58, no. 5, pp. 2130–2136, Jun. 2009.
[8] M. A. Khalighi, et al., “Water filling capacity of Rayleigh MIMO channels”

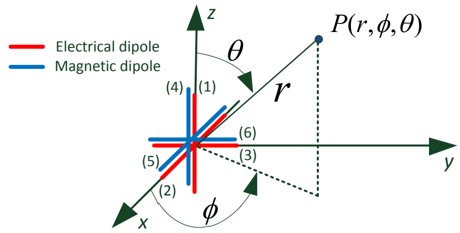

Fig. 1. Point source with orthogonal electric and magnetic dipoles

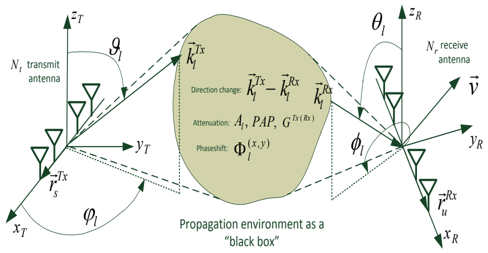

Fig. 2. Depiction of 3D channel modeling.

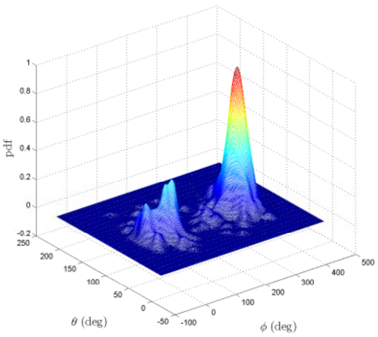

Fig. 3. Angular of arrival profile with mixture of FB5 distributions.

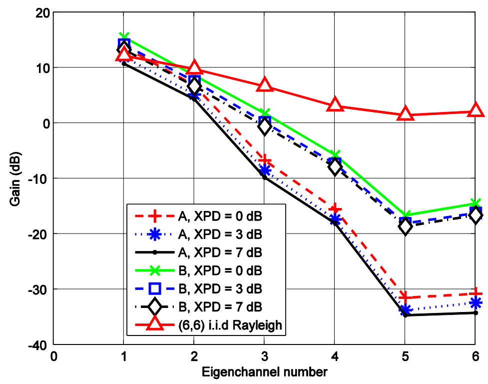

Fig. 4. Comparison of mean gain values of the 6 eigenchannels for various scattering conditions.

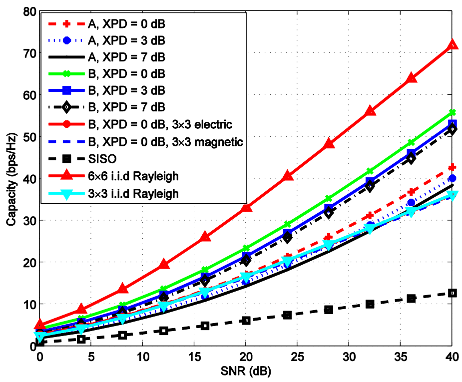

Fig. 5. Comparison of mean capacity versus SNR for various scattering conditions.

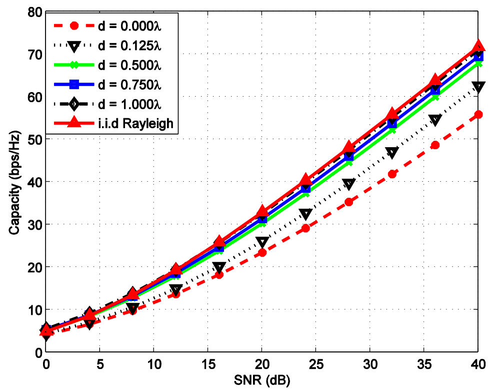

Fig. 6. Comparison of mean capacity for different separation between electric and magnetic dipoles.