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Accuracy Improvement in Moment Method Solutions of the Tensor-Volume Integral Equation for Three-Dimensional Dielectric Scatterers

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1. Introduction

The topic of electromagnetic wave scattering by dielectric bodies is of great interest in numerous areas of applications, such as analysis of radar targets, electromagnetic wave interaction with biological systems and specific absorption rate (SAR) analysis. Two popular numerical techniques for those kinds of problems are finite-difference time-domain (FDTD) method and method of moments (MoM). MoM has great advantages, i.e., smaller memory size and shorter CPU time consumption than FDTD method, and has been applied to many problems including dielectric bodies. Livesay and Chen [1] first presented the formulation of the tensor-volume integral equation (TVIE) for a numerical solution of scattering fields from dielectric bodies illuminated by a plane wave. They used point matching method and first order integration (F.O.I.), which is equivalent to the double point matching method. Other researchers have done similar studies using different basis functions, numerical integration and also different geometric modeling [2]-[4]. The problem of a linear thin wire antenna in the vicinity of a three-dimensional dielectric body has been also studied using integral equation approach in [5] - [9].

In this paper, some improvements are introduced in the previous researches in which MoM has been applied. The tensor-volume integral equation, which includes the free-space dyadic Green's function, is adopted for a scattering problem consisting of only one three-dimensional dielectric body. A system of coupled tensor-volume/line integral equations is also applied for the case of a thin-wire antenna in proximity of a dielectric scatterer. A Galerkin-based moment method is utilized to solve mentioned integral equations. The first major difference of the proposed method compared with the previous works is the extreme care, which has been exercised in calculation of the principle value for singularity of the dyadic Green's function through the Galerkin's method computations to obtain more accurate results. In addition, three-dimensional polynomial function of arbitrary degree with unknown complex coefficients is used as the basis and test functions in MoM procedure. The numerical implementation results are verified by comparison with the results of point matching method, exact method (Mie theory) and FDTD method for different situations and their accuracy and reliability are demonstrated.

2. MoM Formulation

In the TVIE approach, the first step is to replace the dielectric object with a polarization current density. The integral equation, which relates the incident field to the induced field inside the dielectric object, can be

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formulated as [1]:

$$\mathbf{E}^{\text{inc}}(\mathbf{r})+k_0^2\ \text{P.V.}\int_V \overline{\overline{G_0}}(\mathbf{r},\mathbf{r}')\cdot(\varepsilon_r{}^*(\mathbf{r}')-1)\mathbf{E}^{\text{t}}(\mathbf{r}')\ dv'-\overline{\overline{L}}\cdot(\varepsilon_r{}^*(\mathbf{r})-1)\mathbf{E}^{\text{t}}(\mathbf{r})=\ \mathbf{E}^{\text{t}}(\mathbf{r}) \tag{1}$$

where $\varepsilon_r{}^*$ is the complex dielectric permittivity and $\overline{\overline{G_0}}$ denotes the free-space dyadic Green's function. Dyad $\overline{\overline{L}}$ is the extracted singularity of the volume integral of dyadic Green's function in a form of a surface integral over an arbitrary shape, around the observation point, which is given by:

$$\overline{\overline{L}} = \lim_{S_\delta \to 0} \int_{S_\delta} \frac{\hat{\mathbf{R}}'\ \hat{\mathbf{n}}'}{4\pi\ |\mathbf{r}-\mathbf{r}'|^2}\, ds',\ \ \hat{\mathbf{R}}' = \frac{\mathbf{r}'-\mathbf{r}}{|\mathbf{r}-\mathbf{r}'|} \tag{2}$$

It can be shown that when the shape around the singularity point is a square cube or a sphere, $\overline{\overline{L}}$ is reduced to a simpler form of $\overline{\overline{I}}/3$ where $\overline{\overline{I}}$ denotes the unit dyad. Now, the dielectric volume is discretized into N sub-volumes using cubic modeling and according to MoM procedure, the following simultaneous equations are obtained:

$$\left\langle \mathbf{w}_m(\mathbf{r}),\mathbf{f}_m(\mathbf{r})\right\rangle - k_0^2\sum_{\substack{n=1\\ n\neq m}}^{N}\left\langle \mathbf{w}_m(\mathbf{r}),\mathbf{I}_1^n(\mathbf{r})\right\rangle - \left\langle \mathbf{w}_m(\mathbf{r}),\mathbf{I}_2^m(\mathbf{r})\right\rangle + \left\langle \mathbf{w}_m(\mathbf{r}),\overline{\overline{L}}\cdot(\varepsilon_r{}^*(\mathbf{r})-1)\mathbf{f}_m(\mathbf{r})\right\rangle \tag{3}$$
$$= \left\langle \mathbf{w}_m(\mathbf{r}),\mathbf{E}^{\text{inc}}(\mathbf{r})\right\rangle$$

where

$$\mathbf{I}_1^n = \int_{V_n} \overline{\overline{G_0}}(\mathbf{r},\mathbf{r}')\cdot\ (\varepsilon_r{}^*(\mathbf{r}')-1)\ \mathbf{f}_n(\mathbf{r}')\ \ dv',\ \ \mathbf{I}_2^m = \lim_{V_\delta\to 0}\int_{V_m-V_\delta} \overline{\overline{G_0}}(\mathbf{r},\mathbf{r}')\cdot\ (\varepsilon_r{}^*(\mathbf{r}')-1)\ \mathbf{f}_m(\mathbf{r}')\ \ dv' \tag{4}$$

The basis functions used for electric field expansion inside the dielectric body are as follows:

$$\mathbf{f}_m(\mathbf{r}) = \sum_{c=1}^{3}\sum_{i=0}^{n_{mx}}\sum_{j=0}^{n_{my}}\sum_{k=0}^{n_{mz}} a_{ijk}^{mc}\ x^i y^j z^k \hat{c}\quad ,\quad (\hat{1},\hat{2},\hat{3}) = (\hat{x},\hat{y},\hat{z}) \tag{5}$$

When the upper limits of the summations in (5) are assigned as $n_{mx}= n_{my}= n_{mz}=0$, $\mathbf{f}_m$ is equivalent to the pulse basis function. Furthermore, when some of unknowns in (5) are forced to be zero, $\mathbf{f}_m$ is linear basis function, which has been introduced in [3]. This kind of basis function has been introduced as entire-domain function in [2] and applied with point matching method for solution of the potential-volume integral equation. By choosing these kinds of basis functions in Galerkin's method, an acceptable accuracy will be well achieved. In fact, due to the entire-domain nature of these functions as three-dimensional Taylor series, they have analytical characteristics being able to estimate the field's behavior with a good accuracy inside the dielectric body. They satisfy the boundary conditions at interfaces between adjacent cells of different permittivities and prevent the production of fictitious charge density in a homogeneous dielectric body, whereas they happen in conventional MoMs, especially when pulse functions are applied.

In [1] and [7]-[9], the third term of the left-hand side in (3) has been replaced with an approximated term by putting a sphere instead of the original shape of the cube during the volume integration. In this work, this term is evaluated using numerical integration. The same expressions are used for the system of integral equations in the case of radiation from a thin-wire antenna in the presence of the dielectric body.

3. Numerical Results

In this work, the upper limits of the summations in (5) are assumed to be $n_{mx}= n_{my}= n_{mz}=1$ in all calculations. Figs. 1 and 2 show the normalized backscattering cross-section (RCS) and the specific absorption rate (SAR), respectively, for the model of a dielectric sphere which is illuminated by an incident plane wave in different complex permittivities. Numerical results are compared with exact solutions using Mie theory. The results of the proposed approach agree well with the exact Mie solutions confirming the validity of the method.

The model for a thin-wire antenna in the vicinity of a dielectric box is shown in Fig.3. Input impedance of the wire antenna was calculated by the proposed technique and is shown in Figs.4 and 5 and is compared with the solutions of FDTD method results [6] and conventional MoM results. The accuracy of the introduced method is demonstrated through the presented results.

4. Conclusion

A Galerkin-based MoM has been introduced for an accurate solution of the tensor-volume integral equation (TVIE) using three-dimensional polynomial basis and test function. Exact calculation of principle value of the singular volume integral has been also performed. Numerical results show that an acceptable accuracy can be achieved by the proposed method. However, the method uses a time-consuming algorithm and it is required to obtain a reasonable CPU time by modifying the algorithm, which is used for calculations of six-fold integrals in (3) and optimizing the number of unknowns and the size of blocks.

References

[1]. D.E.Livesay , K.M.Chen , " Electromagnetic Fields Induced Inside Arbitrarily Shaped Biological Bodies", *IEEE Trans. MTT -22, No. 12, Dec.1974*

[2]. B.D.Popovic, B.M.Notaros, "Entire-domain Polynomial Approximation of Volume Currents in the Analysis of Dielectric Scatterers", *IEE Proc.-Microw. Antennas Propag.,Vol 142, No.3, June 1995*

[3]. C.T. Tsai, et al., "A Procedure for Calculating Fields inside Arbitrarily Shaped Inhomogeneous Dielectric Bodies Using Linear Basis Functions with Moment Method",*IEEE Trans. MTT, Vol.34, No.11,Nov.1986*

[4]. T.K.Sarkar , et al. , "Electromagnetic Scattering form Dielectric Bodies" , *IEEE Trans. AP , Vol. 37 , No. 5, May 1995*

[5]. B.M.Notaros, B.D.Popovic, "General Entire-domain Galerkin Method for Analysis of Wire Antennas in the Presence of Dielectric Bodies*", IEE Proc.-Microw. Antennas Propag.,Vol 145, No.1, Feb. 1998*

[6]. D.Koizumi, Q.Chen, K.Sawaya,"Galerkin-MoM Analysis for Dielectric Scatterers by Using Sinusoidal Reaction Techniques" *Proc. of the 2001 IEEE-APS Int. Symp.*

[7]. D.P.Nyquist, et al., "Coupling Between Small Thin-Wire Antennas and A Biological Body", *IEEE Trans. AP, Vol.25, No. 6, Nov.1995*

[8]. K. Karimullah, et al., "Electromagnetic Coupling between a Thin-Wire Antenna and a Neighboring Biological Body: Theory and Experiment ", *IEEE Trans. MTT, Vol.28, No. 11, Nov.1980*

[9]. J.S. Colburn, Y. Rahmat-Samii,"Electromagnetic Scattering and Radiation Involving Dielectric Objects", *Journal of Electromagnetic Waves and Applications, Vol.9, No.10, 1995*

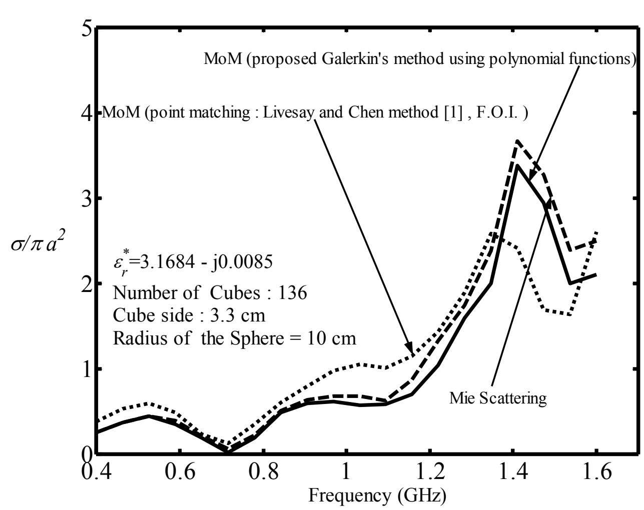

Figure 1. Normalized RCS of the dielectric sphere.

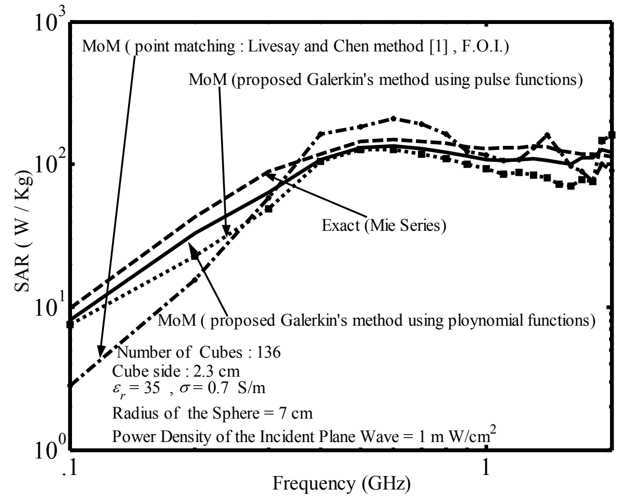

Figure 2. Average SAR rate inside the dielectric sphere.

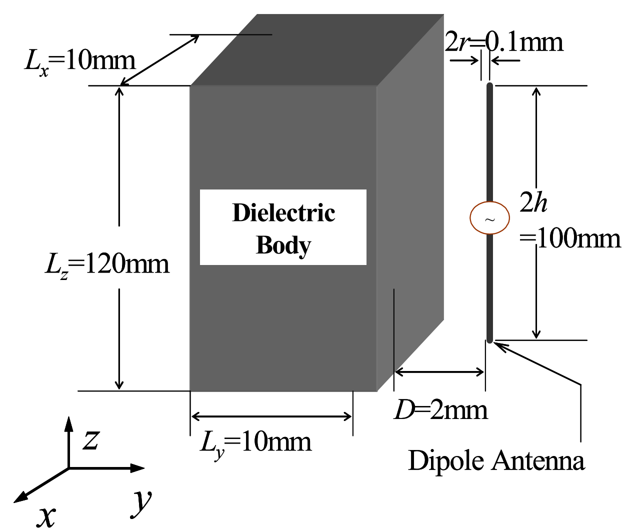

Figure 3. Analysis model for the dipole antenna in vicinity of the dielectric box.

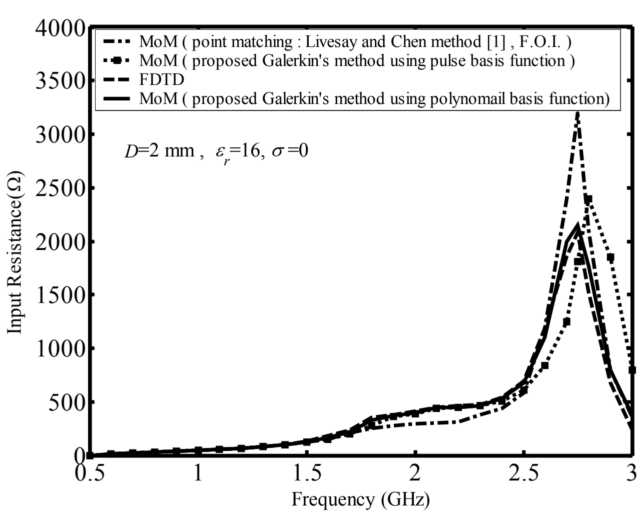

Figure 4. Input resistance of the dipole antenna.

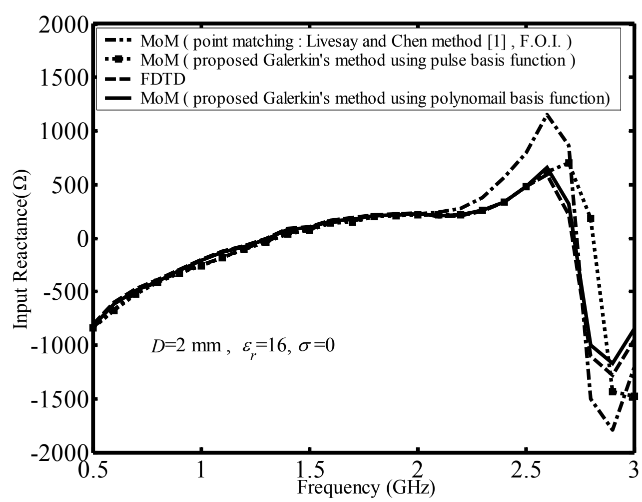

Figure 5. Input reactance of the dipole antenna.