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ANALYSIS OF SPACE DIVERSITY RECEPTION IN DIGITAL RADIO SYSTEMS

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Introduction: High-speed digital radio systems experience severe performance degradations due to the dispersive channel characteristics during periods of multipath fading. These degradations can be mitigated by means of frequency and time domain equalizers and/or by space diversity reception with continuous combining of the received signals. The various types of equalizers will be analyzed in a companion paper /1/, whereas in this work we treat the improvement due to diversity techniques. Traditionally, space diversity reception is applied to improve the S/N ratio in analog FM/FDM systems. In digital systems it is also important that this type of reception tends to reduce amplitude and group delay distortions in the combined signal as compared to the signal without diversity. As a consequence, an improvement results already in cases where the S/N ratio is still sufficiently high and the outage is only caused by the frequency-selective nature of the fading process. An analysis of such an improvement for the digital radio systems being presently introduced in many countries can only be made if the propagation conditions are modelled in an appropriate way. This means that besides a good approximation to measured propagation data, it should also be possible to simulate the propagation model in the laboratory by a fading simulator.

2. Propagation model

In a limited bandwidth ($\Delta f \lesssim 40$ MHz) the transfer function of the propagation channel in periods of multipath activities can be well approximated by a two-path model with fixed delay τ /2/. With space diversity reception there exists a second propagation path. Then the transfer functions $H_1(f,t)$ and $H_2(f,t)$ of the two channels can be written as

$$H_{1,2}(f,t) = a_{1,2}\left\{1 - b_{1,2}\exp\left[-j2\pi(f - f_{1,2})\tau\right]\right\} \tag{1}$$

For the statistical distribution of the parameters $b_{1,2}$ and $f_{1,2}$ it is reasonable to assume that the notch frequencies $f_{1,2}$ are uniformly distributed in the range $-1/2\tau \leq f_{1,2} \leq 1/2\tau$ and that the relative amplitudes $b_{1,2}$ of the delayed waves are Rayleigh distributed. This means that at a fixed frequency f_c in the relation

$$b_{1,2}\exp\left[-j2\pi(f_c - f_{1,2})\tau\right] = x_{1,2} + jy_{1,2} \tag{2}$$

the quantities $x_{1,2}$ and $y_{1,2}$ have a Gaussian distribution with mean value 0 and variance σ. The in-phase and quadrature components of the Gaussian processes are uncorrelated. The correlation between the two diversity branches is determined by

$$\varrho = \langle x_1x_2\rangle = \langle y_1y_2\rangle \tag{3}$$

With these assumptions it can be shown that the joint probability density of the relative amplitudes b_1 and b_2 and the notch frequencies f_1 and f_2 is given by /3/

$$p(b_1,b_2,f_1,f_2) = \frac{4\tau^2 b_1 b_2}{(1-\varrho^2)k_1^2 k_2^2} \exp\left\{ - \frac{1}{1-\varrho^2}\left[\frac{b_1^2}{k_1^2} + \frac{b_2^2}{k_2^2} - 2\varrho\frac{b_1 b_2 x}{k_1 k_2}\right]\right\} \quad (4)$$

with $k_{1,2}^2 = 2\sigma^2/a_{1,2}^2$ and $x = \cos\left[2\pi(f_1 - f_2)\tau\right]$.

The normalized received amplitudes $A_{1,2}$ at a fixed frequency $f = f_c$ are obtained from the transfer function by

$$A_{1,2} = \left|H_{1,2}(f_c,t)\right| \quad (5)$$

The marginal probability density $p(A_{1,2})$ leads to the Rice-Nakagami distribution:

$$p(r_{1,2}) = \frac{2r_{1,2}}{k_{1,2}^2} \exp\left\{ - \frac{1+r_{1,2}^2}{k_{1,2}^2}\right\} I_o\left(\frac{2r_{1,2}}{k_{1,2}^2}\right) \quad (6)$$

with $r_{1,2} = A_{1,2}/a_{1,2}$.

Certain features of this propagation model can be compared with experimental results: it was found /4/ that for line-of-sight links in the 4-GHz range the exceedence probability of the received signal without diversity at a fixed frequency follows eq. (6). The parameter k is then related to the path length d (d 50 km) by

$$k^2 = \exp(d/\Delta) - 1 \quad (7)$$

where the parameter Δ was found to be Δ = 265 km for North-West Europe.

For K = 20 log k $\leq$ - 5 dB it can be shown /5/ that the correlation of the received signals at a fixed frequency is approximately equal to

$$\varrho_A = \langle A_1 A_2 \rangle \approx \varrho \quad . \quad (8)$$

ϱ_A depends on the vertical antenna separation. Measurements of this dependence were performed by Giloi /6/ by means of a vertical antenna array with an antenna spacing of 2m. A typical result for the worst fading month in 1981 is shown in Fig.1. The analytical expression proposed by Hosoya /7/ can be well fitted to the experiments.

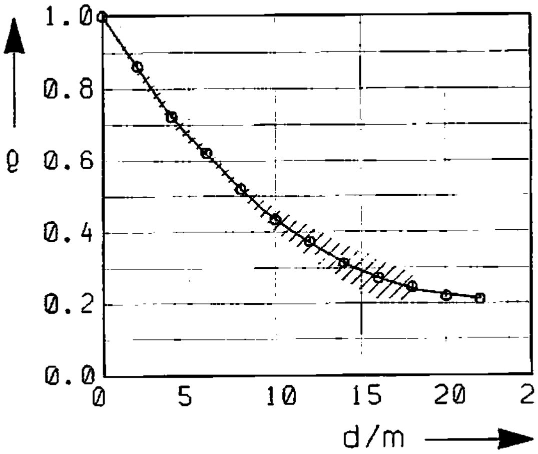

Fig. 1: Measured correlation of the received amplitudes versus vertical separation d of receiving antennas, results for the worst month in 1981 /6/, ▨ : standard deviation

3. Maximum power combining

For a further evaluation of the performance of systems with diversity reception the applied combining strategy has to be considered. In most cases the two diversity signals are combined such that the phase of the diversity signal is adjusted to maximize the total received power. This phase results from the solution of /8/

$$\int_B |H_1(f)|\,|H_2(f)|\,\sin(\psi(f) + \phi)\,df = 0 \tag{9}$$

with $\psi(f) = \arg(H_2) - \arg(H_1)$, the integration being performed over the channel bandwidth B. The total transfer function including the idealized maximum power combiner can now be expressed as

$$H_c = \frac{1}{\sqrt{2}}\,(H_1 + H_2 \exp(j\phi)) \tag{10}$$

4. Outage calculations for 16-QAM 140-Mbit/s systems

The effects of the propagation channel described by the transfer function of eq.(10) can be evaluated by calculating the "signature" /1/ for different notch frequencies f_1 and f_2. A result of such calculations is shown in Fig. 2 where the parameter b_2 is fixed. For the calculations it was assumed that remodulation is applied to recover the carrier and that the timing recovery circuit possesses a non-linear element. The outage probability P_{out} is now simply given by the probability that the channel conditions (parameters b_1, b_2, f_1 and f_2) result in a point below the signature:

$$P_{out} = \iint_{-1/2\tau}^{1/2\tau} \int_{b_2} \int_{b_1^m}^{b_1^n} p(b_1, b_2, f_1, f_2)\, db_1 db_2 df_1 df_2 \tag{11}$$

This expression was evaluated by inserting the probability density of eq. (4) as well as the calculated signature points $b_1^m(b_2, f_1, f_2)(< 1)$ for minimum phase and $b_1^n(b_2, f_1, f_2)(> 1)$ for non-minimum phase fading. In Fig. 3 the outage probability is shown versus the correlation between the diversity branches. It can be observed that maximum power combining results in a remarkable improvement of the outage probability which is due to the reduction of the distortions caused by the frequency-selective nature of the fading.

5. Fading simulation

The propagation model discussed here was realized in hardware for testing the 16-QAM 140-Mbit/s radio systems operating in the 4-GHz range. As shown in the block diagram in Fig. 4, the ring modulators in the inphase and quadrature channels are driven by PRN generators producing a Gaussian distributed output voltage. The correlation between the generators is adjustable. After adding inphase and quadrature channels, a Rayleigh distributed rf signal is generated on the delayed path. Tests performed with radio-relay systems equipped with maximum-power combiners showed an even larger improvement due to the diversity configuration than that expected from the analysis presented here. This can partly be explained by the fact that maximum-power combining improves also the S/N ratio neglected in our analysis.

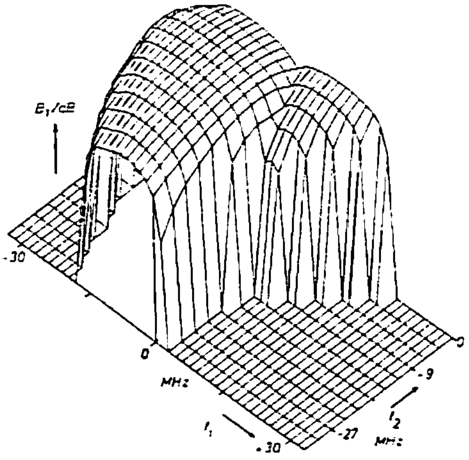

Fig. 2: Calculated signatures for a 16-QAM 140-Mbit/s radio system with space diversity, b_2=0.5, τ = 6.3 ns, roll-off: 0.5.

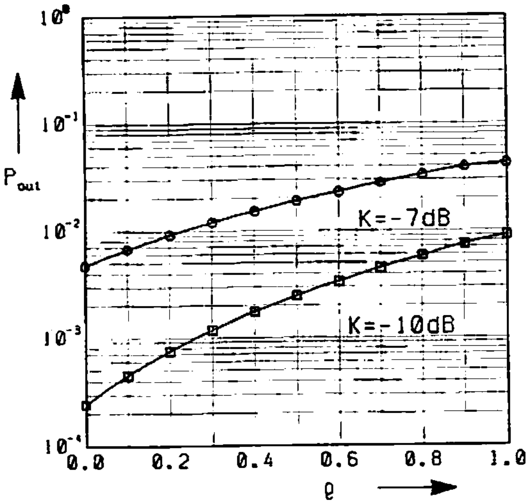

Fig. 3: Calculated outage probability P_{out} in periods of multipath fading for a 16-QAM 140-Mbit/s radio system versus correlation ρ.

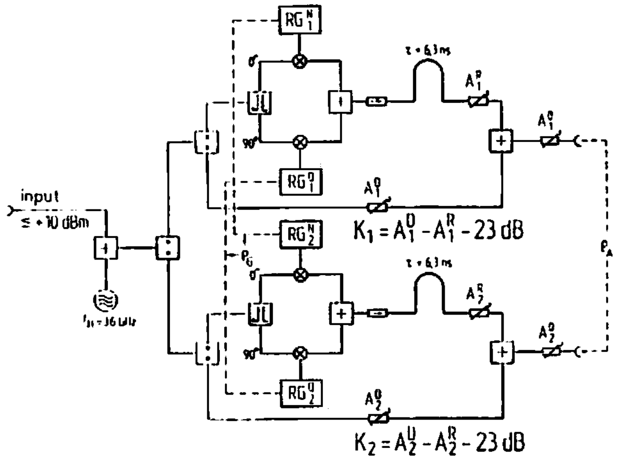

Fig. 4: Block diagram of the fading simulator.

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