Studies about the Input Admittance of Wire/Plate Attachment

Qin Jianjun, Cao Xiangyu, Gao Jun, and Yin Zhaowei
The Telecommunication engineering institute, Air Force Engineering University of CPLA
125# Mail Box, No.1 Feng Hao ROAD Xi'an, Shaanxi, 710077, P.R.CHINA
qinjianjuncz@163.com, hk_cxy@hotmail.com

1. Introduction

A wire attached to a plate is a popular problem for the researchers. When analyzing the electromagnetic characters of the metallic objects using MOM, the singularities can be met in the calculating of the self-impedance and the mutual-impedance. Because the self-impedances and the mutual-impedances of the adjoin elements are the major in the impedance matrix, the calculating accuracy of those is very important, or else those will worsen the whole calculating accuracy. The singular integrals in the self-impedance expressions of the plate and the wire have been treated by several references [1][2]. The reference [3] studied the singular integrals in the mutual-impedance expression between the wires and conductors, on those the surface current is expanded using RWG basis function.

In this paper, the singular integral in the mutual-impedance expression between a wire and a conductor is studied by the analytical method. The surface current is expanded with rooftop basis functions.

2. Problem descriptions

The structure of a wire attached to a plate is shown in figure 1.

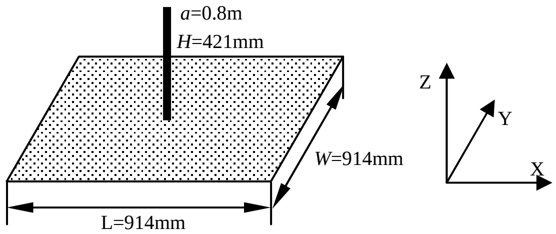

Fig1. The structure of a wire attached to a plate

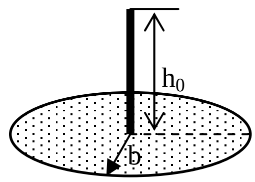

Fig2. Attachment mode

By choosing rooftop functions as basis function and using Galerkin method, the mutual-impedance between the wire and the plate can be expressed as [4][5]:

$$Z_{ml}^{pw,x} = \frac{jk\eta}{4\pi}\frac{1}{hy}\int_{x_m-hx}^{x_m+hx}\int_{y_m-hy/2}^{y_m+hy/2}\int_{l} dl \begin{bmatrix} T(l)T(x-x_m)(\vec{u}_l\cdot\vec{u}_x) \\ -\frac{1}{k^2}T'(l)T'(x-x_m) \end{bmatrix}\frac{e^{-jkR}}{R} \qquad (1)$$

where,

$$R=\sqrt{(x-x')^2+(y-y')^2+z'^2},\ (\vec{u}_l\cdot\vec{u}_x)=0,\ (\vec{u}_l\cdot\vec{u}_y)=0,\ k=2\pi/\lambda,\ \eta=120\pi$$

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$$T_{xn}(x')=\begin{cases}\dfrac{x'-x'_{n-1}}{h_x}, & x'_{n-1}\le x'\le x'_n\\ \dfrac{x'_{n+1}-x'}{h_x}, & x'_n\le x'\le x'_{n+1}\\ 0, & elsewhere\end{cases},\quad T(l)=\begin{cases}\dfrac{l-l_{n-1}}{\Delta l}, & l_{n-1}\le l\le l_n\\ \dfrac{l_{n+1}-l}{\Delta l}, & l_n\le l\le l_{n+1}\\ 0, & elsewhere\end{cases}$$

h_x : the segment length of the plate in x direction, $\vec{u}_x$ the unit vector in x direction.

h_y : the segment length of the plate in y direction, $\vec{u}_y$ the unit vector in x direction.

Δl : the segment length of the wire, $\vec{u}_l$ the unit vector along the wire.

It is well known that as $R\to 0$ the kernel of $z_{ml}^{pw,x}$ in formula (1) changes quickly. If using numerical integral, it will induce large error, also it is time-consuming.

3. Extracting the singularity

In this part, the procedure used to treat the singularities is to add and subtract a singular term from the integrals which can be integrated analytically and also renders the integrals well behaved so that standard numerical integration methods can be applied. By using this method formula (1) can be expressed as:

$$\begin{aligned}Z_{ml}^{pw,x}=&-\frac{j\eta}{4\pi k}\frac{1}{hy}\int_l T'(l)dl\int_{y_m-hy/2}^{y_m+hy/2}dy\int_{x_m-hx}^{x_m+hx}T'(x-x_m)\frac{e^{-jkR}-1}{R}dx\\&-\frac{j\eta}{4\pi k}\frac{1}{hy}\int_l T'(l)dl\int_{y_m-hy/2}^{y_m+hy/2}dy\int_{x_m-hx}^{x_m+hx}T'(x-x_m)\frac{1}{R}dx\end{aligned}\tag{2}$$

Because of $\lim_{R\to 0}[(e^{-jkR}-1)/R]=-jk$, the first part of formula (2) can be integrated by standard numerical integration methods easily. But the second part also contains singularity.

Fortunately, the second part of formula (2) can be integrated analytically:

Set

$$u=x-x',\quad v=y-y'\Rightarrow\quad R=\sqrt{u^2+v^2+l^2},\quad dxdy=dudv$$

So

$$\int_{y_m-hy/2}^{y_m+hy/2}dy\int_{x_m-hx}^{x_m+hx}T'(x-x_m)\frac{1}{R}dx=T'(x-x_m)\int_{v_1}^{v_2}dv\int_{u_1}^{u_2}\frac{1}{R(u,v)}du\tag{3}$$

The double integral $\int_{v_1}^{v_2}dv\int_{u_1}^{u_2}1/R(u,v)du$ in the formula (3) can be integrated analytically [1]. As a result, the second part of formula (2) can be expressed as:

$$\frac{j\eta}{4\pi k}\frac{1}{hy}\int_{l} dl T'(l) T'(x) \left[\begin{array}{l} u\ln(v+R)+v\ln(u+R)-u \\ +l\left(\arctan(u/l)-\arctan(uv/(lR))\right) \end{array} \right]_{u_1,v_1}^{u_2,v_2} \quad (4)$$

Where

$$T'_{xm}(x'-x_m)=\begin{cases} \dfrac{1}{h_x}, & x_{m-1}\le x'\le x_m \\ \dfrac{-1}{h_x}, & x_m\le x'\le x_{m+1} \\ 0, & elsewhere \end{cases}$$

The limits of integration are:

$$x_m - x' - h_x \le u \le x \le x_m - x' + h_x,\ \ y_m - y' - h_y/2 \le v \le y_m - y' + h_y/2 .$$

Studying the property of (4), it can be easily obtained that it has not contained singularities. In sequence, the triple integral is also simplified into a single integral, and it can be integrated by standard numerical integration method quickly.

4. Ascertaining the size of the wire attachment mode

It is well known that the input susceptance will trend to zero at the resonance frequency. Also the input admittance will be changed with the disk size of the attachment mode as shown in Fig.3. In this part, the radii *b* will be ascertained by the curve of input admittance versus disk radii at the resonance frequency. It is easily attained from the curve the input susceptance trends to zero when $b/\lambda \approx 0.3$, i.e. $b \approx 0.3\lambda$.

5. Numerical Results

Using the methods brought in this paper, the popular sample as shown in Fig.1 is re-calculated, and the results are shown in figure 4. The plate dimensions is $914mm \times 914mm$, the wire is $421mm$ long, and the wire radii is $0.8mm$.

It can be obtained clearly that the calculated results of this paper are agreed well with the measured results.

6. Conclusion

When the MOM is used to analyze wire attachment, the MOM matrix elements integral involve singular integrals worsen the whole accuracy. In order to increase the accuracy, the singular integrand is evaluated by analytical evaluation. As shown in Fig.3, the calculated results are agreed well with those of the reference and the measured respectively.

To ascertain the size of attachment mode is also very important. The method put forward this paper is very efficient.

The analytical evaluation express and the method to ascertain the size of attachment radii can be applied to analyze and design the antennas on the plane, vehicle, and ship.

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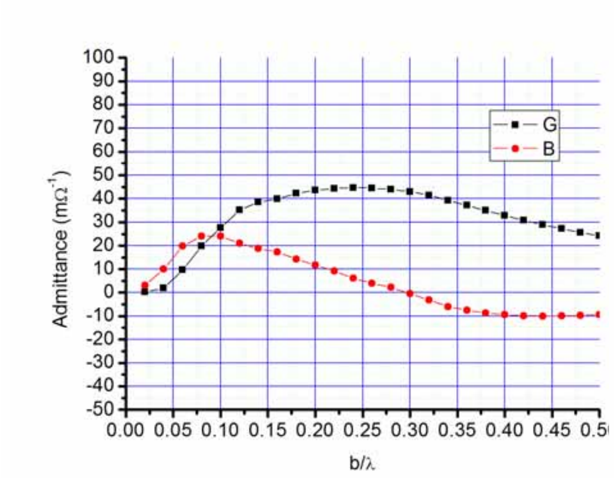

Fig3. The relationship between input admittance and attachment disk radii b at *f*=170MHz

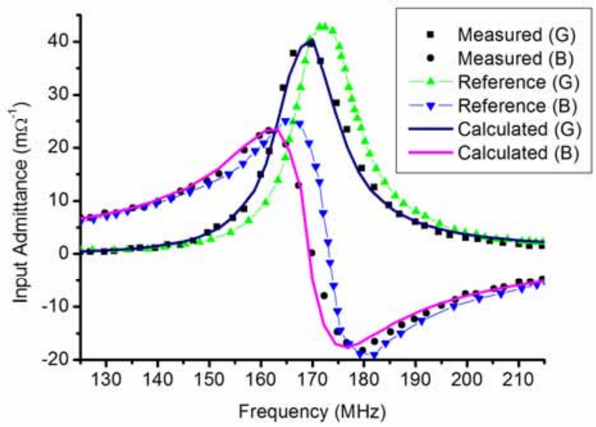

Fig4. Results of analytical evaluation, measured and reference [5]