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FIELD DESCRIPTION WITH COMPLEX VARIABLES AND ITS APPLICATION TO DISCONTINUITIES ON ANTENNA PROBLEMS

M.Hashimoto and K.Fujisawa
Department of Electrical Engineering
Osaka University
Toyonaka, Osaka, Japan

Introduction

In solving the field diffracted by an aperture on a screen or the radiation field from an open-end of waveguide, some field-matching technique will be employed at the aperture or the open-end; the field functions defined in both sides must be continued to one another. The most common techniques for field-matching are the variational methods, the point-matching method, and the methods of integral equations. When the aperture structures are complicated, one will remark the fact that much analytical or computational effort is required, and furthermore will notice that the cause of the difficulty is complication of the field functions with real variables x,y.

Vekua[1] has shown a new field description where the field $u(z,\bar{z})$ ($z=x+iy$, $\bar{z}=x-iy$) is related to a regular function Φ of z as follows:

$$u = \mathcal{H}\Phi \qquad (1)$$

where $\mathcal{H}$ is an operator. Henrici[2] also has discussed about $\mathcal{H}$ and has concluded that Φ is uniquely determined by

$$\Phi(z) = \mathcal{H}^{-1}\, u = u(z,0) - \tfrac{1}{2}u(0,0) \qquad (2)$$

The Φ is often observed to be simpler than u.
In this paper, instead of u, the regular function Φ is manipulated to reduce our computational effort. The field-matching is therefore achieved by regular-function-matching.
An example will be presented at the lecture.

Regular-function-matching

We imagine in Fig.1 that a guided wave in the region R_1 is radiated in the horn region R_2. Both do not overlap except at P and Q. The reference points of R_1 and R_2 on a complex plane are z_1 and z_2, respectively. The guided wave u_1 and the radiation field u_2 are written as modal expansions:

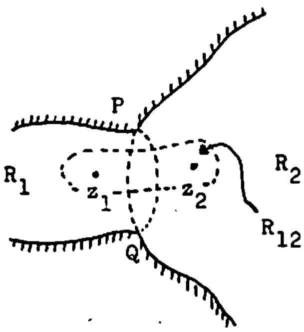

Fig.1. Construction of regions for field-matching.

$$u_1 = \sum a_{1n}\psi_{1n}, \quad u_2 = \sum a_{2n}\psi_{2n} \qquad (3)$$

The u_1 and u_2 can be related to the regular functions Φ_1 and Φ_2 through the operators $\mathcal{H}_1$ and $\mathcal{H}_2$ which are defined at the points z_1 and z_2, respectively. The corresponding regular functions of ψ_{1n} and ψ_{2n} can be obtained by inversion.

$$\phi_{1n}(z) = \psi_{1n}(z,\bar{z}_1) - \tfrac{1}{2}\psi_{1n}(z_1,\bar{z}_1)$$
$$\phi_{2n}(z) = \psi_{2n}(z,\bar{z}_2) - \tfrac{1}{2}\psi_{2n}(z_2,\bar{z}_2) \qquad (4)$$

Then

$$\Phi_1 = \sum a_{1n}\phi_{1n},\ \Phi_2 = \sum a_{2n}\phi_{2n} \qquad (5)$$

Of course, Φ_1 and Φ_2 are restricted in R_1 and R_2, respectively.

Suppose that Φ_2^c is analytic continuation of Φ_2, in R_{12}. The analytic continuation of u_2 can then be expressed, in R_{12}, by

$$u_2^c = \mathcal{H}_2\Phi_2^c \qquad (6)$$

The u_2^c is also expressible in the other form:

$$u_2^c = \mathcal{H}_1\widetilde{\Phi}_2 \qquad (7)$$

where $\widetilde{\Phi}_2$ is regular in R_{12} and is given by ***

$$\widetilde{\Phi}_2(z) = \mathcal{H}_1^{-1}\mathcal{H}_2\Phi_2^c \qquad (8)$$

The u_2^c must coincide with u_1 in the neighborhood of z_1. The matter can not be altered by inversion of $\mathcal{H}_1$. Thus, the field-matching can be replaced by the regular-function-matching such that

$$\Phi_1(z) = \widetilde{\Phi}_2(z) \quad \text{at } z_1 \qquad (9)$$

Actually, (9) is established by

$$b_{1n} = b_{2n} \qquad (10)$$

where

$$\begin{aligned}\Phi_1(z) &= \sum b_{1n}(z-z_1)^n \\ \widetilde{\Phi}_2(z) &= \sum b_{2n}(z-z_1)^n\end{aligned} \qquad (11)$$

The b_{1n} and b_{2n}, of course, contain the unknown coefficients a_{1n} and a_{2n}, respectively. The number of equations of (10) should be chosen as the number of the unknown coefficients.

Provided that both the regions overlap partly, we may take the same reference point z_1 at the common region. The Φ_2 can then be substituted for Φ_2^c. Thus,

$$\Phi_1(z) = \Phi_2(z) \quad \text{at } z_1 \qquad (12)$$

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The $\widetilde{\Phi}_2$ is given, at z_1, by inverting the Vekua's description. The result is

$$\begin{aligned}\widetilde{\Phi}_2(z) = &-\widetilde{\Phi}_2(z_1) + \Phi_2^c(z) \\ &-\int_{z_2}^{z}\frac{\partial}{\partial t} J_o(2\sqrt{(\bar{z}_1-\bar{z}_2)(z-t)}\ ) \\ &\cdot \Phi_2^c(t)dt \quad + \overline{\Phi_2^c(z_1)} \\ &-\int_{\bar{z}_2}^{\bar{z}_1}\frac{\partial}{\partial \bar{t}} J_o(2\sqrt{(z-z_2)(\bar{z}_1-\bar{t})}\ ) \\ &\cdot \overline{\Phi_2^c(t)}d\bar{t}\end{aligned}$$

where the bar denotes the complex conjugate, and

$$\begin{aligned}\widetilde{\Phi}_2(z_1) = &\ \Phi_2^c(z_1) \\ &-\int_{z_2}^{z_1}\frac{\partial}{\partial t} J_o(2\sqrt{(\bar{z}_1-\bar{z}_2)(z_1-t)}\ ) \\ &\cdot \Phi_2^c(t)dt\end{aligned}$$

References

1) I.N.Vekua, New Methods for Solving Elliptic Equations, North-Holland Pub. (1967).

2) P.Henrici, Zur Funktionentheorie der Wellengleichung, Comm. Math. Helv. 27 (1953) 235-293.