C–5–4

PARTIAL COHERENCE PROPERTIES OF OPTICAL WAVE PROPAGATION IN DISPERSIVE WAVEGUIDE

Yasumitsu Miyazaki
Department of Electrical Engineering, Nagoya University
Furo-cho, Chikusa-ku, Nagoya, Japan

1. Introduction

Low loss and small dispersion are required to optical fibers for high capacity communications[1]. Radiation waves and higher modes excited by incoherent or partially coherent sources or caused by index fluctuations and geometrical irregularities in fibers lead to large power losses and mode group delay.[2] LED and semiconductor lasers, which seem to be prominent optical sources, radiate partially coherent waves. Scattering losses due to randomnesses, which are comparable to absorption losses in recent fibers, are evaluated by using correlation functions of randomnesses.[3] Randomnesses in optical fibers also decrease coherence degrees of propagating waves. Hence, study on spatial and temporal coherence properties of optical waves in optical waveguides is indispensable for evaluation of channel capacities and transmission losses. In optical fiber systems optical signals at input and output are pulses of power envelopes that are second moments of the fields[2],[4],[5]. Transmission rates of information can be estimated by transient correlation characteristics of optical pulses concerned with interference effects of signals.

In this paper, general transmission theory on transient coherence of the optical field in dispersive fibers is shown. Based on this theory, optical pulse signals in gradient fibers with radially inhomogeneous lossy cladding that give excellent filtering properties are discussed. Scattering and pulse broadening, base band power response and signal interference are discussed.

2. Transient Coherence Function in Random Waveguides

The electromagnetic field in random optical fiber with core index $\epsilon = \epsilon^{(1)}(\mathbf{r}) + \epsilon^{(2)}(\mathbf{r})$, where $\epsilon^{(2)}$ shows index fluctuations, is shown as in eq.(1), by Green's dyadic for the unperturbed fiber[2]

$$\hat{\mathbf{E}}_{totl} = \int (\omega^2 \epsilon^{(2)} \mu \hat{\mathbf{E}}_{totl}) \cdot \mathbf{\Gamma}\, dv - \int_{S,S_1} \mathbf{n} \cdot \left\{ (\nabla \times \hat{\mathbf{E}}_{totl}) \times \mathbf{\Gamma} + \hat{\mathbf{E}}_{totl} \times (\nabla \times \mathbf{\Gamma}) \right\} dS. \quad (1)$$

If the correlation function for index fluctuations is given as $\langle \epsilon^{(2)}(\mathbf{r}) \epsilon^{(2)}(\mathbf{r}') \rangle = \langle \epsilon^{(2)}(\mathbf{r}_t) \epsilon^{(2)}(\mathbf{r}_t') \rangle \langle \epsilon^{(2)}(z) \epsilon^{(2)}(z') \rangle$, by using eigenfunction expansions of the field in the unperturbed fiber[2],

$$\hat{\mathbf{E}}(\mathbf{r}, \omega) = \sum_{\alpha} a_{\alpha}(z, \omega)\, \mathbf{\Phi}_{\alpha}(\mathbf{r}_t, \omega) e^{-j\beta_{\alpha}(\omega) z},$$

the differential equations for double Fourier components

$$\langle b_{\nu}(z, \omega_1, \omega_2) \rangle = \langle a_{\nu}(z, \omega_1) a_{\nu}^{*}(z, \omega_2) \rangle \quad e^{-j(\beta_{\nu}(\omega_1) - \beta_{\nu}^{*}(\omega_2))z}$$

that yield transient temporal coherence functions are derived,

$$\begin{aligned} &\frac{d}{dz}\langle b_{\nu}(z, \omega_1, \omega_2) \rangle + j(\beta_{\nu}(\omega_1) - \beta_{\nu}^{*}(\omega_2)) \langle b_{\nu}(z, \omega_1, \omega_2) \rangle \\ &= \sum_{\eta} \langle b_{\nu}(z, \omega_1, \omega_2) \rangle \mathcal{F}\, \epsilon^{(2)} (\beta_{\eta}^{*}(\omega_2) - \beta_{\nu}^{*}(\omega_2)) \langle \delta_{\nu\eta:\eta\nu} \rangle_{(i)} + \text{c.c}\Big\}_{1\rightleftarrows 2} \\ &+ \sum_{\mu} \langle b_{\mu}(z, \omega_1, \omega_2) \rangle \left\{ \mathcal{F}\, \epsilon^{(2)} (\beta_{\mu}^{*}(\omega_2) - \beta_{\nu}^{*}(\omega_2)) + \text{c.c} \right\}_{1\rightleftarrows 2} \langle \delta_{\nu\mu:\nu\mu} \rangle_{(ii)} \end{aligned} \quad (2)$$

where $\langle \delta_{\nu\mu:\nu\mu} \rangle_{(ii)} = -\langle \delta_{\nu\mu:\mu\nu} \rangle_{(i)}$ are transverse fluctuation factors. Hence, we have assumed that $\frac{\partial}{\partial z} a(z) \ll \frac{\partial}{\partial z} \langle \epsilon^{(2)}(z'+z)\ \epsilon^{(2)}(z')\rangle$, $\frac{\partial a}{\partial z} | z_{s_0} - z_{s_1} | \ll 1$.

The frequency is in the range of $\frac{\partial \beta_\nu}{\partial \omega} |\omega_1 - \omega_2| \ll \frac{\partial}{\partial z} \langle \epsilon^{(2)}(z'+z) \epsilon^{(2)}(z')\rangle$.
$\mathcal{F}\epsilon^{(2)}$ is the Fourier transformation of the correlation function as

$$\mathcal{F}\epsilon^{(2)}(\beta) = \int_0^\infty \langle \epsilon^{(2)}(z+z')\ \epsilon^{(2)}(z')\rangle e^{j\beta z}\, dz.$$

The coupling equations (2) that yield temporal coherence functions are generalized forms of coupled power equations.[1] Solutions of eq. (2) can be derived by substitution of $b_o = b_{o\nu}\, e^{-\gamma z}$ and $\gamma = \lambda + j(\beta_\nu(\omega_1) - \beta_\nu^*(\omega_2))$. The temporal correlation function of the field in the case of Gaussian modulation $h(t) = e^{-(\frac{2t}{T})^2}$

$$\langle E_i(z,t_1) E_i^*(z,t_2)\rangle = \sum_{\alpha,m,n} \langle |c_{\alpha i}|^2\rangle b_{o\alpha}^{(m)}\ R_{0n}\ e^{-\lambda_m(\omega_{on})z + 2\mathrm{Im}\,\beta_\alpha z}$$
$$\frac{T^2\left(2(D+\mathrm{Re}\,A_2)-F\right)\frac{1}{8}}{\left[(4D^2-F^2)+4|A_2|^2+8D\,\mathrm{Re}A_2\right]^{1/2}} e^{-\frac{(D+A_2^*)(t_1-A_1)^2+(D+A_2)(t_2-A_1^*)^2-F(t_1-A_1)(t_2-A_1^*)}{\left[4D^2-F^2+4|A_2|^2+8D\,\mathrm{Re}A_2\right]}} \tag{3}$$

where $A_1 = -j\lambda_m'(\omega_{on})z + \beta_\alpha'(\omega_{on})z$, and $2A_2 = \lambda_m''(\omega_{on})z + j\beta_\alpha''(\omega_{on})z$. Further, if the temporal correlation function of optical sources is $R_{nn'}(t_1-t_2) = R_n e^{j\omega_{on}\tau} e^{-b_n^2\tau^2} \delta_{nn'}$, $\tau = t_1 - t_2$, we have $D = F(^4/_{T^2} + b_n^2)/(2b_n^2)$, and $F = b_n^2/(^8/_{T^2}(4/T^2+2b_n^2))$. It is found from eq. (3) that, if λ_m is small compared with other $\lambda_{m'}$ and λ_m'' or α_m'' is large, $8D\mathrm{Re}A_2 \gg 4D^2-F^2+4|A_2|^2$ and the pulse width increases with $\sqrt{z}$. The coefficients $c_{\alpha i}$ are evaluated by the spatial coherence function at the incident plane $z=0$.

3. Pulses in Random Gradient Fiber with Inhomogeneous Lossy Cladding

In the optical fiber with the graded index core of parabolic profile and radially inhomogeneous lossy cladding of the following index distribution

$$\epsilon = \epsilon'\left(1-(\ell_t r)^2 + D_{IV}(\ell_t r)^4 + D_{VI}(\ell_t r)^6\right) \tag{4}$$

where the complex coefficients D_{IV}, D_{VI} are $D_{IV} = D_{IV}^{(r)} - j D_{IV}^{(i)}$ (and likewise for VI), phase constants β_{mnx}, β_{mny} and attenuation constants α_{mnx}, α_{mny} of the guided (m,n) modes are

$$\beta_{mn\,x/y} = \beta_1\Big[1 - (2n+2m+2)\,\ell_t/\beta_1 - \ell_t^2/\beta_1^2 - \frac{\ell_t^3(1-2D_{IV}^{(r)})}{\beta_1^3}\left(3\binom{m}{n}+\binom{n}{m}+2\right) + \frac{D_{IV}^{(r)}}{4}\frac{\ell_t^2}{\beta_1^2}A_{mn} + \frac{D_{VI}^{(r)}}{8\beta_1^3}\ell_t^3 B_{mn}\Big]^{1/2}$$

$$\alpha_{mn\,x/y} = \frac{D_{IV}^{(i)}}{2}\left[\frac{\ell_t^2}{\beta_1}\frac{A_{mn}}{4} + \frac{\ell_t^3}{\beta_1^2}\left(3\binom{m}{n}+\binom{n}{m}+2\right)\right] + \frac{D_{VI}^{(i)}}{16\beta_1^3}\ell_t^3 B_{mn}$$

Optimum coefficients for minimum dispersion are $D_{IV}^{(r)} = 0.8$ and $D_{VI}^{(r)} = 0$. In the case of fluctuation correlation

$$\langle\delta\epsilon_t^{(2)}\rangle\langle\delta\epsilon_z^{(2)}\rangle e^{-\frac{(x-x')^2+(y-y')^2}{\gamma_t^2}} e^{-\frac{|z-z'|}{\gamma_t}}$$

the output pulse power is given by substitution of $t_1 = t_2 = t$ in eq. (3).

Index fluctuations of short correlation length lead to scattering losses due to Rayleigh scattering, and those of long correlation length lead to pulse broadenings due to Mie scattering [6],[7]. If the maximum mode number of guided mode is $M+N+1 \cong \beta_1 \ell_t a^2/2$ in the fiber of graded core index $\epsilon_1(r)$ and cladding index $\epsilon_2 = \epsilon_1(a)$, scattering loss due to mode conversions in the case of incident fundamental mode is

$$\langle L_{loss}\rangle = \frac{\beta_1^3 \ell_t\, e^2}{\pi}\gamma_t^2\gamma_z\langle\delta\epsilon^{(2)}\rangle\left\{\frac{1}{M+N} + \ell_t\ell_z\left(\tan^{-1}\ell_t\ell_z(M+N) - \frac{\pi}{2}\right)\right\} \tag{5}$$

When $\beta_1 = 10^7 (m^{-1})$, $\gamma_t = \gamma_z = 0.2\mu m$, $\sqrt{\langle \delta\epsilon^{(2)} \rangle} = 4\times10^{-5}$, $\ell_t = 10^3 (m^{-1})$, radius of the fiber $a = 50\mu m$, scattering loss is 3.77 dB/km.

By using the results of eq.(3), numerical examples of pulse signal are shown in Fig.1, in case of $\gamma_z = 10\mu m$, $\gamma_t = 0.1\mu m$, $\beta_1 = 10^7 (m^{-1})$, $\ell_t = \sqrt{10}\times10^3 (m^{-1})$ $\langle \delta\epsilon^{(2)} \rangle = 2/\sqrt{10}\times10^{-9}$, $a = 50\mu m$, $M+N = 50$ and $D_{Vl}^{(i)} = \sqrt{10}\times10^{-2}, \sqrt{10}\times10^{-4}$. As shown in Fig.1, suitable inhomogeneous absorption cladding yields short pulse signal without loss of transmitted power.

4. Base Band Power Transfer Function

Input-output transfer functions with regard to voltages or currents in linear circuits are very useful for systematic estimation of circuits. Here, based on the transient coherence functions of eq.(3), we define power transfer function by a ratio of Fourier transformations of optical input and output intensities $I_i(t)$ and $I_o(t)$ that are second moments of the fields as

$$H(\omega) = \sum_{m,n} e^{-j\omega A_1} h_{mn} e^{-\lambda_{mn} z - 2\alpha_m z} \left[\frac{4D^2 - F^2}{(4D^2-F^2) + |2A_2|^2 + 4DR_e(2A_2)} \right]^{1/2} \cdot \left[\frac{Re(D+A_2) - F/2}{ReD - F/2} \right]^{1/2} e^{-\frac{\omega^2}{4}\left[\frac{(4D^2-F^2)+|2A_2|^2+4DRe(2A_2)}{2Re(D+A_2^*)-F} - \frac{4D^2-F^2}{2D-F} \right]} \quad (6)$$

It is found that the power transfer function depends on the input pulse form if the second dispersion can not be neglected and optical fibers have low pass filter properties. If propagation factors are given by $\lambda_{mn} + 2\alpha_{mn} \cong \alpha + C(m+n+1)^2$ and excitation factors $-\log h_{mn} = C'(m+n+1)^2$, normalized transfer function is shown as

$$H_1(\omega) = e^{j\omega \frac{\partial \beta_1}{\partial \omega}\left(1 + \frac{\ell_t^2}{2\beta_1^2}\right) z + \alpha z} H(\omega)$$
$$= \frac{C'}{\xi} e^{-j\theta} \left[\frac{1 - e^{-v(M+N)}}{1 - e^{-c'(M+N)}} \right] \quad (7)$$

where $v = \xi e^{j\theta} = Cz + C' + j\omega \frac{\partial \beta_1}{\partial \omega} \frac{\ell_t^2}{2\beta_1^2} z$. In Fig.2(a) and (b), $H_1(\omega)$ is shown for lossy cladding coefficients $D_{1v}^{(i)} \cong 10^{-6}$ or 10^{-7} corresponding to $Cz_o = 2.5\times10^{-2}$ or 2.5×10^{-3} when $\ell_t = 10^4 (m^{-1})$ and z (z_o=1km), $\sqrt{\epsilon'} = 1.5$, $\beta_1 = 10^7 (m^{-1})$. The cut off frequency of base band power transfer function decreases with the fiber length as $2\pi f_c \cong (C + C'/z) / \left(\frac{\partial \beta}{\partial \omega} \frac{\ell_t^2}{2\beta_1^2} \right)$. Although the power transfer function contain less information comparing with pulse analysis, it gives useful global data for system design.

5. Interference Effects between Pulses

By substituting t_i and $t_i - T_w$, and t_i and $t_i - T_w$, where T_w is time interval between neighbouring pulses, into t_1 and t_2 in transient coherence functions and calculating cross terms related with t_i and $t_i - T_w$, interference effects of signals can be evaluated for neighbouring pulses carried by partially coherent waves.

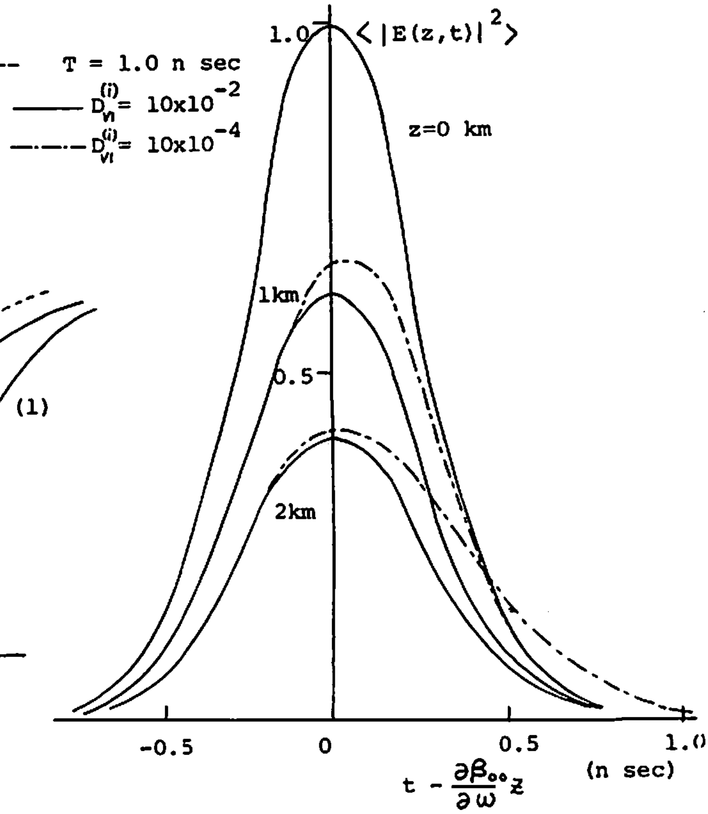
6. Conclusion

It is found that transient coherence functions of eq.(3) yield exact evaluation of pulses carried by partially coherent waves. Inhomogeneous lossy cladding gives excellent mode filtering for random fibers. Base band power transfer function shows global characteristics.

References

(1) D. Marcuse: Theory of dielectric optical waveguides (Academic Press, New York, 1974)
(2) Y. Miyazaki: AEÜ 28(1974) 60, 29(1975) 205.
(3) Y. Miyazaki: Trans. of IECE of Japan, 59-C(1976) 37.
(4) S.D.Personick: B.S.T.J., 52(1973) 1175.
(5) C.C.Timmermann and K.Petermann: AEÜ 29(1975) 235.
(6) Y.Suematsu and K.Furuya: IEEE,Trans., MTT-20(1972) 524.
(7) Y.Miyazaki: Proc. Internal. Electromag. Sym. Stanford, 5-1 (1977) 124.

Fig.1-- T = 1.0 n sec



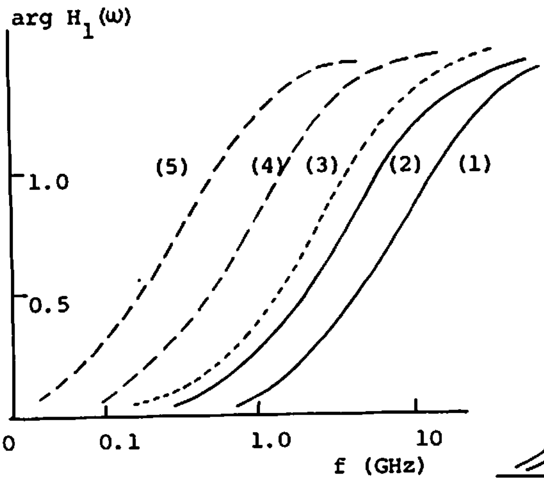


Fig.2 (a)

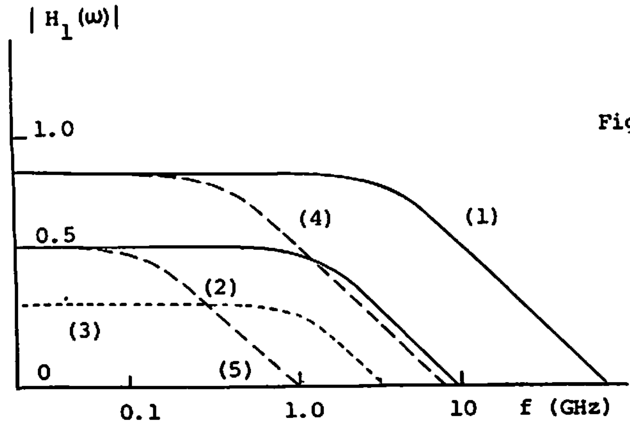


Fig2 (b)

(1) $C'=10^{-1}$, $C=2.5\times10^{-5}$, z= 1km
(2) $C'=10^{-1}$, $C=2.5\times10^{-5}$, z= 4km
(3) $C'=10^{-2}$, $C=2.5\times10^{-5}$, z= 1km
(4) $C'=10^{-2}$, $C=2.5\times10^{-6}$, z= 1km
(5) $C'=10^{-2}$, $C=2.5\times10^{-6}$, z= 4km