CYLINDRICAL RESONATORS WITH INHOMOGENEOUS DIELECTRICS

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1 Introduction

Cylindrical cavities formed by filling a circular waveguide section with inhomogeneous dielectrics have been used as resonators in microwave circuits [1] - [4]. Several coupled dielectric rod or ring resonators can be arranged coaxially in a circular waveguide to form a bandpass filter. The resonant frequencies of the axisymmetric $TE_{01\delta}$ and $TM_{01\delta}$ modes have been calculated by using a mode-matching technique [1], [2]. For both modes, the resonant frequencies are below the cutoff frequency of the TE_{01} waveguide mode. In [3], the resonant frequency of non-axisymmetric hybrid modes is calculated by using a similar technique.

In [4], a finite integration technique (FIT) is proposed to calculate the resonant frequencies of a cavity filled with inhomogeneous dielectrics. A brief summary of mode nomenclature is also provided. Other methods like variational method, finite-difference method in the frequency domain, finite-difference time-domain (FDTD) method, and finite element method have also been used.

Mode matching method proves to be efficient for many canonical geometries. Usually, the eigenmodes in a stratified medium need to be solved first to represent the field distribution. If the dielectric ring consists of many layers or if a dielectric rod has a continuous permittivity profile, conventional mode matching method becomes tedious or impossible. For such structures, finite element method, FDTD method, and FIT method can be used at the expense of finer grids to express the fields accurately.

In this work, a generic numeric scheme is proposed to solve such problems. First, the eigenmodes in each waveguide section loaded with inhomogeneous dielectrics are obtained by solving a symmetric eigenvalue problem. Reflection matrices at the junctions of waveguide sections are defined to reduce the number of unknowns. A determinantal equation is obtained by imposing the continuity conditions at layer interfaces. The resonant frequencies are calculated by solving the determinantal equation.

2 Formulation

In Figure 1, we show the configuration of a cylindrical resonator which consists of several sections of circular waveguides loaded with inhomogeneous dielectrics. The cylinder radius is a. The permittivity in each layer is a piecewise continuous functions of ρ and is independent of ϕ and z. Axisymmetric modes exist in such a medium, and are categorized into TM (to z) and TE (to z) modes [4].

First, consider the TM modes in an infinitely long circular waveguide with an inhomogeneous dielectric profile which is uniform along the axial direction. The existing field components are E_z, E_ρ, and H_ϕ, and

$$\left(\rho\epsilon\frac{\partial}{\partial\rho}\frac{1}{\rho\epsilon}\frac{\partial}{\partial\rho}+k^2+\frac{\partial^2}{\partial z^2}\right)\rho H_\phi=0 \quad (1)$$

Next, express H_ϕ of the nth eigenmode by a set of basis functions $S_m(\rho)$ as

$$H_{n\phi}=\phi_n(\rho)e^{\pm ik_{nz}z}=\sum_{m=1}^{N} b_{nm}S_m(\rho)e^{\pm ik_{nz}z} \quad (2)$$

Choose the H_ϕ distribution in an empty circular waveguide as the basis functions. i.e., $S_m(\rho)=J_1(\xi_m\rho/a)$ with $J_0(\xi_m)=0$. Substitute (2) into (1), then take the inner product of $S_p(\rho)$ with the resulting equation to have a symmetric matrix equation. The propagation constant k_{nz} can then be solved numerically. The tangential field components in the waveguide can be expressed in terms of these eigenmodes.

The field components in each inhomogeneous layer of the resonator can be represented by one set of eigenmodes. Imposing the continuity condition on the tangential field components at the interfaces between layers, we obtain a determinantal equation to be solved for the propagation constants.

For the TE modes, the existing field components are H_z, H_ρ, and E_ϕ, and

$$\left(\rho\mu\frac{\partial}{\partial\rho}\frac{1}{\rho\mu}\frac{\partial}{\partial\rho}+k^2+\frac{\partial^2}{\partial z^2}\right)\rho E_\phi=0 \quad (3)$$

Express E_ϕ of the nth eigenmodes by a set of basis functions $\tilde{S}_m(\rho)$ as

$$E_{n\phi}=\tilde{\phi}_n(\rho)e^{\pm i\tilde{k}_{nz}z}=\sum_{m=1}^{N} \tilde{b}_{nm}\tilde{S}_m(\rho)e^{\pm i\tilde{k}_{nz}z} \quad (4)$$

Next, choose the E_ϕ distribution in an empty circular waveguide as the basis functions, i.e., $\tilde{S}_m(\rho)=J_1(\zeta_m\rho/a)$ with $J_1(\zeta_m)=0$. Substitute (4) into (3), then take the inner product of $\tilde{S}_p(\rho)$ with the resulting equation to have another symmetric matrix equation. Same procedure applied to the TM modes also applies to the TE modes.

3 Numerical Results

First, we show the resonant frequencies of the TE_{01}, TE_{02}, TM_{01}, and TM_{02} modes of a cylindrical dielectric loaded resonator as shown in Table I. The results from [4] are also shown for comparison. The convergence rate for the TE_{01} and TE_{02} modes are faster than that for the TM_{01} and TM_{02} modes in this case.

Next, we calculate the resonant frequencies of two symmetrically coupled dielectric ring resonators in a circular waveguide. The permittivity of the ring is assumed to have a parabolic profile with an extreme value ϵ_m at $\rho=(a+b)/2$. The resonant frequencies are below the cutoff frequency of the circular waveguide. Due to the structure symmetry, either an electric wall or a magnetic wall can be inserted in the middle plane to form two equivalent problems. The resulting resonant frequency are denoted by f_{oe} (electric wall) and f_{om} (magnetic wall), respectively. The resonant frequencies as a function of resonator separation are shown in Figure 2. The results with a flat profile in the ring match well with those in [1]. The resonant frequency decreases as ϵ_m increases. The difference between f_{oe} and f_{om} increases as the two resonators move closer to each other.

Finally, we calculate the resonant frequencies of two symmetrically coupled dielectric rod resonators in a circular waveguide. The permittivity of the rod is assumed to have a parabolic profile with an extreme value ϵ_m at $\rho = 0$. The resonant frequencies are below the cutoff frequency of the circular waveguide. The results with a flat profile in the rod match well with those in [2]. The resonant frequency decreases as ϵ_m increases. The difference between f_{oe} and f_{om} increases as the two resonators move closer to each other. Note that f_{oe} is lower than f_{om} in this case, and f_{oe} is higher than f_{om} for the coupled dielectric rings in the previous case.

4 Conclusions

A general numeric scheme combining the eigenvalue method and the mode matching technique has been developed to calculate the resonant frequencies of a cylindrical resonator consisting of cascaded sections of circular waveguides loaded with inhomogeneous dielectrics. The results obtained by using this approach compare favorably with those in the literatures. The resonant frequencies with continuous dielectric profiles have also been calculated, which can not be done by using conventional mode matching method.

References

[1] Y. Kobayashi and M. Minegishi, "Precise design of a bandpass filter using high-Q dielectric ring resonators," *IEEE Trans. Microwave Theory Tech.*, vol.MTT-35, pp.1156-1160, December 1987.

[2] Y. Kobayashi and M. Minegishi, "A low-loss bandpass filter using electrically coupled high-Q $TM_{01\delta}$ dielectric rod resonators," *IEEE Trans. Microwave Theory Tech.*, vol.MTT-36, pp.1727-1732, December 1988.

[3] K. A. Zaki and C. Chen, "Coupling of non-axially symmetric hybrid modes in dielectric resonators," *IEEE Trans. Microwave Theory Tech.*, vol.MTT-35, pp.1136-1142, December 1987.

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Mode	$N = 10$	$N = 15$	$N = 20$	$N = 25$	$N = 30$	[4], measured	[4], computed
TE_{01}	7.02	7.02	7.02	7.02	7.02	7.037	6.943
TE_{02}	11.39	11.39	11.39	11.39	11.39	11.391	11.316
TM_{01}	9.54	9.45	9.43	9.42	9.41	9.296	9.185
TM_{02}	11.44	11.30	11.27	11.25	11.23	11.113	10.943

Table I : Comparison of resonant frequencies (GHz) computed by using this approach with those in [4]. N is the number of basis functions, $\epsilon_r = 37.3$, $\epsilon_s = 3.78$, $u = 0.427$ mm.

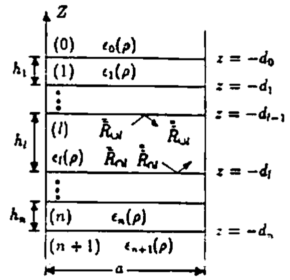

Figure 1: Geometrical configuration of a cascade of circular waveguide sections loaded with inhomogeneous dielectrics.

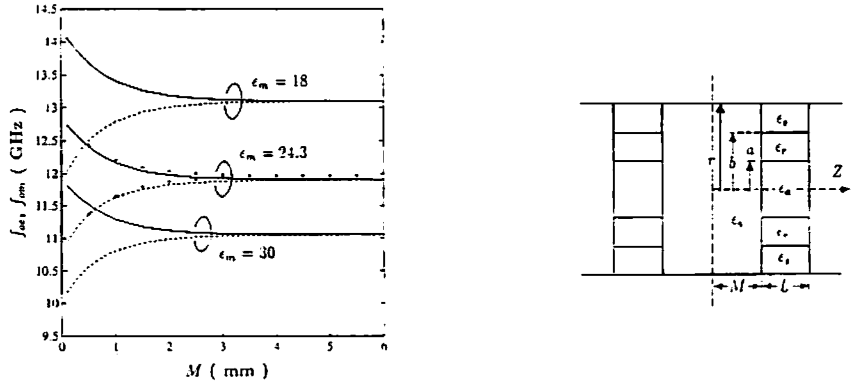

Figure 2: Resonant frequencies of the $TE_{01\delta}$ mode of two symmetrically coupled dielectric ring resonator, $\epsilon_r(\rho) = 24.3 + 4(\epsilon_m - 24.3)(\rho - a)(b - \rho)/(b - a)^2$, $\epsilon_s = 1.031$, $\epsilon_a = 1$, $b = 2.455$ mm. $a = 0.3b$, $r = 2.39b$, $(b/L)^2 = 0.4625$. ——: electric wall in the middle, - - - -: magnetic wall in the middle, * : results from [1].

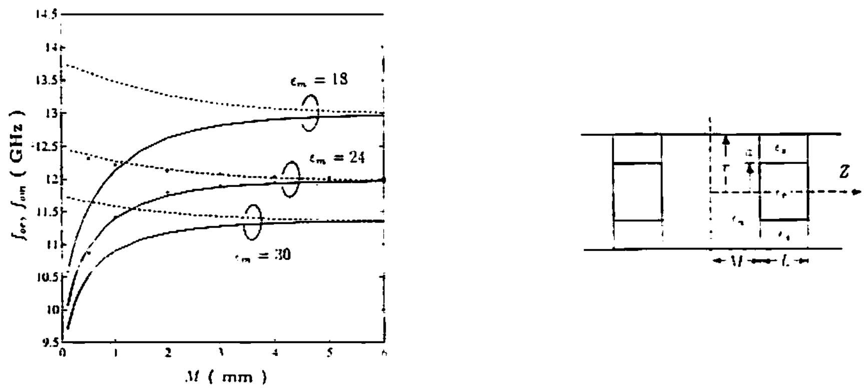

Figure 3: Resonant frequencies of the $TM_{01\delta}$ mode of two symmetrically coupled dielectric rod resonator, $\epsilon_r(\rho) = 24 + (\epsilon_m - 24)(1 - \rho^2/a^2)$, $\epsilon_s = 1.031$, $a = 3.635$ mm, $r = 5.45$ mm, $L = 1.04$ mm, ——: electric wall in the middle, - - - -: magnetic wall in the middle, *: results from [2].