2-III A3

DIFFRACTION OF ELECTROMAGNETIC WAVES BY CYLINDRICAL STRUCTURES CHARACTERIZED BY VARIABLE CURVATURE AND SURFACE IMPEDANCE

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In this paper, we consider the problem of radiation from an electric, line-source $\bar{J} = J(x,y)\bar{a}_z$ parallel to a convex, cylindrical boundary of finite, cross-sectional area. The boundaries of the diffracting object are characterized by an arbitrarily varying surface impedance $Z_s(x)$ and radius of curvature $R(x)$, where the variable x is the distance measured along the surface of the cross-section (see Fig. 1). An exp $\{i\omega t\}$ excitation is assumed.

In view of the geometry of the diffracting object, we construct a natural coordinate system (x,y,z) around the surface of the straight convex cylinder and express the electric field (z component only) in terms of a complete set of local basis functions,

$$E_z(\xi,x)=\sum_{n=1}^{\infty} e_n(x)H_{\nu_n}^{(2)}(\xi)/N_n \quad .$$

$$\equiv \sum_n [a_n(x)+b_n(x)]H_{\nu_n}^{(2)}(\xi)/N_n \qquad (1)$$

The basis functions $H_{\nu_n}^{(2)}(\xi)$ are Hankel function of the second kind whose orders are determined by the modal equation

$$H_{\nu_n}^{(2)'}(\xi_R) - iy_s H_{\nu_n}^{(2)}(\xi_R) = 0, \qquad (2)$$

in which y_s is the normalized surface admittance. Using the orthogonal properties of the basis functions, the electric field nth mode amplitude $e_n(x)$ is recognized to be the transform of the function $E_z(\xi,x)$, i.e.

$$e_n(x)=\int_{\xi_R}^{\infty} E_z(\xi,x)[H_{\nu_n}^{(2)}(\xi)/M_n]\nu_n \frac{d\xi}{\xi} . \qquad (3)$$

In the expressions above, which are recognized to be the Watson transform pair, N_n and M_n are normalization coefficients. The expansion of the solution in terms of the local cylindrical modes provides the basis for the transformation of Maxwell's equations into a set of coupled first order differential equations for the forward and backward wave amplitudes $a_n(x)$ and $b_n(x)$ respectively.

$$\frac{-da_n(x)}{dx} + i\frac{\nu_n}{R} a_n(x) \; .$$
$$= \sum_{n=1}^{\infty} \frac{dT_{nm}}{dx} a_m(x) + \frac{dR_{nm}}{dx} b_m(x) \; .$$
$$+ J_o\delta(x - x_o) \qquad (4)$$

and

$$\frac{-db_n(x)}{dx} + i\frac{\nu_n}{R} b_n(x) \; .$$
$$= \sum_{n=1}^{\infty} \frac{dT_{nm}}{dx} b_m(x) + \frac{dR_{nm}}{dx} a_m(x) \; .$$
$$- J_o\delta(x - x_o), \qquad (5)$$

where $\frac{dT_{nm}}{dx}$ and $\frac{dR_{nm}}{dx}$ are identified as transmission and reflection scattering coefficients respectively and J_o is the normalized current density. Using a generalized Bessel transform, the discrete Watson expansions of the fields are shown to merge with the Fourier-type, (plane wave) expansions above plane boundaries ($R \to \infty$), and with the Kontorowich-Lebedev transform for $R \to 0$. Thus, in our analysis, no restrictions are made on the local radius of curvature R(x); however, the surface impedance variations must be compatible with the surface impedance concept. As an illustrative example, consider the launching of a surface wave in the region $x < 0$ ($R \to \infty$, see Fig. 2),

$$E(y,x) = E_o \exp - ik(S_o x + C_o y) \quad (6)$$

where E_o is a constant and $S_o^2 + C_o^2 = 1$. Using (5), the WKB-type expression for the electric field in the region of the bend where R is finite can be shown to be,

$$E(\xi,x) = \frac{a_o(x) H^{(2)}_{\nu_o}}{N_o(x)} = E_o \left[\frac{iS_o}{2y_s}\right]^{\frac{1}{2}} H^{(2)}_{\nu_o}(\xi) \cdot$$

$$\exp\{-i \int \frac{\nu_o}{R} du\} / [M_o(x) N_o(x)]^{\frac{1}{2}} \quad , (7)$$

where the product of the normalization coefficients is

$$M_o(x) N_o(x) = - \frac{\xi_R}{2} H^{(2)}_{\nu_o}(\xi_R) \cdot$$

$$\frac{\partial}{\partial\nu} [H^{(2)'}_{\nu}(\xi_R) - iy_s H^{(2)}_{\nu}(\xi_R)]_{\nu_o}, \quad (8)$$

and ν_o is the surface wave parameter satisfying (2). Similarly the higher order mode amplitudes are,

$$a_n(x) = - \int_o^x \frac{dT_{no}}{du} a_o(u) \cdot$$

$$\exp - \{ \int_u^x [\frac{dT_{nn}}{dv} + i \frac{\nu_n}{R}]dv\} du \quad (9)$$

and

$$b_n(x) = \int_x^\infty \frac{dR_{no}}{du} a_o(u) \cdot$$

$$\exp - \{ \int_u^x [\frac{dT_{nn}}{dv} - i \frac{\nu_n}{R}]dv\} du. \quad (10)$$

It can be shown that an alternative expansion of the solution in terms of local plane waves results in stronger coupling between the component waves. In the local plane wave expansion, we characterize the boundary by the reflection coefficients for the local "tangent" planes.

Although, we have restricted our attention to two dimensional problems, this analysis may be applied to three dimensional problems, if the variations of the boundary curvature and surface impedance, transverse to the path of propagation, are small compared to the variations along the path.

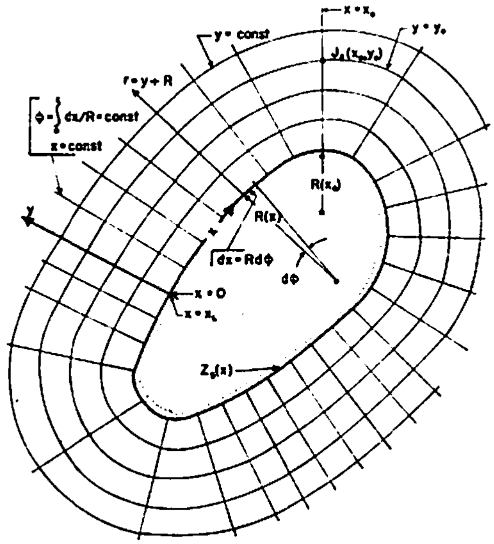

FIG. 1. Radiation by an electric line source parallel to a convex cylinder of arbitrary cross-section and variable surface impedance boundary.

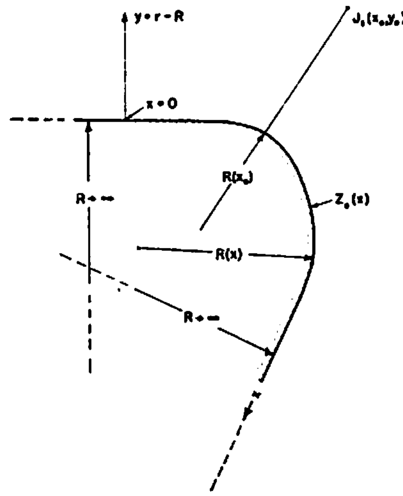

FIG. 2. Propagation over an infinite wedge with a rounded corner.