Physical Optics Inverse Scattering Identity Using Single Frequency, Multistatic Data

Atsuhiko Hashigaya and Akira Noguchi
Department of Electrical Engineering
Faculty of Science and Technology, Keio University

1. Introduction

The reconstruction problem of perfectly conducting bodies from far-field data has been discussed, in the context of physical optics, by Lewis[1] and Bojarski[2]. But these works are restricted to use backscattering data or monostatic data . S.Ronsenbaum-Raz[3] extended the theory to the bistatic case. Though, he also refer to single frequency measurements, his work is a formal one because it violates the monostatic-bistatic equivalence principle, and some of the quantities necessary for reconstruction are not measurable. In this paper we will show a Identity using the single frequency, multistatic data. Some numerical experiment results will be presented.

2. The electromagnetic physical optics direct scattering approximation

By the Stratton-Chu direct integration of the vector electromagnetic wave equation, the magnetic fields scattered by closed surface of a scatterer is

$$Hs = \oint \{(\mathbf{n} \times \mathrm{H}) \times \nabla G + (\mathbf{n} \cdot \mathrm{H}) \nabla G - j\omega\varepsilon (\mathbf{n} \times \mathrm{E}) G\} dS. \quad (1)$$

Here, the integral is over the entire surface S of the target, **n** denotes the unit vector normal to the surface S, and G is the Green's function. For the perfect conductor, the surface boundary condition are

$$\mathbf{n} \times \mathrm{E} = 0 \qquad \mathbf{n} \cdot \mathrm{H} = 0, \quad (2)$$

and we assume that the scattering processes are adequately describable by the theory of physical optics. Appropriate expression for the far scattered field due to an incident field Hi is

$$Hs = 2j \oint_{Ki \cdot n < 0} G \{Hi (\mathbf{ks} \cdot \mathbf{n}) - \mathbf{n} (\mathbf{ks} \cdot Hi)\} dS. \quad (3)$$

Here, the surface integral in (3) is over the illuminated portion of the target surface, **Ks** is the wavevector characterizing the scattered wave. From (3), Bojarski and Lewis led the far-field scattering amplitude on condition that the incident field in the vicinity of the scatterer is a plane wave, and that the scattering data is restricted to the backscattering data or monostatic data. In this paper, we remove the second condition, or use multistatic data.

We consider an incident field is a plane, time-harmonic wave of the form

$$Hi = I(\mathbf{ki}) \exp(j\, \mathbf{ki} \cdot \mathbf{x}')\, \mathbf{hi}, \quad (4)$$

where **Ki** is the wavevector characterizing incident radiation, I(**Ki**) is the complex amplitude and **hi** is the unit vector representing the incident wave polarization. And we assume the scattered field is the far field or spherical wave. The substitution of (4) into (3) yields

$$\mathbf{E}s = \rho\,(\mathbf{k}s,\ \mathbf{k}i)\ I\ (\mathbf{k}i)\ \frac{\exp\,(j\,\mathbf{k}s\cdot\mathbf{x})}{\sqrt{4\pi}\ x} \tag{5}$$

where

$$\rho\,(\mathbf{k}s,\ \mathbf{k}i) = j/\sqrt{\pi} \oint_{Ki\cdot n<0} e^{-j(ks-ki)\cdot x}\, \{\mathbf{h}i\,(\mathbf{k}s\cdot\mathbf{n} - \mathbf{n}\,(\mathbf{k}s\cdot\mathbf{h}i)\}\ ds \tag{6}$$

Note that, in this case, the normalized complex scattering amplitude $\rho\,(\mathbf{ks},\mathbf{ki})$ is a vector.

3. Inverse scattering identity

For the negative argument of (6), i.e., geometrically opposite viewing direction, (6) yields

$$\rho\,(-\mathbf{k}s,\ -\mathbf{k}i) = -j/\sqrt{\pi} \oint_{Ki\cdot n>0} e^{j(ks-ki)\cdot x}\, \{\mathbf{h}i\,(\mathbf{k}s\cdot\mathbf{n}) - \mathbf{n}\,(\mathbf{k}s\cdot\mathbf{h}i)\}\ ds. \tag{7}$$

From (6) and (7), it follows that

$$\rho\,(\mathbf{k}s,\ \mathbf{k}i) + \rho^{*}\,(-\mathbf{k}s,\ -\mathbf{k}i) = j/\sqrt{\pi} \oint_{s} e^{-j(ks-ki)\cdot x}\, \{\mathbf{h}i\,(\mathbf{k}s\cdot\mathbf{n}) - \mathbf{n}\,(\mathbf{k}s\cdot\mathbf{h}i)\}\ ds. \tag{8}$$

Here the asterisk denotes complex conjugate, and the integral is over the entire surface S of the target. To apply the divergence theorem to (8), decompose (8) into three components(**hi**,**ki**, and **ei** component).

First, as for **hi** component of (8) is

$$\mathbf{h}i\cdot\{\rho\,(\mathbf{k}s,\ \mathbf{k}i) + \rho^{*}\,(-\mathbf{k}s,\ -\mathbf{k}i)\} = j/\sqrt{\pi} \oint_{s} e^{-j(ks-ki)\cdot x}\, \{\mathbf{k}s - \mathbf{h}i\,(\mathbf{k}s\cdot\mathbf{h}i)\}\cdot \mathbf{ds}. \tag{9}$$

By the divergence theorem, (9) reduce to

$$\frac{\sqrt{\pi}\,\mathbf{h}i\cdot\{\rho\,(\mathbf{k}s,\ \mathbf{k}i) + \rho^{*}\,(-\mathbf{k}s,\ -\mathbf{k}i)\}}{(\mathbf{k}s - \mathbf{k}i)\cdot\{\mathbf{k}s - \mathbf{h}i\,(\mathbf{k}s\cdot\mathbf{h}i)\}} = \int_{v} e^{-j(ks-ki)\cdot x}\, dv \tag{10}$$

Following bojarski[1], we define the characteristic function of the scatterer as

$$\gamma\,(\mathbf{x}) = \begin{cases} 1 & \mathbf{x} \in V \\ 0 & \mathbf{x} \notin V. \end{cases} \tag{11}$$

And we define the left hand side of (10) as $\Gamma 1\,(\beta)$. Then (10) reduce to

$$\Gamma 1\,(\beta) = \int_{-\infty}^{\infty} \gamma\,(\mathbf{x})\ e^{-j\beta\cdot x}\, d\beta, \tag{12}$$

where $\beta = \mathbf{k}s - \mathbf{k}i$. It shows that $\Gamma 1\,(\beta)$, which is discribed in term of $\rho\,(\mathbf{k}s,\ \mathbf{k}i)$, is the Fourier transform of the characteristic function $\gamma\,(\mathbf{x})$. Thus, by the three-dimensional spatial inverse Fourier transform of (12),

$$\gamma(x) = (2\pi)^{-3} \int_{-\infty}^{\infty} \Gamma 1(\beta)\, e^{js\cdot x} dx. \tag{13}$$

From other component of (8), ie., ki,ei component, we can obtain similar relation.

In order to recover $\gamma(x)$ from (13), we must know the value of $\Gamma 1(\beta)$ in the whole β space. Bojarski and Lewis recovered the β space by means of the knowledge of the backscattered field for all frequencies at all aspects of the target . By their procedure, the low frequency scattering informations are included even though in that range the physical optics approximation is not valid. The reason why the good result obtained is due to the fact that the physical optics inverse scattering identity (13) is heavily weighted by the reciprocal of the square of the frequency in favor of the low frequency. But if the target shape is more complicated, the range that the physical optics approximation is not valid become more wide. Such effect may not arise in this case, and good results may not be obtained by Bojarski's method.

Now, we present a way to recover β space from a single frequency scattering data. When a harrmonic plane is transmitted at all aspects and the scattered field is obtained at all aspects, we can recover the β space within range of a sphere with radius 2k(see fig.1). Though the obtainable information is limited, this two things can be pointed out.

1) If the wavenumber of the transmitted wave is enough large, all of the obtained information hold the physical optics approximation.

2) The main contribution to the Fourier integral comes from the low frequency information of $\Gamma 1(\beta)$.

Then we can obtain $\gamma(x)$ by means of the limited band information of $\Gamma 1(\beta)$.

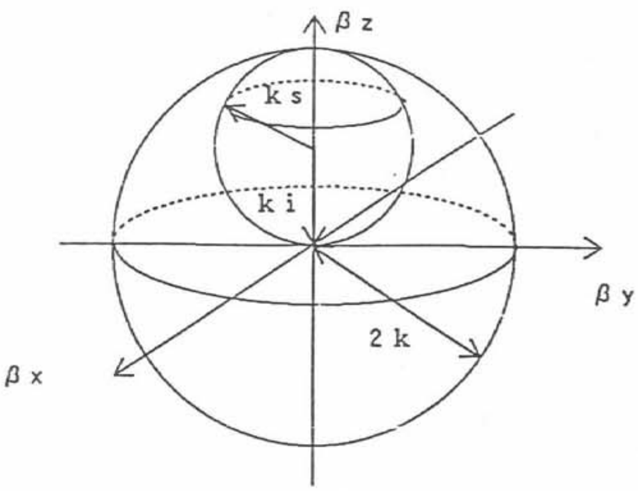

Fig.1. Volume of integration obtained from single frequency k, multistatic data.

3. Numerical expariment results.

Some numerical results are presented here in case of two dimensional scatterer or the perfectly conducting elliptic cylinder. The scattered fields are calculated by the equivalent source method. In the presentation of the results the following notation are used: major axis of elliptic = a, minor axis = b, incident wavenumber = k.

Fig.2 showns the $\gamma(x)$ of a elliptic cylinder of a=1.0, b=0.5 with k=10.0 constructed according to (13). In this case, The result can be considered as very satisfactry. But when the same wavenumber k=10.0 is transmitted to the a=1.0, b=0.2 elliptic cylinder, we couldn't obtain a recognizable reconstruction(fig.3). This may be come from the physical optics approximation is not valid for this complicated shape with K=10.0. But if we choose larger k, the physical optics approximation become valid. Fig.4 shows γ (**x**) of the same elliptic cylinder as fig.3 with k=30. In this case, as expected, a reasonable reconstruction is obtained.

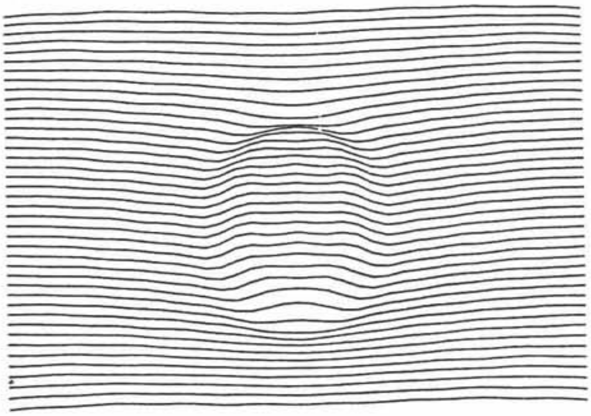

Fig.1. characteristic function of reconstructed a=1.0, b=0.5 elliptic cylinder with k=10.0.

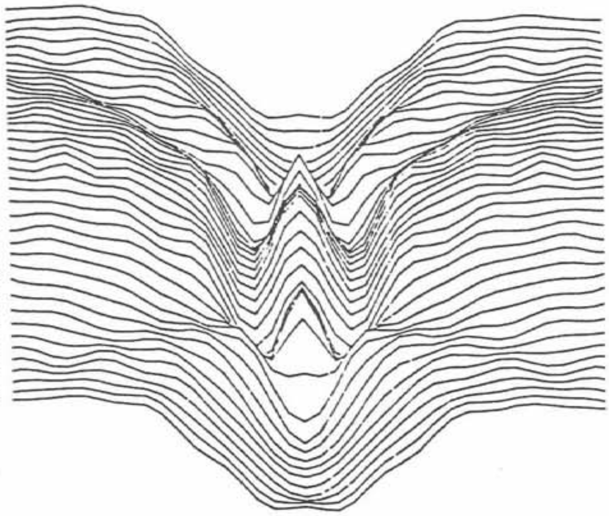

Fig.2. characteristic function of reconstructed a=1.0, b=0.2 elliptic cylinder with k=10.0.

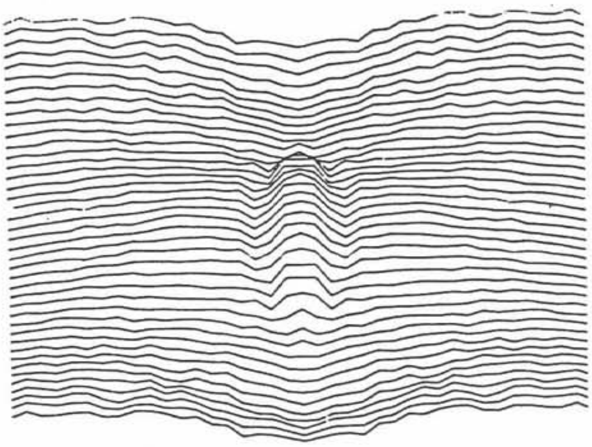

Fig.3. characteristic function of reconstructed a=1.0, b=0.2 elliptic cylinder with k=30.0.

4. conclusion

We have extended the Bojarski's physical optics scattering identity for single frequency measurements case. And numerical experiments have confirmed the validity of this method.

reference

[1] R.M.Lewis, "Physical Optics Inverse Diffraction," IEEE Trans. Antennas Propagat., vol.AP-17, pp.308-314, May 1969.

[2] N.N.Bojarski, "A Survey of the Physical Inverse Scattering Identity," IEEE Trans. Antennas Propagat., vol.AP-30, pp.980-989, Sept 1892.

[3] S.Rosenbaum-Raz, "On Scaqtterer Reconstruction from Far-Field Data," IEEE Trans. Antennas Propagat., vol.AP-24, pp.66-70, Jan 1976.