A NEW METHOD FOR PERMITTIVITY INVERSION OF DISCRETE LAYERED MEDIA

E. A. HASHISH M. M. ZAHRAN AND M. N. I. FAHMY
ELECTRONICS AND COMMUNICATION DEPARTMENT
FACULTY OF ENGINEERING , CAIRO UNIVERSITY , GIZA EGYPT

1. INTRODUCTION :

This paper presents a new method for inverting the permittivity of systems of discrete layered media where each individual layer is homogeneous,and characterized by its own permittivity;the method also determines the physical thicknesses of individual layers.Compared to other methods previosly introduced by different authors [1,2,3] , the new method is more accurate and yet much simpler.Besides,the new method which is based on fast Fourier transformation of surface-impedance data measured on top of layered system ,is associated with a new and elegant physical picture where the transformed data are directly related to layer parameters. One thus proceeds , in an iterative manner, determining the parameters of one layer after the other.The new method can be modified to treat the inversion of lossy dielectric media following modified spectrum of surface impedance.

2. FORMULATION OF THE PROBLEM; SPECTRUM OF SURFACE IMPEDANCE :

Consider the two-layer earth model shown in fig.(1) with horizontal boundaries;the upper plane boundary (earths surface) exists between the air and the first layer,the second plane boundary occurs at a depth L below the earths surface and define the upper boundary of a semi-infinite layer.

With normal incidence of a plane wave on the layered system,the surface impedance variation with frequency is measured on top,assuming lossless media and $\mu_r=1$

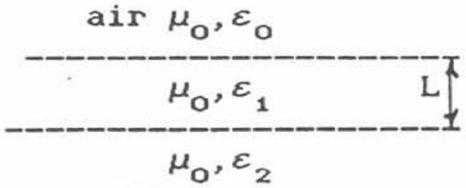

Fig.(1)

for all layers.The surface impedance Z_S can be written as [4] :

$$Z_S = E_o/H_o\big|_{z=o} = \eta_1 \frac{1 + R_1^{-2j\beta_1 L}}{1 - R_1 e^{-2j\beta_1 L}} \quad \ldots\ldots\ldots(1)$$

where $R_1=(\eta_2-\eta_1)/(\eta_2+\eta_1)$ is the reflection coefficient at the boundary between layers 1,2 looking downwards , β_1 is the phase constant of medium 1 , L is its thickness and η_1,η_2 are the intrinsic impedances of layers 1and 2 respectivly.

Thus Z_{S1} can be expanded in the form :

$$Z_{S1} = \eta_1 [\ 1 + 2 \sum_{n=1}^{\infty} R_1\ e^{-2jn\beta_1 L}\ ] \quad ....(2)$$

The term $\beta_1 L$ can be put in the form :

$$\beta_1 L = \omega\ T_o \quad ....(3)$$

where $\omega = 2\Pi f$, $T_o = \sqrt{\mu_o\ \varepsilon_1}\ L$

hence $Z_{S1}(\omega) = \eta_1\ [\ 1 + 2 \sum_{n=1}^{\infty} R_1 e^{-2jn\omega T_o}]$ ......(4)

Define the spectrum $Z_{S1}(T)$ to be the Fourier transform of $Z_{S1}(\omega)$,to get [5]:

$$Z_{S1}(T) = (1/2\Pi) \int_o^{\infty} Z_{S1}(\omega)\ e^{j\omega T}\ d\omega \quad ......(5)$$

The inverse spectrum $Z_{S1}(T)$ can be put in the form :

$$Z_{S1}(T) = \sum_{n=0}^{\infty} A_n\ \delta(\ T-2nT_o) \quad ....(6)$$

where $A_o = \eta_1$, $A_n = 2\ \eta_1 (R_1)^n$, $n = 1,2,...$

The inverse spectrum on the T-scale is shown in fig.(2).

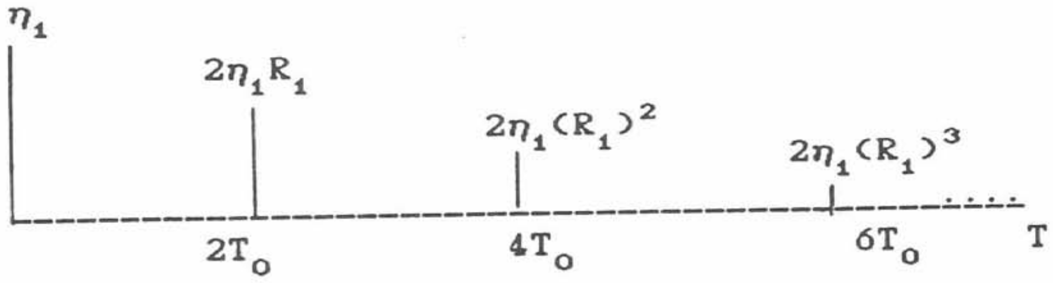

Fig.(2)

3. INTERPRETATION ALGORITHM :

From the measurement of surface impedance at different frequencies,one can draw the results to get a contineous curve representing surface impedance variation with a specified frequency range starting from frequency f_1 up to frequency f_2. The objective now is to sample the resulting curve and transform the result to T-domain using the fast Fourier transform (FFT) algorithm to get a discrete components like those shown in fig.(2). From the inverse spectrum $Z_{S1}(T)$ one can immediately deduce the following :

1-The layer depth L is given by :

$$L = (n/2)\ (1\ /\ \sqrt{\mu_o\ \varepsilon_1}\ )\ (1/BW) \quad ....(7)$$

where n is the ratio of the interval between two significant components and the sampling interval in the T-domain.

2-The intrinsic impedance of the first layer, can be deduced from the D.C. component, viz,:

$\eta_1 = \sqrt{\mu_0/\varepsilon_1}$, and hence one can get the value of ε_1.

3-Now the relation between the surface impedance at the surface of the first layer Z_{S1} and the surface impedance of the second layer Z_{S2} can be deduced from the analogy between the model and a lossless transmission line as follows [4] :

$$Z_{S2} = \eta_1 \frac{\eta_1 \tanh(j\beta_1 L) - Z_{s1}}{Z_{s1} \tanh(j\beta_1 L) - \eta_1} \quad \ldots\ldots\ldots(8)$$

Using the above relation, $Z_{S2}(\omega)$ can be obtained with η_1, β_1 and L are known.

4-In thecase of two-layer model, the significant component will be the D.C. value only which will equall to η_2 .

$\eta_2 = \sqrt{\mu_0/\varepsilon_2}$ and one can get ε_2 .

5-In the case of multi-layer model, the spectrum will contain many components beside the D.C. with interspacing between the first two components related to the depth of the subsequent layer by equation (7), while the characteristic impedance is obtained from the first component (D.C.).

6-With the parameters of that layer known we repeat steps 3-5 until we reach the last semi-infinite layer.

4. ILLUSTRATIVE EXAMPLE OF THREE-LAYER EARTH :

In this example the surface impedance data are simulated in the frequency range f=40-44 MHZ for the model parameters shown in the RHS of table (1), and the variation of surface impedance magnitude and phase are shown in figs. (3,4). The results of the inverse problem are shown in the RHS of table(1) where it is clear that the results are very close to the model parameters which indicates the high accuracy to the new method.

	model parameters	inverse problem
ε_{r1}	9	8.998
L1 m	75	75
ε_{r2}	36	36.0038
L2 m	25	25
ε_{r3}	8	7.9999

table (1)

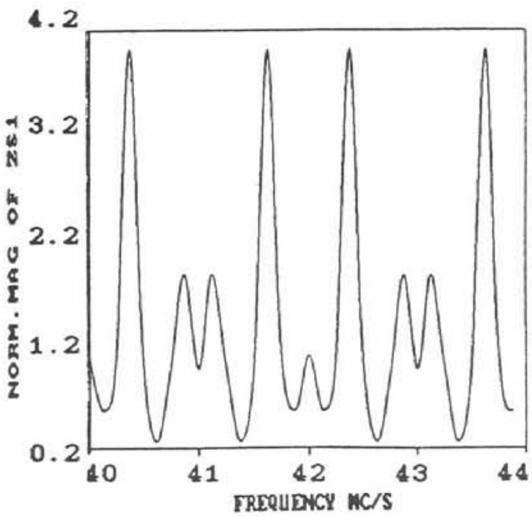

Fig.(3)

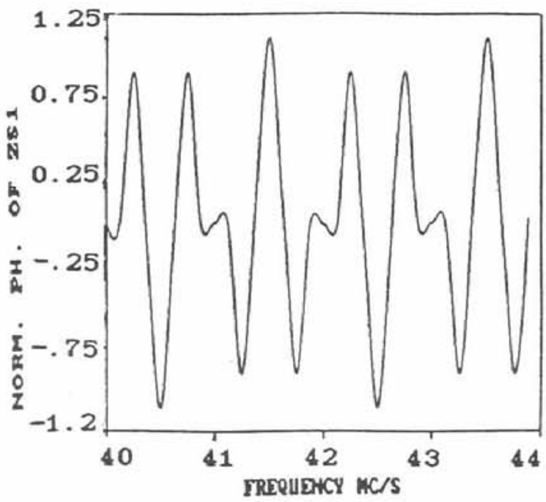

Fig.(4)

5. CONCLUSION :

The accuracy of the new method is evident and it is interesting to point out that, following modifications of surface impedance spectrum, inversion of lossy dielectric media has also been successfully achieved, with examples given elsewhere [6].

REFERENCES

1-T.M.Habashy and R.Mittra "On some inverse methods in electromagnetics " Journal of Electromagnetic Waves and Applications, vol.1 no.1 p25-28 , 1987.

2-Panayioties V.Frangos "The reconstruction of stratified dielectric profiles using successive approximations" IEEE, Transactions of Antennas and Propagation , vol.AP-35 no.11 , nov.1987.

3-Shimon Coenand Kenneth K.Mell "Inverse scattering technique applied to remote sensing of layered media " IEEE, Transactions of Antennas and Propagation, vol.AP-29 no.2 , march 1981.

4-G.K.BHAGAVAT "Electromagnetic Theory and Microwave Circuits " Academic Book Ltd.Bombay,Newdelhy 1970

5-NASIM AND T.NATARAJAN "Discrete-time Signals and Systems " Reston Publishing Company.Inc. Virginia 1983.

6-M.M.ZAHRAN "Contributions To The Forward And Inverse Problems Of Layered Media " PH.D thesis, Cairo University 1989.