

INTEGRAL EQUATION METHOD FOR ANALYZING

A CLASS OF PLANAR FEEDER LINES

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With the recent development of planar array antennas,¹ planar transmission lines of various types are being conceived as feeder lines suitable for these radiators.

This paper describes an integral equation method for analyzing transmission properties of a class of planar lines which are characterized by multiple insulating layers, planar conductor, and shielding plates. Examples of the cross-sectional view of these feeder lines are shown in Fig. 1 (a) through (h).

The fundamental propagation mode of a feeder line with a homogeneous medium is the TEM wave whose potential distribution $\phi(x,y)$ rigorously satisfies Poisson's equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\phi(x,y) = -\frac{1}{\epsilon} \rho(x,y) \quad (1)$$

and boundary conditions where ϵ is the dielectric constant and $\rho(x,y)$ is the charge density distribution.

Even with an inhomogeneous medium, the fundamental mode is considered to be nearly the TEM wave, and therefore, to satisfy Poisson's equation as far as structural dimensions of the cross-section are much smaller than the wavelength to be employed.

In this paper, Green's function which satisfies

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)G(x,y; x_0, y_0) = -\frac{1}{\epsilon} \delta(x-x_0)\delta(y-y_0) \quad (2)$$

and boundary conditions similar to those of the potential is derived to solve (1). It is noted that the conditions of a rectangular shielding conductor with zero potential as shown in Fig. 1 (a) are also imposed on Green's function in this paper.

The rectangular boundary can be used as either real one, or fictitious one by taking large dimensions. Effects of the shielding conductor on transmission characteristics should be negligible when the dimensions of the shielding conductor are very large. The derivation procedure of Green's function suitable for this structure has been discussed by the authors recently.² Once Green's function is found, the potential can be expressed by a superposition integral

$$\phi(x,y) = \int G(x,y; x_0, y_0) \rho(x_0, y_0) d\ell_0 \quad (3)$$

where the integral is defined over the inner conductor surface. Suppose, the potential on the surface of one

conductor, S_1 , be V_1 , and that on the surface of the other conductor, S_2 , be V_2 . Then the following two equations are derived from (3):

$$V_1 = \int G(x, y; x_0, y_0) \rho(x_0, y_0) dl_0$$

(x, y on S_1) (4)

and

$$V_2 = \int G(x, y; x_0, y_0) \rho(x_0, y_0) dl_0$$

(x, y on S_2) (5)

Since $\rho(x_0, y_0)$ is unknown and included in the integrals, these equations constitute the simultaneous Fredholm integral equations of the first kind. These integrals can be approximately written in a series form by deviding the surface of the inner conductor and taking an average potential in each section to represent the potential of each conductor. Then, the resulting equations are linear, simultaneous, and inhomogeneous. These equations are simple enough to be numerically solved by an electronic computer.³ Therefore, the solution of the charge density distribution is obtained in a discrete form. The total charge and the line capacitance per unit length are consequently estimated.

The characteristic impedance and the line wavelength are deduced from the line capacitance in equivalent circuits of TEM lines. When the dissipation of the energy flowing the line

due to the dielectric loss and conductor loss is very small, the attenuation constant of the line can be calculated by using a perturbation theory combined with the above solution of the charge density distribution.

The structure⁴ as shown in Fig. 1 (a) was chosen as an example case of this analytical method. Numerical results of the transmission characteristics are shown in detail.

References

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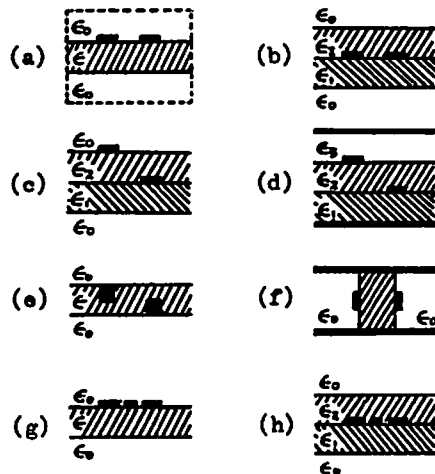


Fig. 1.