Design of Metamaterial Lens for Antenna Array

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Abstract – **In this paper, a metamaterial lens is proposed to enable a linear array to possess the radiation properties of circular array in free space. The Quasi-Conformal Transformation Optics (QCTO) methodology is used to derived the required refractive index distribution of the lens. The radiation characteristics of linear array with the metamaterial lens are obtained by full-wave simulation. The design approach is expected to apply to another conformal array.**

Index Terms — **Metamaterial lens, Antenna array, Quasi-Conformal Transformation Optics （QCTO）, refractive index distribution.**

1. Introduction

Lenses have been received considerable attention for microwave and millimeter wave applications, urged by advances in antennas and metamaterials. Metamaterials are artificial media that derive their extraordinary properties from the arrangement of their unit cells rather than the composition of the base materials and have been widely used to control the propagation of electromagnetic waves [1]. In particular, metamaterials with tailored distribution of refractive index have been explored to design various types of lenses such as negative index metamaterial (NIM) lenses, zero-index metamaterial (ZIM) lenses, and gradient refractive index (GRIN) lenses [2] to realize invisibility cloaks, high-gain broadband antennas and beam shifters.

Circular antenna arrays are very popular in phase array since they have axisymmetric radiation properties. However, in satellites and airborne systems, not all carriers have circular profiles. Therefore, in order to keep the radiation patterns of arbitrary-shape conformal arrays in accordance with those of circular antenna arrays, several techniques including projection techniques, iterative least square techniques and global optimization techniques have been proposed to address this problem. More recently, metamaterial lens as an alternative approach to build a bridge between arbitrary-shape conformal arrays and circular arrays.

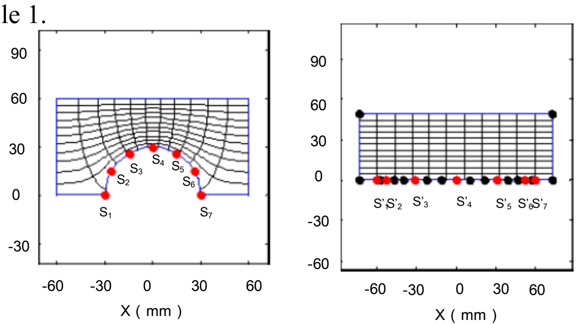
In this work, the Schwarz-Christoffel (SC) mapping as a QCTO [3] optimization algorithm is employed to transform a circular array in the virtual region into a linear array in the physical region. A GRIN metamaterial lens covering the linear array is designed to provide the array with the radiation features of the circular array during scanning.

2. QCTO and Refractive Index Distribution

In QCTO, the geometrical transformation between the virtual and physical regions are converted into the transformation between their physical properties such as permittivities ($\overline{\overline{\varepsilon}}$, $\overline{\overline{\varepsilon'}}$) and permeabilities ($\overline{\overline{\mu}}$, $\overline{\overline{\mu'}}$) by the transformation rules $\overline{\overline{\varepsilon'}} = \overline{\overline{\Lambda}}\,\overline{\overline{\varepsilon}}\,\overline{\overline{\Lambda}}^T / \det(\overline{\overline{\Lambda}})$ and $\overline{\overline{\mu'}} = \overline{\overline{\Lambda}}\,\overline{\overline{\mu}}\,\overline{\overline{\Lambda}}^T / \det(\overline{\overline{\Lambda}})$ [4],where $\overline{\overline{\Lambda}}$ is the Jacobian matrix of the transformation. According the boundary conditions of the two regions, the medium parameters of metamaterial are numerically solved.

To reduce the residual error in QCTO optimization process, the SC mapping is adopted to compute the conformal transformation.

Fig. 1 shows the modified SC mapping between a semi-circular array in the virtual region in Fig. 1(a) and its corresponding linear array in the physical region in Fig. 1(b). The semi-circular array of $N = 7$ elements denoted by the red dots S_i (i = 1,2,…,7) is transformed to the corresponding elements denoted by the red dots S'_i (i = 1,2,…,7). It's obviously that the elements of the semi-circular array are arranged axisymmetrically and equidistantly while the elements of transformed linear array are not arranged equidistantly as the regular linear array. Considering the operating frequency is 10GHz, the radius of the semi-circular array is set as $1\lambda_0$ or 30mm. The geometries of virtual region covering the semicircular array are set as $4\lambda_0 \times 2\lambda_0$ or 120mm × 60mm while the geometries of the physical region covering the linear array are $4.8\lambda_0 \times 1.6\lambda_0$ or 144mm × 48mm. The x-coordinate values of the elements of the linear array are listed in the Table 1.



(a) Semi-circular array (the virtual region)
(b) linear array (the physical region)

Fig. 1.The modified SC map ping with mesh lines.

TABLE I
The Distance and Phase of Elements of Linear Array

Elements number	x-coordinate (mm)	Phase （degree）
S'_1	-60	0
S'_2	-52.5	180
S'_3	-31	311.8
S'_4	0	360
S'_5	31	311.8
S'_6	52.5	180
S'_7	60	0

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Applying the SC mapping, the refractive index of the physical region or metamaterial lens can be given [5] by

$$n' = n/|dw/dz| \tag{1}$$

where w and z are coordinates of the w domain (the virtual region) and the z plane (the physical region) in complex analysis. n and n' denotes the refractive indexes of the virtual and physical region. With the semi-circular array among free space, the refractive index of virtual region equals to 1. Then n' is computed numerically based on (1).

However, it should to be noted that some values of refractive index on the upper boundary of the lens are far less than 1. This mismatch will lead to a strong reflection when the electromagnetic wave propagates from the lens to the free space. Therefore, multiple matching layers are coated on the top of the lens. For the fabrication of metamaterial lens in the future, the refractive index distribution is dispersed in the interval of 6mm shown as in Fig. 2.

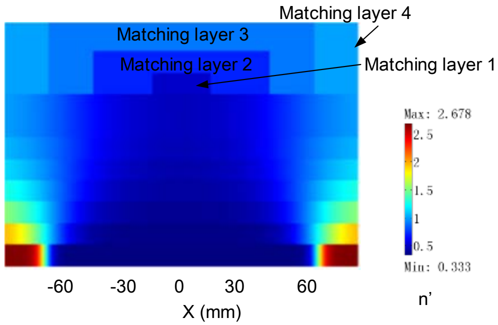
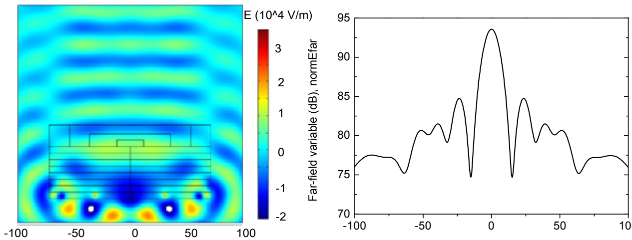


Fig.2. The dispersed refractive index distribution of the metamaterial lens with multiple matching layers.

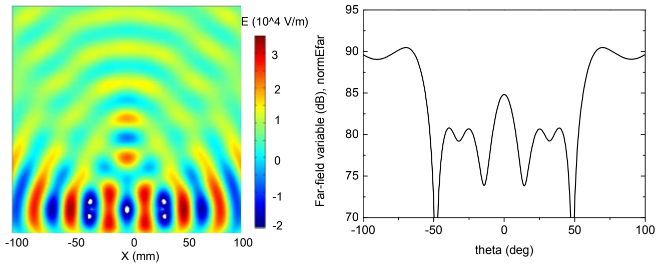
3. Simulation and Analysis

To explore the radiation characteristics of the linear array with the metamaterial lens, the electric field distributions and radiation patterns of the array are simulated for 10GHz with by full-wave software COMSOL Multiphysics in Fig. 3(a) and (b) compared with the same arranged linear array without lens in Fig. 3(c) and (d). In this case, the feed phases of antenna elements listed in Table 1 are same as the circular array with the beam pointing to 0°. The simulation results show that the linear array with the metamaterial lens radiates a high directive beam in the direction of 0° while there is no directive radiation in the same direction without the lens.

Fig. 4(a) and (b) show the electric field distribution and radiation pattern of the linear array with the lens when beam scanning to the direction of 20°. It indicates that the metamaterial lens also endows the linear array with the capability of beam scanning as the semi-circular array.

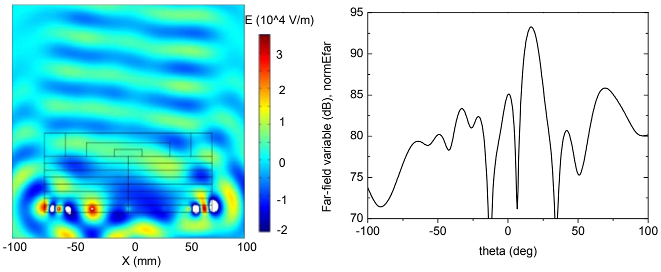


(a) Electric filed (b) Radiation pattern



(a) Electric filed (b) Radiation pattern

Fig.3. The electric field distributions and radiation patterns of linear array with the metamaterial lens and without the metamaterial lens (the beam direction of 0°).



(a) Electric filed (b) Radiation pattern

Fig.4. The electric field distributions and radiation patterns of linear array with the metamaterial lens (the beam direction of 20°).

4. Conclusion

A metamaterial lens based on the SC mapping is presented in this paper. The required refractive index is numerically derived for the conformal transformation. The simulation results of electric field distributions and radiation characteristics of the linear array verify the transformation function of the lens.

Acknowledgment

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