A NEW METHOD TO DETERMINE ROTATING SCATTERERS BY USE OF SOME VALUES OF HIGH FREQUENCIES RCS

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1. INTRODUCTION

According to Bojarski's identity, Lewis gave a computational formula to determine scatterers by use of all of $\rho(k)$ (κ from 0. to ∞) in order to determine rotating scatterers. But, because radar systems are restricted by band-limited frequencies, and the computational formula given by Lewis needs $\rho(k)$ on all of frequencies, it is impossible to use Lewis' conclusion. This paper will propose a new method to determine rotating scatterers by use of some values of high frequencies RCS.

2. SCATTERING CHARACTERS AND DETERMINING METHODS ON ROTATING SCATTERERS

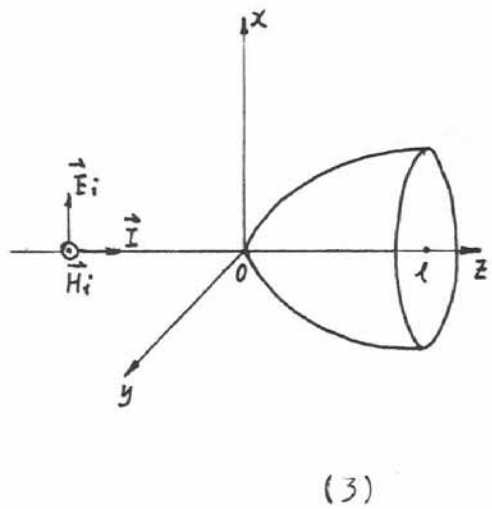
Suppose a scatterer B be a rotating body about z axis, whose surface be

$$x^2 + y^2 = f(z) \qquad (1)$$

where $f(z) \geqslant 0.$, $0 \leqslant z \leqslant 1$, $f(0)=0, f'(z) \geqslant 0$.

A time harmonic electromagnetic fields which propagate along z axis is

$$\vec{E}_i(t, \vec{x}) = \vec{E}_0 e^{-i\omega t} \qquad (2)$$



According to Lewis' paper, it is known that [1]

$$Q(z) = \frac{2}{\sqrt{\pi}} R_e \int_{-\infty}^{\infty} \frac{\rho(0,0,p_3)}{p_3^2} e^{ip_3 z} dp_3 \qquad (3)$$

where $Q(z)$ presents the area of the scatterer at z, that is $\pi f(z)$.

According to formula (3) it is known that all of $\rho(0,0,p_3)$ must be abtained in order to determine B. But, because radars are restricted by band-limited frequencies, the method proposed by Lewis can not be employed in order to determine B. So, it is important to find out the other method.

Bojarski's identity can be changed into [2]

$$\rho(\vec{p}) + \rho^*(-\vec{p}) = \frac{|\vec{p}|^2}{2\sqrt{\pi}} \int_B e^{-i\vec{p}\cdot\vec{x}} d\vec{x} \tag{4}$$

and

$$\rho(-\vec{p}) = \frac{-i}{2\sqrt{\pi}} \int_{\vec{p}\cdot\vec{n}<0} \vec{p}\cdot\vec{n}\, e^{i\vec{p}\cdot\vec{x}}\, dS(\vec{x}) \tag{5}$$

when $\vec{p}\cdot\vec{x} < 0$, the surface projected on B is $x^2 + y^2 = f(1)$ and $z=1$, then

$$\rho(-\vec{p}) = ik\sqrt{\pi}\, f(l)\, e^{2kil} \tag{6}$$

On the other hand

$$\int_B e^{-i\vec{p}\cdot\vec{x}} d\vec{x} = \int_0^l e^{2ikz} dz \int_B dx\,dy = \int_0^l \pi f(z) e^{2ikz} dz \tag{7}$$

Let $\rho(\vec{p}) = \rho(k)$, formula (4) can be simplified into

$$\rho(k) - ik\sqrt{\pi}\, f(l)\, e^{2ikl} = 2k^2\sqrt{\pi} \int_0^l f(z)\, e^{2ikz} dz$$

that is

$$\rho(k) = ik\sqrt{\pi} \int_0^l f'(z)\, e^{2ikz} dz \tag{8}$$

If f(z) is n+1 times differentiable, and $|f^{(n+1)}(z)| \leq M$, then

$$\rho(k) = -ik\sqrt{\pi} \sum_{j=1}^{n} \frac{f^{(j)}(l)e^{2ikl} - f^{(j)}(0)}{(-2ik)^j} + R_N(k) \tag{9}$$

where

$$R_N(k) = \frac{ik\sqrt{\pi}\,(-1)^n}{(2ik)^n} \cdot \int_0^l f^{(n+1)}(z)\, e^{2ikz} dz \tag{10}$$

and

$$|R_N(k)| \leqslant \frac{\sqrt{\pi}\, M l}{2(2k)^{n-1}} \tag{11}$$

In general, k is greater than 100, then it is feasible to let n=4. Then

$$\rho(k) = \frac{\sqrt{\pi}}{2} \Big\{ e^{2ikl} \Big[f'(l) - \frac{f''(l)}{2ki} - \frac{f'''(l)}{4k^2} - \frac{f^{(4)}(l)}{8k^3 i} \Big] -$$

$$-\left[f'(0)-\frac{f''(0)}{2ki}-\frac{f'''(0)}{4k^2}-\frac{f^{(4)}(0)}{8k^3 i}\right]\Bigg\} \tag{12a}$$

RCS is [3]

$$\sigma(k)=|\rho(k)|^2 \tag{13a}$$

when $k\to\infty$, the above formulas become

$$\rho(k)=\sqrt{\pi}/2\,[f'(l)e^{2ikl}-f'(0)] \tag{12b}$$

$$\sigma(k)=\pi/4\,[(f'(l))^2+(f'(0))^2-2f'(l)f'(0)\cos 2kl] \tag{13b}$$

The formula (13b) demonstrates that when $k\to\infty$, $\sigma(k)$ is a periodic vibrating function whose period T is $\frac{\pi}{2l}$, whose amplitude is $\frac{\pi}{2}f'(l)f'(0)$, whose vibrating center is $\frac{\pi}{4}[(f'(l))^2+(f'(0))^2]$. l can be determined by the period T.

According to the formula (12a), let k take four different values, it can construct four complex linear equations or eight real linear equations and $f^{(j)}(l)$, $f^{(j)}(0)$ (j=1,2,3,4) can be determined. Then, suppose $f'(z)$ be a seventh order polynomial

$$f'(z)=a_0+a_1z+a_2z^2+\cdots+a_7z^7 \tag{14}$$

then

$$a_0=f'(0),\ a_1=f''(0),\ a_2=\tfrac{1}{2}f'''(0),\ a_3=\tfrac{1}{6}f^{(4)}(0)$$

others a_j (j=4,5,6,7) can be determined from other four conditions. Then

$$f(z)=\int_0^z f'(z)dz \tag{15}$$

3. DETERMINING METHOD ON THREE SPECIAL SCATTERERS

(A) Rotating paraboloids. Suppose

$f(z) = cz \qquad 0\leq z\leq l$, then

$f'(z) = c,\ f^{(j)}(z) = 0 \qquad j\geq 2$

$\rho(k) = \frac{1}{2}\sqrt{\pi}\,c(e^{2ikl}-1)$

$\sigma(k) = \pi c^2\sin^2 kl$

$\rho(k)$ and $\sigma(k)$ are undamped vibrating periodic functions.

(B) Cones. Suppose

$f(z) = cz^2 \qquad 0\leq z\leq l$, then

$f'(z) = 2cz,\ f''(z) = 2c,\ f^{(j)}(z) = 0 \quad j\geq 3$

$\rho(k) = \sqrt{\pi}\,c\,[e^{2ikl}(l-\frac{1}{2ki})+\frac{1}{2ki}]$

$$\sigma(k)=\pi c^2\left[l^2+\frac{1}{2k^2}+\frac{1}{k}\left(l\sin 2kl+\frac{\cos 2kl}{2k}\right)\right]$$

$\rho(k)$ and $\sigma(k)$ are damped vibrating periodic functions, and vibrating center is variable.

(C) Rotating ellipsoids. Suppose

$f(z) = a^2-(a/b)^2(z-b)^2 \quad 0 \leq z \leq b$, then

$f'(z) =-2(a/b)^2(z-b)$, $f''(z)=-2(a/b)^2$, $f^{(j)}(z)=0 \quad j \geq 3$

$$\rho(k)=\frac{\sqrt{\pi}}{2}\left[\frac{a^2}{ikb^2}(e^{2ikb}-1)-2\frac{a^2}{b}\right]$$

$$\sigma(k)=\pi\left(\frac{a^2}{b}\right)^2\left[1-\frac{\sin 2kb}{kb}+\frac{1-\cos 2kb}{2(kb)^2}\right]$$

$\rho(k)$ and $\sigma(k)$ are damped vibrating periodic functions. But, the vibrating center is not variable.

The above three scatterers have their different scattering characters. It is easy to distinguish them from $\rho(k)$ or $\sigma(k)$ and determine the parameters of the scatterers.

4. CONCLUSION

This paper discussed how to determine rotating scatterers by use of their scattering far-fields of some high frequencies and deduced the relations between the scatterers and $\rho(k)$. and propose a method to determine rotating scatterers. And the concrete method is given in order to determine rotating paraboloids, cones and rotating ellipsoids.

REFERENCES

[1] R. Lewis, IRE Trans. AP-17, 1969, pp308-314.

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[3] M.I. Skolnik, Introduction to radar systems, McGraw-Hill, N.Y., 1969.