

Method of Moment for Local Correction of Physical Optics  
to eliminate Fictitious Penetrating Rays

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### 1. Introduction

Diffraction analyses for electromagnetic waves are generally reduced to the problems to derive secondary sources on the scatterers. In low frequency, it's well known that Method of Moment (MoM) is effective and has high accuracy. In HF, when you get the current distribution, the size of the matrix becomes larger, which makes the computational load  $O(N^3)$ . When we deal with high frequency problems, we can use locality to approximate currents. Typical example of this technique is Physical Optics (PO). When using PO, we determine the currents at the point of interest by assuming the tangential infinite plane and independent from macroscopical shape of the scatterer. Because the current distribution does not be derived but given, the order of the computational load is described as  $O(N^1)$ . The accuracy of PO current is high except the edges, corners, and surfaces with small curvature. There are two error factors in PO. One is the error of the diffracted wave, and the other is the fictitious penetrating rays, which is penetrated EM fields in the region where you cannot see the source directly. The motivation of this study is to dissolve these errors. In the region where PO current has good accuracy, by substituting the unknown currents for PO currents, we can reduce the computational load without losing the valid accuracy.

The hybrid method like this is proposed before. But these had some problems in past research: definition of the switching point between HF technique and numerical solution was obscure, and the region where they put PO currents didn't adequate. We solve the problem by considering the character of PO error and Locality phenomena in HF. We optimize the position of switching point, keeping the valid accuracy and using the Locality phenomena.

We show the indicator of the switching point for curvature radius. We check the accuracy of both the current distribution on the scatterer and radiation pattern. We estimate the computational time and conform the effectiveness of hybrid method.

### 2. Locality phenomenon in HF

We consider the scattering problem that EM fields from the electric line current is diffracted by 2 dimensional strip. We indicate the locality phenomena in HF by changing the width of the strip to see the behavior of the current around the edge. We show the current distribution on the scatterer when changing  $L$ , the width of the strip. For  $L \leq 0.5\lambda$ , we can see the mutual coupling of both edges. For  $L \geq 1.0\lambda$ , the current distribution near the left edge doesn't change for all width. In HF, the perturbation near edges, corners, and so on is independent from the scatterer structure.

For this model, when you calculate it by MoM in  $L \leq 0.5\lambda$  where the perturbation exists and by PO in elsewhere, you can reduce the computational time having the valid accuracy. Since the unknown currents are assigned not the entire but only the critical regions, computational load is not increasing so fast with the frequency

### 3. Hybrid Method Formulation

In MoM, we divide the scatterer into many pieces and assume the unknown currents on each of them. Then we set up the matrix equation as regard to it and calculate the inverse matrix of this equation for the unknown currents.

$$[Z_{NN}][I_N] = [V_N] \Rightarrow [I_N] = [Z_{NN}]^{-1}[V_N] \quad (1)$$

The size of the matrix in the equation (1) is  $N \times N$ . The unknown currents in the equation (1) are

substituted by the PO currents, that is  $2\mathbf{n} \times \mathbf{H}$ , only in the areas where PO currents are reliable. If  $M (< N)$  unknown currents out of total  $N$  are replaced by PO currents, we can get equation (2).

$$[Z_{N,N-M}][I_{N-M}] = \left[ \begin{array}{c} V_1 - \sum_i z_{1i} I_i^{PO} \\ \vdots \\ V_N - \sum_i z_{Ni} I_i^{PO} \end{array} \right] \quad N \quad (2)$$

These are  $N$  equations for  $(N-M)$  of unknowns. Mathematically, we should approximately satisfy this by least-square method. But we have calculation errors when we use this method. The reason for this is as follows.  $z_{mn}$  can be written like this:

$$z_{mn} = - \int_{-w/2}^{w/2} H_0^{(2)}(\beta|\rho_m - \rho_n|) dx_s \propto \frac{1}{\sqrt{(x_m - x_n)^2 + \left(\frac{t}{2}\right)^2}} \quad (t \ll \rho_m, \rho_n) \quad (3)$$

When we consider as  $m \approx n$ , which means that the equivalent source and the point of observation on the scatterer is close each other, the value of  $z_{mn}$  becomes very large. If we calculate this equation in the whole part of the scatterer, or both the area for PO currents and that for the unknown currents, the change in the magnitude of  $z_{mn}$  is very large, which causes serious numerical errors. We solve  $(N-M)$ 's equations only in the area for  $(N-M)$  unknown currents.

$$[Z_{N-M,N-M}][I_{N-M}] = \left[ \begin{array}{c} V_1 - \sum_i z_{1i} I_i^{PO} \\ \vdots \\ V_N - \sum_i z_{Ni} I_i^{PO} \end{array} \right] \quad N-M \quad (4)$$

The size of the matrix is reduced from  $(N \times N)$  to  $(N-M) \times (N-M)$ , which enable us to ease the calculation load. As the frequency becomes higher, the critical region is much smaller than the whole area and  $N-M \ll N$ , the reduction is significant.

#### 4. Numerical results and discussions

We show the relationship of the switching point and the errors in hybrid method. When the switching point is put at  $0.5\lambda$ . We can see great difference between PO's result and MoM's especially in the shadow region. Good agreement between MoM's result and the hybrid's is seen for all the observation angles. We show the result when you choose the curved surface. For  $r=0$  the scatterer is identified to the corner scatterer. Here we consider the expanded MoM's region from the curved or bended structure to get the valid solution. The important point to evaluate only the influence caused by the curvature is as follows. PO currents have good accuracy. Influence of the scattering wave can be neglected. The amplitude of the fictitious penetrating rays doesn't change greatly.

We investigate the switching point which gives us the accuracy about -40dB. The vertical axis shows the maximum value of radiation pattern difference between the hybrid method and MoM in shadow region, which is normalized by the incident wave.

$$Error \triangleq 20 \log \left( \left| \frac{E^{HYBRID} - E^{MoM}}{E^{in}} \right| \right) \quad (5)$$

When the curvature ratio is set as  $0.25\lambda$ , we have to put the additional MoM's region by  $1\lambda$ . Then we check the accuracy of each method. The current distribution and the radiation pattern is the

result of the case when you choose the curvature ratio as  $0.5\lambda$ . We can see the continuity in current distribution, and also see the good agreement with the MoM's result.

Lastly, we investigate the computational time. The analysis model is chosen as 2 dimensional strip. For relatively large scatterer, the computational time is almost the same. When you deal with the very large scatterer, remarkable difference is seen. The result of PO is much smaller than the others. The results are close to the expectation from the analytical point of view. The result of hybrid method is proportional to  $L^2$ , not  $L^1$

## 5. Conclusions

The hybrid method of PO and MoM is proposed by bringing MoM in the locality in HF. We realized the computational reduction with valid accuracy. We check the relationship between radiation pattern error and the switching point for curved structure with small curvature radius.

The application to curved surfaces and three dimensional scatterers are important topics in the future.

## References

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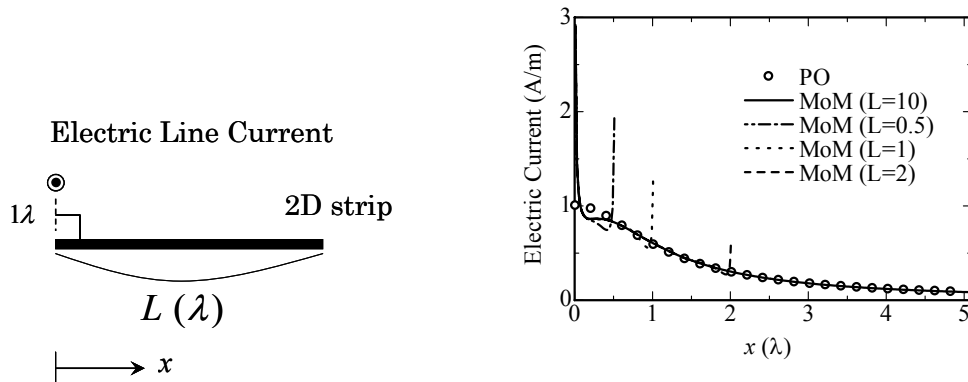


Fig.1 Locality in HF

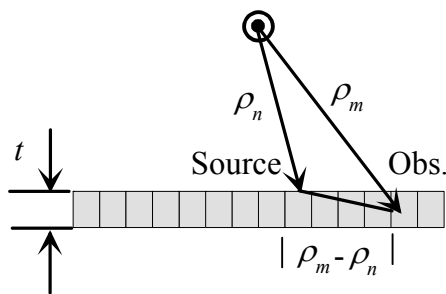


Fig.2 Analysis model in MoM

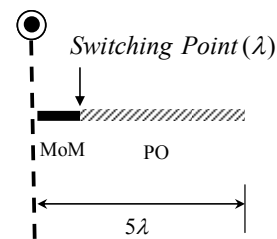


Fig.3 (a) Analysis model

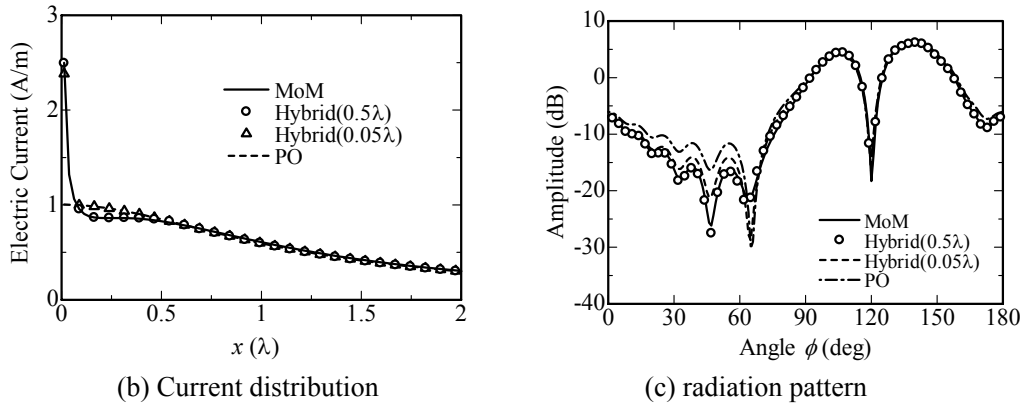


Fig.3 Accuracy check of 2-D strip

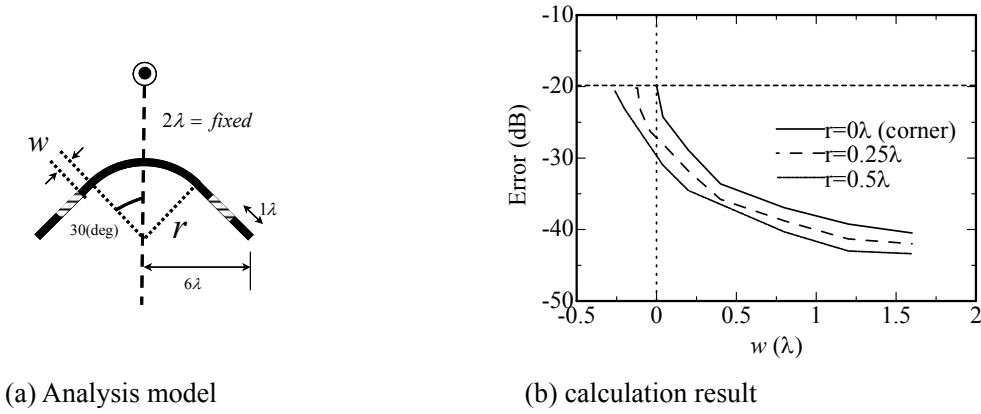


Fig.4 relationship between switching point and radiation errors

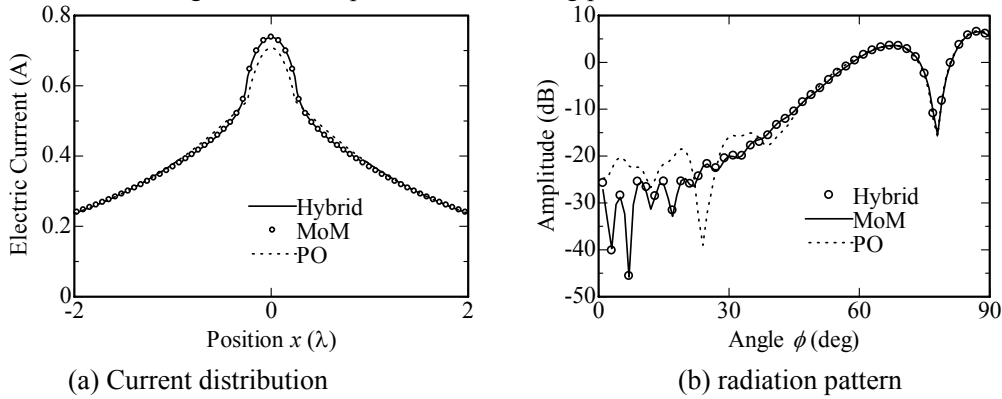


Fig.5 Accuracy check of curved structure (error:-40dB)

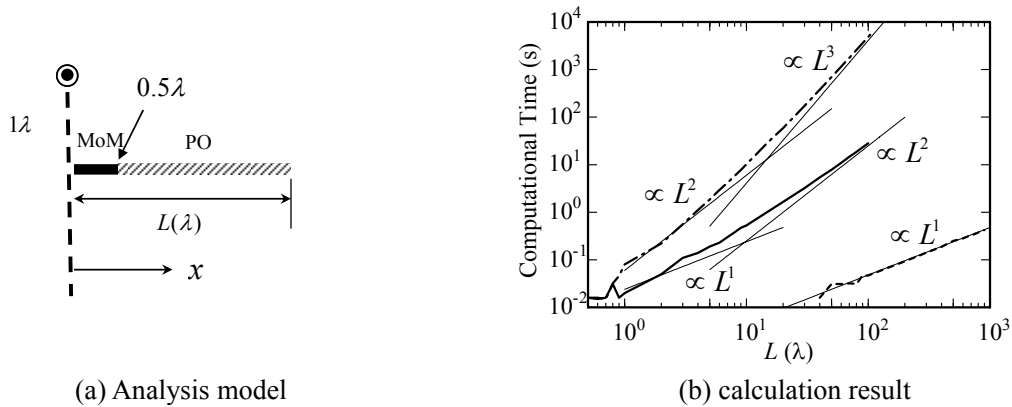


Fig.6 computational time