# C–5–2 PROPAGATION OF PULSED BEAM WAVES IN RANDOM MEDIA

C. H. Liu and K. C. Yeh
Department of Electrical Engineering
University of Illinois at Urbana-Champaign
Urbana, Illinois 61801

Summary

The study of the effects of turbulence and random scatterers on the propagation of pulsed signals has received increasing attention in recent years [1], [2]. This interest is sustained by applications in areas such as high data rate communication systems, precise ranging and navigation systems as well as remote sensing using radars and lidars. In this paper, a newly developed technique based on the idea of temporal moments of transient signals will be applied to investigate the problem of propagation of pulsed beam waves through turbulent media. It will become clear that this approach enables us to study the pulse problem analytically even under strong turbulence conditions. Furthermore, the technique is applicable for both the continuous turbulent media and media with discrete scatterers.

Let us consider a pulsed beam wave propagating in the z-direction

$$p(z,\vec{\rho},t) = \int_{-\infty}^{+\infty} f(\omega)u(z,\vec{\rho},\omega)\ \exp\{j[\omega t-k(\omega)z]\}d\omega \tag{1}$$

where $f(\omega)$ is the temporal spectrum of the pulsed signal and $u(z,\vec{\rho},\omega)$ is the complex amplitude of a monochromatic beam wave propagating in the medium. The n-th temporal moment of the signal is defined by

$$M^{(n)}(z,\vec{\rho}) = \int_{-\infty}^{+\infty} t^n \langle p^2(z,\vec{\rho},t)\rangle dt \tag{2}$$

where the bracket < > indicates ensemble averaging.

Substituting (1) into (2), we obtain

$$M^{(n)}(z,\vec{\rho}) = 2\pi(j)^n \int_{-\infty}^{+\infty} f^*(\omega_2)\ e^{jk(\omega_2)z}\ \frac{\partial^n}{\partial\omega_1{}^n}\left[f(\omega_1)\Gamma e^{-jk(\omega_1)z}\right]_{\omega_1=\omega_2} d\omega_2 \tag{3}$$

where $\Gamma$ is the one-position, two-frequency mutual coherence function defined by $\Gamma(z,\vec{\rho},\omega_1,\omega_2) = \langle u(z,\vec{\rho},\omega_1)\ u^*(z,\vec{\rho},\omega_2)\rangle$.

In practice, the receiving aperture effects have to be considered. Therefore, we define the aperture averaged temporal moments by

$$\overline{M^{(n)}}(z,\vec{\rho}) = \iint M^{(n)}(z,\vec{\rho})G(\vec{\rho})d\vec{\rho} \tag{4}$$

where $G(\vec{\rho})$ is the aperture function of the receiving systems.

From (3) and (4) we note that the $\overline{M}^{(n)}$'s depend on the derivatives of the coherence function evaluated at $\omega_1=\omega_2$. If we express the coherence function $\Gamma$ in power of $\delta=(\omega_2-\omega_1)/\omega_2$, we have

$$\Gamma = \Gamma_0+\delta\Gamma_1+\delta^2\Gamma_2+ \ . \ . \ . \tag{5}$$

where $\Gamma_0=\Gamma|_{\delta=0}$, $\Gamma_1=\frac{\partial\Gamma}{\partial\delta}|_{\delta=0}$, etc. The moments $\overline{M}^{(n)}$ depend on $\Gamma_0$, $\Gamma_1$, $...\Gamma_n$ and not on $\Gamma_{n+1}$ or beyond. It turns out that under fairly general conditions, these coefficients $\Gamma_i$'s can be solved analytically from the transport equations governing the mutual coherence functions for both the contineous and discrete turbulent media under multiple scattering conditions [3], [4].

Once we have the solutions for the $\Gamma_i$'s it is possible to reconstruct the pulse as it propagates through the random medium by applying the technique of orthogonal polynomial expansion. Let us consider the average intensity of the received signal $I(t)=\langle p^2(t)\rangle$. We approximate this function by the expansion

$$I_N(t) = \sum_{n=0}^{N} b_n w(t)\phi_n(t) \tag{6}$$

where $w(t)$ is a non-negative weighting function and $\phi_n$'s are the polynomials given by

$$\phi_n(t) = \sum_{k=0}^{n} \alpha_{nk}t^k \qquad \alpha_{nn}\neq 0 \tag{7}$$

These polynomials satisfy the orthogonal relation

$$\int_{-\infty}^{+\infty} w(t)\phi_n(t)\phi_m(t)dt = 0 \qquad m\neq n \tag{8}$$

The criterion to determine the expansion coefficients $b_n$'s in (6) is to minimize the weighted mean square error defined by

$$\int_{-\infty}^{+\infty} \{[I(t)-I_N(t)]^2/w(t)\}dt \tag{9}$$

The coefficients are then obtained as

$$b_n = [ \sum_{k=0}^{n} \alpha_{nk} \bar{M}^{(k)} ] / \int_{-\infty}^{+\infty} w(t)\phi^2{}_n(t)dt \quad (10)$$

We note that they are related to the aperture averaged temporal moments $\bar{M}^{(n)}$.

In this approach the weighting function is chosen to approximate the original pulse intensity function closely. The corresponding polynomials can be found using the Schmidt orthogonalization procedure. Therefore, we have developed a rather general technique to find the mean pulse shape as it propagates through the turbulent medium. The technique has been used to study the case of a pulsed plane wave signal [5]. In this paper, we apply the technique to investigate pulsed beam waves propagating through contineous or discrete random media. For an incident Gaussian beam wave of the form

$$u(0,\vec{\rho},\omega) = \exp[-\rho^2/2a^2-jk(\omega)\rho^2/2F] \quad (11)$$

explicite analytic expressions for the first few temporal moments $\bar{M}^n$'s for an arbitrary pulse are derived for both the contineous turbulence and discrete scatterer cases. We then apply these results to a special case of a Gaussian pulse in time domain. For this signal, the weighting function is chosen as a two parameter Gaussian function and the associated othogonal polynomials are the Hermite polynomials. Using these polynomials the first few terms of the expansion of the average intensity of the pulse are derived to be

$$I_N(t) = B[1 - 2(\bar{M}^{(3)}/\bar{M}^{(0)})\lambda^4 t(1-2\lambda^2t^2/3) + \ldots] \cdot \exp(-\lambda^2t^2) \quad (12)$$

where

$$B = \lambda\bar{M}^{(0)}/\sqrt{\pi} \quad (13)$$

$$\lambda = \sqrt{\bar{M}^{(0)}/2\bar{M}^{(2)}} \quad (14)$$

The time origin of this expression is taken at the time of arrival of the signal at the receiver given by

$$t_a = \bar{M}^{(1)}/\bar{M}^{(0)} \quad (15)$$

These results are then applied to study several examples. It is found that at optical frequencies, perhaps except for pico second pulses, the atmospheric turbulence does not affect severely the pulse shape of the signal. However, the effects of cloud or fog on optical beams waves can be substantial. The pulse will become highly unsymmetric after passing through the medium. At microwave frequencies, scattering by rain may have very large effects on the pulses. Some numerical examples will be presented.

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References

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