Quasi-Isotropic Chiral Particles Composed of Twisted Thin-Wire Staples

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Abstract – **Effective constitutive characteristics of three-dimensional periodic array of quasi-isotropic chiral particles composed of thin-wire twisted staples are analyzed. The Lorentz' approach is applied with the method of moments to evaluate the isotropy of the resultant medium. The cancellation of electric quadrupole moments is also checked for evaluation.**

Index Terms — **Artificial medium, chiral medium, electromagnetic theory, medium design.**

1. Introduction

The artificial electromagnetic medium has been of great interest in the fields of radio waves and optics. They are fabricated from an aggregation of artificial small particles and are designed to show required macroscopic characteristics. The sizes and spacings between the particles are to be much smaller than the wavelength. The first artificial medium was a so-called Pasteur medium i.e. a reciprocal isotropic chiral medium. It was considered as the magnified scale model of the organic compounds which K. F. Lindman investigated on the dispersion characteristics of the rotatory power in the microwave frequency region in 1914 through 1922 [1]. Artificial bi-isotropic and bi-anisotropic media including simple Pasteur media as well as artificial dielectrics and artificial magnetic materials have been intensively studied [2]. This field was given great attention on the negative index of refraction of the left-hand materials (LHM) by J. B. Pendry, D. R. Smith *et al.* [3] by around 2000, and the term 'metamaterials' has taken the place of the name 'artificial media'. In this situation however, the studies on the artificial chiral media such as helical structures have been intensively reported not only on the rotatory power but on the interesting absorption characteristics based on the circular dichroism, the bio-mimetic functions, and the medical effects such as suppression of cancer proliferation etc. [4]-[5]. Most typical structures of isotropic chiral media are thought to be random or homogeneous distributions of randomly-oriented chiral particles. In the practical fabrications however, it is actually very hard to have the chiral particles randomly-oriented to avoid any kind of anisotropy. It is instead more realistic and steady to design and fabricate a three-dimensional periodic array of quasi isotropic chiral particles.

In this work, we study a quasi-isotropic chiral particle by combining the structures with 4-fold rotational symmetry whose symmetric axes are parallel to the Cartesian coordinate axes. Each rotationally-symmetric structure is composed of four perfectly-conducting thin-wire twisted staples. To evaluate the property of isotropy of the composed mediun, the effective constitutive characteristics are analyzed numerically by the quasi-static Lorentz theory combined with the method of moments for thin-wire approximation [6]. The electric and magnetic dipole moments and that of the electric quadrupoles are to be taken into account in the analyses of constitutive characteristics for anisotropic chiral media [7],[8]. For the perfect isotropic chiral media however, the electric quadrupole moments are cancelled out. In the present analysis they are also checked as one of the criteria for the isotropy.

2. Structure of the medium and particles

In this work, three-dimensional periodic array of particles along the Cartesian coordinate system in free space is assumed as the medium structure.

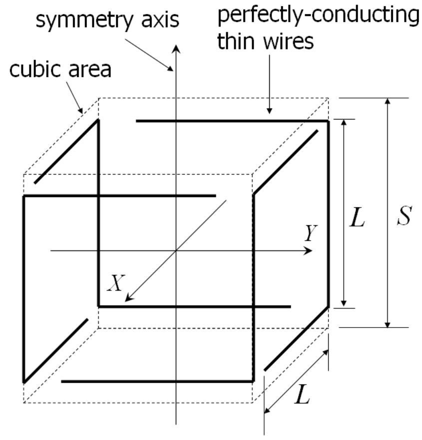

Fig. 1 Rotational symmetry on z-axis

The periodicities along the axes x, y and z are d_x, d_y and d_z respectively. Fig. 1 shows a wired structure with 4-fold rotational symmetry where perfectly-conducting thin-wire twisted staples are sequentially rotated by $\pi/2$ [rad] on a certain symmetry axis. Each staple is made by folding a thin wire with length l into three straight lines with identical length L and radius W. The chiral particle considered in this work is composed of the structures same as Fig. 1 whose symmetry axes are parallel to the coordinate axes x, y and z as shown in Fig. 2. The envelope of the particle becomes a cube with side length S.

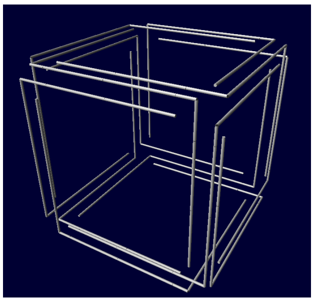

Fig. 2 Structure of a chiral particle

3. Effective Constitutive Parameters

The macroscopic constitutive relations in terms of the average electric and magnetic flux densities and fields $\boldsymbol{D}$, $\boldsymbol{B}$, $\boldsymbol{E}$ and $\boldsymbol{H}$ respectively in the medium can be expressed in $\boldsymbol{E}-\boldsymbol{H}$ form as follows [6]:

$$\begin{bmatrix} \boldsymbol{D} \\ \boldsymbol{B} \end{bmatrix} = \begin{bmatrix} \bar{\boldsymbol{\varepsilon}} & \bar{\boldsymbol{\xi}} \\ \bar{\boldsymbol{\zeta}} & \bar{\boldsymbol{\mu}} \end{bmatrix} \begin{bmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{bmatrix} \quad (1)$$

$\bar{\boldsymbol{\varepsilon}}$, $\bar{\boldsymbol{\mu}}$, $\bar{\boldsymbol{\xi}}$ and $\bar{\boldsymbol{\zeta}}$ are 3×3 matrices for the effective constitutive parameters. They are obtained by quasi-static Lorentz' approach combined with the method of moments with thin-wire approximation. Fig. 3 show numerical results of normalized effective chiral parameters as follows:

$$\kappa_{ij} = \mathrm{Im}\left(c\bar{\xi}_{ij}\right) = \mathrm{Im}\left(c\bar{\zeta}_{ij}\right) \; (i,\, j = x,y,z) \quad (2)$$

where c is the velocity of light in vacuum. The structural conditions are that $l = 25$ mm, $L = l/3$, W=0.075 mm and $d_x = d_y = d_z$ =2S. In the graphical representations, κ_D implies diagonal element $\kappa_{xx} = \kappa_{yy} = \kappa_{zz}$, and κ_{ND}, κ_Q imply average of the absolute values of non-diagonal elements and that of electric quadrupole's contributions respectively. It can be seen that κ_D draws the line of typical Cotton's effect for Pasteur medium while κ_{ND} and κ_Q stay almost zero implying the isotropy of the chirality. The particle size however is found to be $S = 0.14\lambda_R$ (λ_R is the resonant wavelength) i.e. the granularity is poot. The same things hold for the permittivity and permeability. Fig. 4 shows the results for the case that each straight wire composing a staple is compressed into five-turn wire helix where the length of each helix is 40% of that of straight line. Results show that κ_{ND} and κ_Q are not negligible any more implying the anisotropy, even though the granularity is improved to be $S = 0.11\lambda_R$.

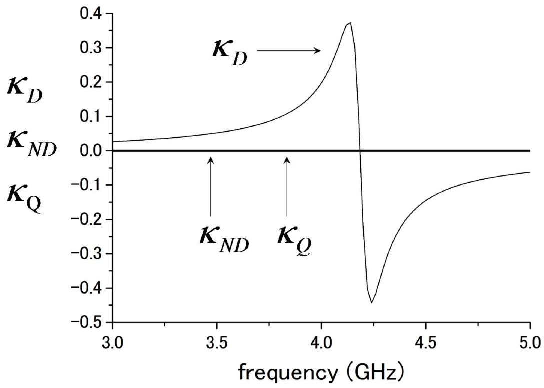

Fig.3 chiral parameters (without compression)

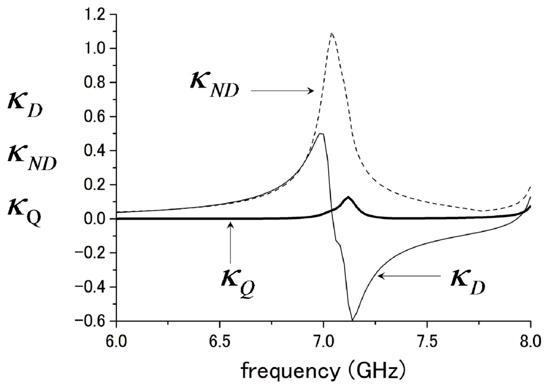

Fig.4 chiral parameters (with compression)

4. Conclusion

For the present strucrure (without compression of wires), non-diagonal elements and contribution of electric quadrupoles are found to be negligible. Granularity of the medium on the other hand is poor which is to be studied in the future.

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