

MULTIPATH FADING ON A SPACE DIVERSITY LINK

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1. INTRODUCTION

Terrestrial Line-Of-Sight paths at frequencies below 10 GHz are primarily limited by the effects of multipath propagation /1/ which may cause destructive interference at the receiving antenna. Depending on pathlength-difference  $l$  (fig. 1) and the amplitude of the refracted wave relative to the direct wave, frequency selective fading may occur. Therefore, high capacity digital radio systems are often limited, not by the flat fade margin, but by frequency dependent attenuation within transmitted bandwidth, and system outage may occur at input levels 20 dB or more above the flat fade margin /2/.

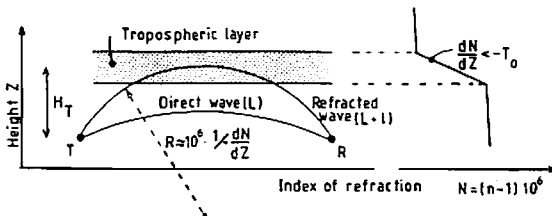


Fig. 1 Multipath mechanism.

2. PROPAGATION MODEL

A theoretical study of different structures of tropospheric layers /1/ shows that 3 rays are sufficient to represent all fades of practical interest, generated by single layers. The simple reduced 3-way model /3/ also represents the characteristics of multipath fading well, and is practical to handle. The transfer function of the reduced 3-way model, relative to the unfaded one, is given by (minimum phase)

$$\tilde{H}(j2\pi f) = a(1 - re^{-j2\pi(f-f_0)\tau}) \quad (1)$$

where  $0 < a < 1$  controls the fade level and  $0 < r < 1$  the shape of the fade. The delay corresponding to the path length difference between main and refracted rays is represented

by  $\tau$  and the frequency of the transmission minimum is represented by the notch frequency  $f_0$ .

3. MEASUREMENTS

Transmission of a modulated signal with powerspectrum  $S(f)$  over a radio channel  $H(f)$  with multipath fading results in a received spectrum  $R(f)$ .

$$R(f) = S(f) |H(f)|^2 \quad (2)$$

From the 3-way model (1) it follows that received signal, at inband distortion, is amplitude distorted in a characteristic way (fig. 2). Paths with short delays  $\tau \lesssim 1$  ns produce essentially flat fades while increasing delays, at sufficient notch depth  $-20 \log(1-r)$ , correspond to

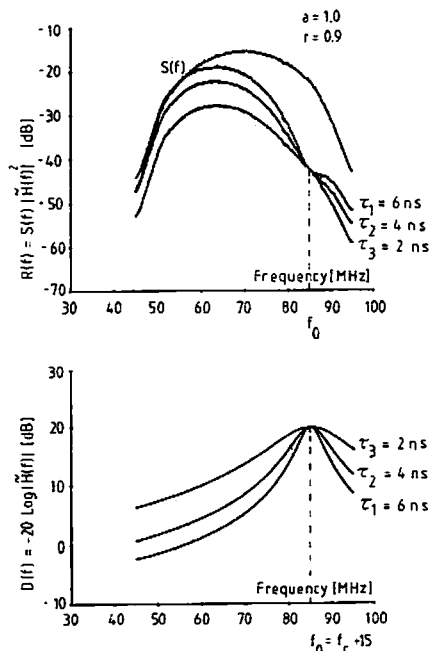


Fig. 2 Received powerspectra  $R(f)$  at transmission of a MSK-spectrum  $S(f)$  over a radio channel modeled by (1), and corresponding discrepancy spectra  $D(f) = S(f) - R(f)$ .

more pronounced frequency selective fades.

The narrowband sweep-measurements were event-controlled. Each scan was compared to the non-faded powerspectrum  $S(f)$ , and recorded only if the slope of the discrepancy spectrum  $D(f)=S(f)-R(f)$  exceeded 0.3 dB/MHz in the vicinity of the notch frequency  $f_0$ .

From laboratory simulations of multipath fading this slope was found to represent a spectrum distortion of concern.

From May 20 to Sept 12, 1983 and from May 15 to Sept 10, 1984 propagation measurements were conducted on a 52.8 km path H-G (first Fresnelzone clear) with two identical 3.0 m parabolic receiving dishes 8 m apart. Radiated power was +30 dBm resulting in a flat fade margin of 34 dB. Transmitted signal was a 34 Mb/s MSK-signal with a 3 dB-bandwidth of 55 MHz operating at 7.2 GHz.

#### 4. SPACE DIVERSITY IMPROVEMENT

The multipath fading probability  $Pr(w < W)$  during "worst season" related to clear LOS-paths with negligible earth reflection is given by CCIR-recommendations (1978) based on a method developed by Morita /4/.

$$Pr(w < W) = p 10^{-F/10}; F > 15 \text{ dB} \quad (3)$$

where the occurrence probability of Rayleigh fading

$$p = KQ f^B d^C$$

$K$ =climatic factor  $Q$ =terrain factor  
 $f$ =frequency (GHz)  $d$ =path length (km)  
 $F=-10\log (W/W_0)$   $W_0$ =received power

at non-fading conditions.

A fit of (3) with  $B = 1$ ,  $C = 3.5$  to experimental data measured during worst season June to August, 1984

(fig. 3) gives  $K = 1.2 \cdot 10^{-8}$  (assuming  $Q = 0.3$ ) which is in agreement with the CCIR-rec. for North West

Europe  $K = 1.4 \cdot 10^{-8}$ . Corresponding cumulative distribution during the period June to August, 1983 is identical.

Simplification of the geometry in Fig. 1 results in the pathlength difference

$$l \approx 2(H_T H_R)/d \quad (4)$$

where  $H_{T,R}$  = Height of layer above Transmitter, Receiver. Vertically separated antennas will consequently receive the multipath signal with different phases, where  $180^\circ$  phase difference gives maximum and minimum simultaneously in main and secondary antenna.

The improvement factor  $I_0(F)$ , at a given fadedepth  $F=-10\log (W/W_0)$ , obtained by spacediversity depends strongly on antenna separation  $s$ . Vigants /5/ proposes a quadratic relationship between improvement and separation.

$$I_0(F) = ks^2 fd^{-1} 10^{F/10} \quad (5)$$

which predicts an improvement

$I_0(F=34 \text{ dB})=56$  with  $k=1.5 \cdot 10^{-4}$ ,  
 $s = 26$  feet,  $f = 7.2$  GHz, and  
 $d = 33$  miles. This is in agreement with measured improvement  $I_0 \approx 63$

at corresponding fade depth  $F=34$  dB (Fig. 3).

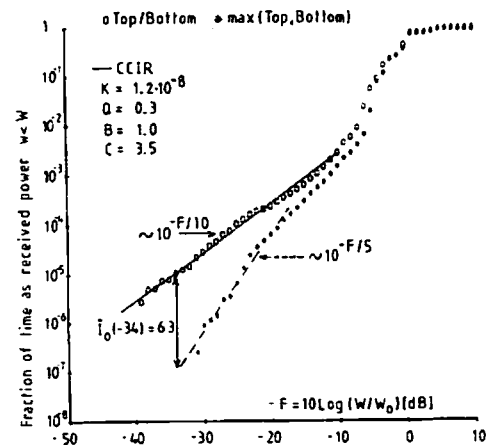


Fig. 3 Space diversity on path H-G during worst season June-August, 1984. ( $W_0$  is received power at free space, corresponding to -41 dBm).

## 5. EXTRACTION OF MULTIPATH PARAMETERS

The stochastic model  $\tilde{H}$  given by (1) may be used to calculate the performance of radio link systems. From narrowband sweep-measurements the parameters are obtained from a minimum square fit of synthetic power-spectra  $\tilde{R}(f) = S(f) |\tilde{H}(f)|^2$  to received powerspectra  $R(f)$

$$\text{Minimize } \sum_{n=1}^N g_n (R(f_n) - \tilde{R}(f_n))^2 \quad (6)$$

where  $g_n$  = weight function,  $N$  = number of spectral lines. The solution  $\{a_0, r_0, \tau_0\}$  which minimizes (6) is unfortunately not unique, i.e. several solutions exist beside the "true" solution.

From (1) it follows ( $\Delta f$  small)

$$|\tilde{H}(f_0 + \Delta f)|^2 - |\tilde{H}(f_0)|^2 \approx 4\pi^2 (\Delta f)^2 r(a\tau)^2 \quad (7)$$

( $\cos x \approx 1 - x^2/2$  for small arguments  $x$ )

It is evident from (7) that the solution of (6) is not unique, which is confirmed by multipath simulations. The solution of (6) is unique only if the fade level parameter  $a$  or the delay  $\tau$  is kept fixed. An alternative to fixed delay is to estimate the fade level  $-20\log a$  immediately before a frequency selective event. During the period May to Sept., 1983-1984 essentially two days with severe frequency selective fading occurred, June 7, 1983 and August 26, 1983. The typical fading activity during 4 seconds is illustrated in Fig. 4 where the notch  $f_0$  moves  $\Delta f = 5$  MHz. In general

$\Delta f = f_0 (\Delta\tau/\tau)$  in the 3-way model where  $\Delta\tau$  corresponds to variations in refractive index alternatively to variations in the height of the layer.

The distribution of extracted delays during a short period of severe fading activity (Fig. 5) shows a short term mean value of about 7 ns. In general the problem in narrowband sweep-measurements is multipath fades with short delays where it is difficult to separate the fade level

$-20\log(a)$  from the notchdepth  $-20\log(1-r)$  with a shape of the notch which is approximately flat in the measuring bandwidth. In spite of these problems, the expected delay  $\langle\tau\rangle$  can be calculated, based on the fraction of time  $P$  during the measurement period (May to Sept., 1983 on path H-G) with amplitude distortion within the measuring bandwidth  $B$   $\langle\tau\rangle = P/B \approx 0.5$  ns

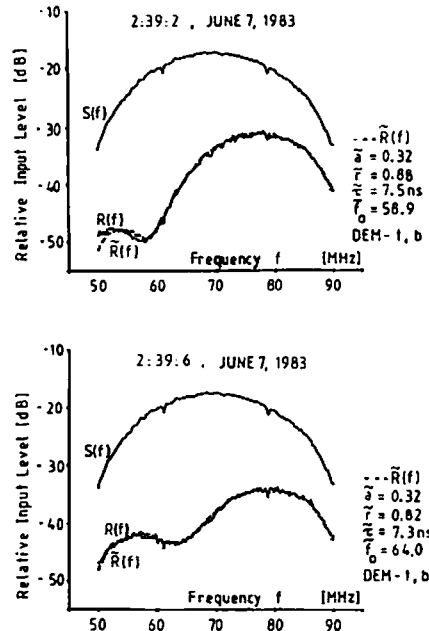


Fig. 4 Typical event of severe frequency selective fading on path H-G. The least square fit  $\tilde{R}$  according to (6), including corresponding multipath parameters are indicated for each scan.

Maximum estimated delay  $\tau = 12$  ns (Fig. 5) during the same period is below the upper limit  $1/\tau$

$$\tau_{\max} = 3.7 (d/20)^3 = 16 \text{ ns}$$

Based on Sodar-measurements in Sweden, the distribution of the height of layers is nearly exponential with a mean of about 125-150 m above ground. (in summer). Since the pathlength difference  $l \approx 2H_T H_R / d$  (4) is dependent

on the height  $H_T, H_R$  of the layer, a natural distribution of the delay is the exponential  $p(\tau) = 1/\tau_0 \exp(-\tau/\tau_0)$

where  $\tau_0 = \langle\tau\rangle$ .

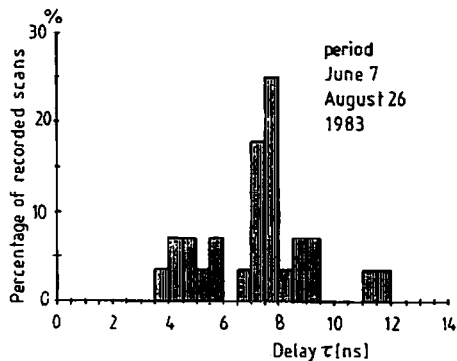


Fig. 5 Distribution of relative delay on path H-G.

## 6. DESIGN OF SPACE DIVERSITY

From the pathlength difference  $l \approx 2H_T H_R / d$  (4) the pathlength difference  $\Delta l$  between main and secondary antenna separated  $\Delta H$  is

$$\Delta l = 2H_T \Delta H / d$$

Maximum and minimum simultaneously in the two antennas corresponds to  $\Delta l = n\lambda/2$  ( $n=1,3,5,\dots$ ) where  $\lambda$  = wavelength.

Consequently

$$2H_T \Delta H / d = n\lambda/2 \quad (n=1,3,5,\dots) \quad (8)$$

"Optimal" antenna separation at layer-height  $H_T$  (horizontal path) corresponding to  $\tau=1/0.3$  ns is received from (4) and (8).

$$\Delta H(\tau) = c \cdot \lambda \sqrt{d} \sqrt{1/\tau} \cdot n \quad (n=1,3,5,\dots) \quad (9)$$

where the constant  $c=1/\sqrt{16 \cdot 0.15}$

The simplified relationship (9) implies that to "combat" a fade with delay  $\tau$  by space diversity, the required separation depends on  $\tau$  (Fig. 6). Since the separation is fixed, optimal choice depends on which criterion is used. Commonly the criterion is to minimize the effects of multipath fading in long terms, where the separation  $\Delta H$  corresponds to the expected delay  $\langle \tau \rangle$ . Estimated value of  $\langle \tau \rangle \approx 0.5$  ns (path H-G) corresponds to  $\Delta H \approx 8.5$  m ( $H_T \approx 70$  m) according to

(9) which may be compared to Vigants /5/ "optimal" separation  $\Delta H \approx 9$  m at 4GHz, 39 km.

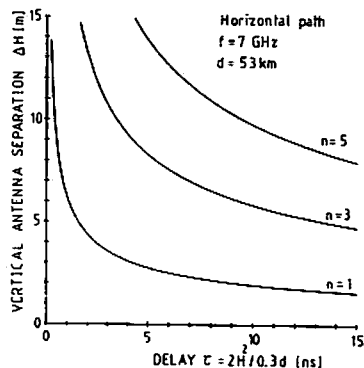


Fig. 6 Simplified relationship between "optimal" antenna separation and delay at a 2-ray fade.

The improvement of space-diversity at multipath fading using antenna separations of 8-10 m is primarily the effect of "combating" the frequent fades with delay  $\leq 1$  ns. If the fade-margin is large enough to ensure sufficiently low probability of outage due to flat multipath fades with short delays, antenna separations of 2-3 m will be more effective since they should reduce the effects of the severe frequency selective fading with long delays.

## REFERENCES

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