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MODELING OF FREQUENCY CORRELATION CHARACTERISTICS FOR LINE-OF-SIGHT MOBILE RADIO SYSTEMS

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1. INTRODUCTION

In digital mobile-radio communication systems, to understand the effect of frequency selective fading is very important for assessment of digital transmission characteristics in terms of delay profile, delay spread and correlation bandwidth (or coherent bandwidth). Moreover, there is a trend toward smaller service areas for each nodal radio station, (namely, micro-cellular systems) to cope with the effective utilization of limited frequencies. These kinds of systems will be available in the near future in urban areas, underground towns, company premises and indoors. In such an environment, it is expected that Nakagami-Rice fading conditions, where various scattered waves interfere with the line-of-sight wave, will be more common. However, a practical model treating correlation characteristics useful for the analysis of digital transmission under Nakagami-Rice fading conditions has not yet available. Considering that Rayleigh fading is one of the extremes of Nakagami-Rice fading, development of the Nakagami-Rice fading model is equivalent to generalization of the existing Rayleigh fading model [1].

In this paper, with the help of the theoretical method developed by Clarke on his analysis of correlation characteristics for Rayleigh fading [1], we present a general model for such correlation statistics under Nakagami-Rice fading conditions which include Rayleigh fading. Using this model, we then analyze the frequency correlation characteristics in the typical line-of-sight environments.

2. A FORMULA FOR CORRELATION OF AMPLITUDE VARIATIONS

2.1 Definitions and Assumptions

In Appendix B of Reference [1], Clarke theoretically examined the relation among the correlation of complex fields and those of amplitude and squared magnitude of received signals under Rayleigh fading conditions. Here we will extend his logic to cope with the analysis of Nakagami-Rice fading.

First, we give some definitions and relations of the terms and symbols used here (Definition: $\equiv$, Relation: $=$).

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x: "Variable" representing time (t), frequency (f) and spatial location (r)

$\dot{E}_D(x)$: Direct wave component; $\dot{E}_R(x)$: Scattered wave component

$\dot{E}_{R0}(x)$, $\dot{a}_1$ and $\dot{a}_2$: Normalized scattered wave component defined by:

$$\dot{E}_{R0}(x) \equiv \dot{E}_R(x)/\dot{E}_D(x); \quad \dot{a}_1 \equiv \dot{E}_{R0}(x); \quad \dot{a}_2 \equiv \dot{E}_{R0}(x+\Delta x) \tag{1}$$

P_R: Mean power of scattered waves; $P_R = \langle \dot{a}_1^* \dot{a}_1 \rangle = \langle \dot{a}_2^* \dot{a}_2 \rangle \equiv 2\sigma^2$ (2)

$\dot{A}_1$ and $\dot{A}_2$: Normalized signal amplitude at x and x+Δx

$$\dot{A}_1 \equiv 1+\dot{a}_1; \; \dot{A}_2 \equiv 1+\dot{a}_2; \; A_1 \equiv |\dot{A}_1|; \; A_2 \equiv |\dot{A}_2|; \; \langle A_1^2 \rangle = \langle A_2^2 \rangle \equiv \langle A^2 \rangle \tag{3}$$

$\dot{R}_a$: Complex covariance of normalized scattered waves ($\dot{a}_1$ and $\dot{a}_2$)

$$\dot{R}_a(\Delta x) = \langle \dot{a}_1^* \dot{a}_2 \rangle \tag{4}$$

$\dot{\rho}_a$: Complex correlation coefficient of normalized scattered waves

$$\dot{\rho}_a(\Delta x) \equiv \langle \dot{a}_1^* \dot{a}_2 \rangle / \sqrt{\langle \dot{a}_1^* \dot{a}_1 \rangle \langle \dot{a}_2^* \dot{a}_2 \rangle} \tag{5}$$

$\rho_{\delta A}$: Correlation coefficient of amplitudes (A_1 and A_2)

$$\rho_{\delta A}(\Delta x) \equiv (<A_1A_2>-<A_1><A_2>)/\sqrt{(<A_1^2>-<A_1>^2)(<A_2^2>-<A_2>^2)} \tag{6}$$

$\rho_{\delta A2}$: Correlation coefficient of squared magnitudes (A_1^2 and A_2^2)

$$\rho_{\delta A2}(\Delta x) \equiv \frac{\langle A_1{}^2A_2{}^2\rangle-\langle A_1{}^2\rangle\langle A_2{}^2\rangle}{\sqrt{\langle A_1{}^4\rangle-\langle A_1{}^2\rangle^2}\sqrt{\langle A_2{}^4\rangle-\langle A_2{}^2\rangle^2}} = \frac{\langle A_1{}^2A_2{}^2\rangle-\langle A^2\rangle^2}{\langle A^4\rangle-\langle A^2\rangle^2} \tag{7}$$

2.2 Relation between $\rho_{\delta A}$ and $\rho_{\delta A2}$

First we consider the mutual correlation of amplitude "$\rho_{\delta A}$". From the definition of $\rho_{\delta A}$ given by Eq. (6), the derivation of covariance between A_1 and A_2 (namely, $<A_1A_2>$) seems to require the most complicated manipulation among the terms in the equation. The probability density function (PDF) of the two-dimensional Nakagami-Rice distribution with a complex correlation coefficient is given by:

$$p(A_1,A_2,\phi_1,\phi_2;\dot{\rho}_a) = \frac{A_1A_2}{4\pi^2\sigma^4(1-\rho^2)}\exp\left[-\frac{1}{2\sigma^2(1-\rho^2)}\left\{A_1{}^2+A_2{}^2-2A_1\cos\phi_1-2A_2\cos\phi_2+2 -2\rho\left[A_1A_2\cos(\theta+\phi_1-\phi_2)-A_1\cos(\phi_1+\theta)-A_2\cos(\phi_2-\theta)+\cos\theta\right]\right\}\right] \tag{8}$$

$$(\rho=|\dot{\rho}_a|;\ \theta=\arg[\dot{\rho}_a])$$

The covariance, $<A_1A_2>$, is therefore given by

$$\langle A_1A_2\rangle = \int_0^\infty\int_0^\infty\int_0^{2\pi}\int_0^{2\pi} p(A_1,A_2,\phi_1,\phi_2;\dot{\rho}_a)\,d\phi_1 d\phi_2 dA_1 dA_2 \tag{9}$$

The above expression may have a possibility of certain simplification, but here the actual calculation is carried out by means of numerical integrations while keeping sufficient accuracy.

Other terms in Eq.(6) such as $<A>$ and $<A^2>$ are easily obtainable; therefore, $\rho_{\delta A}$ can be determined.

Because of the complexity of calculation of $\rho_{\delta A}$ as can be noticed from the above equations, the amplitude correlation coefficient in terms of $\rho_{\delta A}$ in Nakagami-Rice fading conditions is not suitable for engineering applications. Therefore, we try to formulate the correlation coefficient of the squared magnitude $\rho_{\delta A2}$ which is known as a good approximation of $\rho_{\delta A}$ in Rayleigh fading conditions [1].

As for $\rho_{\delta A2}$ in Eq. (7), all terms in the equation can be obtained analytically after statistical manipulations. They are given by:

$$\langle A^2\rangle = \langle(1+\dot{a}_1)(1+\dot{a}_1{}^*)\rangle = 1+2\sigma^2 \tag{10-a}$$

$$\langle A^4\rangle = \langle(1+\dot{a}_1)^2(1+\dot{a}_1{}^*)^2\rangle = 1+8\sigma^2+8\sigma^4 \tag{10-b}$$

$$\langle A_1{}^2A_2{}^2\rangle = \langle(1+\dot{a}_1)(1+\dot{a}_1{}^*)(1+\dot{a}_2)(1+\dot{a}_2{}^*)\rangle = 1+4\sigma^2+4\sigma^4+\dot{R}_a+\dot{R}_a{}^*+\dot{R}_a\dot{R}_a{}^* \tag{10-c}$$

Finally, the following extremely simple relation is obtained.

$$\boxed{\rho_{\delta A2} = \frac{\mathrm{Real}[\dot{\rho}_a]+\sigma^2|\dot{\rho}_a|^2}{1+\sigma^2}} \tag{11}$$

Rayleigh fading conditions correspond to $2\sigma^2 >> 1$, while certain Nakagami-Rice fading conditions where reflected waves are comparatively weak correspond to $2\sigma^2 << 1$. For both extremes, Eq. (11) can be simplified to:

$$\rho_{\delta A2}=\begin{cases} |\dot{\rho}_a|^2 & (2\sigma^2>>1) \quad (12\text{-a}) \\ \mathrm{Real}[\dot{\rho}_a] & (2\sigma^2<<1) \quad (12\text{-b}) \end{cases}$$

In these cases, Eq.(12-a) is identical to the equation derived previously under Rayleigh fading conditions [1], and Eq.(12-b) is identical to the equation derived by the authors for multipath fading due to sea surface reflection in maritime mobile-satellite systems [2]. Eq.(11) may be regarded as a generalization of results which were so far found independently under two extreme conditions.

Then, we examine the relation between $\rho_{\delta A}$ and $\rho_{\delta A2}$. It has been theoretically shown by Clarke that $\rho_{\delta A}$ and $\rho_{\delta A2}$ are approximately equal under Rayleigh fading conditions (namely, $2\sigma^2>>1$) [1]. (Clarke's proof is however limited to the case where ρ_a is real, and it is sufficient in that case.). Moreover, it is obvious that $\rho_{\delta A}$ and $\rho_{\delta A2}$ are nearly the same for $2\sigma^2<<1$.

Based on our calculation for comparison between $\rho_{\delta A}$ and $\rho_{\delta A2}$ for three cases where P_R ($=2\sigma^2$) is 10 dB, 0 dB and -10 dB, the difference between $\rho_{\delta A}$ and $\rho_{\delta A2}$ is within 0.1 for almost all values of $\rho_{\delta A}$ ranging from 1.0 to -1.0. Therefore, we can conclude that the following relation holds even in the case of Nakagami-Rice fading conditions over the whole range of $2\sigma^2$;

$$\boxed{\rho_{\delta A}\approx\rho_{\delta A2}} \quad (13)$$

Based on the above result, we can therefore calculate the correlation characteristics using $\rho_{\delta A2}$ in place of $\rho_{\delta A}$.

3. FREQUENCY CORRELATION CHARACTERISTICS OF NAKAGAMI-RICE FADING

In this section, we will try to formulate the frequency correlation with respect to an arbitrary delay profile without losing generality. We will consider two cases as specific examples of its application. The respective delay profiles of two cases are given by the following expressions:

$$p(\tau) = \delta(\tau) + p_s(\tau) \quad (14)$$

where

$$p_s(\tau) = \begin{cases} \dfrac{P_R}{\sigma_\tau}\exp\left[-\dfrac{\tau}{\sigma_\tau}\right] & \text{(Case 1)} \quad (15\text{-a}) \\ \dfrac{P_R}{\sqrt{2\pi}\sigma_\tau}\exp\left[-\dfrac{(\tau-\tau_O)^2}{2\sigma_\tau^2}\right] & \text{(Case 2)} \quad (15\text{-b}) \end{cases}$$

where P_R is the averaged value of scattered-wave power relative to the direct wave power, τ_o is the average delay of scattered waves relative to the direct wave and σ_τ is standard deviation of each delay of scattered waves. In Eq. (14), the delta function (δ) represents the direct wave component and it has a fixed value (namely, unit impulse function). Case 1 may represent a line-of-sight mobile-radio environment in micro-cellular systems, while Case 2 represents conditions close to multipath fading due to sea surface scattering in maritime and aeronautical mobile-satellite systems. In both cases, Rayleigh fading as an extreme of Nakagami-Rice fading is included assuming that P_R is much larger than unity (=direct wave power).

We will now determine the frequency correlation characteristics for the two cases stated above.

Case 1: A direct wave + scattered waves having an exponential delay profile

Considering that frequency correlation function of scattered waves is given by the Fourier transformation of delay profile of scattered waves the complex correlation of scattered waves is given by:

$$\dot{\rho}_a(\Delta f) = \frac{\langle \dot{a}_1{}^* \dot{a}_2 \rangle}{\sqrt{\langle \dot{a}_1{}^* \dot{a}_1 \rangle \langle \dot{a}_2{}^* \dot{a}_2 \rangle}} = \frac{\int_0^{\infty} p_s(\tau) \exp[-j2\pi\Delta f\tau] d\tau}{2\sigma^2} = \frac{1-j2\pi\Delta f\sigma_\tau}{1+(2\pi\Delta f\sigma_\tau)^2} \tag{16}$$

In this case, the following relation is obtained:

$$\mathrm{Real}[\dot{\rho}_a] = |\dot{\rho}_a|^2 \tag{17}$$

Substituting Eq.(16) into Eq.(11), we obtain the relation:

$$\rho_{\delta A2} = |\dot{\rho}_a|^2 = \frac{1}{1+(2\pi\Delta f\sigma_\tau)^2} \tag{18}$$

which leads to the perhaps surprising conclusion that the frequency correlation coefficient does not depend on the ratio of scattered wave power to direct wave power, but is constant (exactly the same as the case of Rayleigh fading [1]).

Case 2: <u>A direct wave + scattered waves having a Gaussian delay profile</u>

Based on the same procedure as Case 1, we can obtain the following relation.

$$\dot{\rho}_a(\Delta f) = \exp[-2\sigma_\tau{}^2\pi^2\Delta f^2 + j2\pi\Delta f\tau_O] \tag{26}$$

$$\rho_{\delta A2} = \begin{cases} \{2\exp(-2\sigma_\tau^2\pi^2\Delta f^2)\cos(2\pi\Delta f\tau_O) + P_R\exp(-4\sigma_\tau^2\pi^2\Delta f^2)\} / (2 + P_R) & \text{(27-a)} \\ \exp[-4\sigma_\tau{}^2\pi^2\Delta f^2] \qquad (2\sigma^2 >> 1) & \text{(27-b)} \\ \exp[-2\sigma_\tau{}^2\pi^2\Delta f^2]\cos(2\pi\Delta f\tau_O) \qquad (2\sigma^2 << 1) & \text{(27-c)} \end{cases}$$

Frequency correlation characteristics will be key information for designing systems with fading reduction functions such as frequency diversity or an equalizer. Moreover, they are also very important for assessing digital transmission quality in terms of BER under frequency-selective fading conditions, and for examining CDMA performance having a path-diversity function.

4. CONCLUSIONS

First, theoretical analysis of envelope correlation for the Nakagami-Rice fading was carried out, and a simple but general formula was obtained. Then, by using the formula, we analyzed the frequency correlation characteristics assuming two typical cases. Since this model has considerably wide engineering applications, correlation analysis of amplitude variations encountered in environments ranging from terrestrial mobile-radio systems to maritime and aeronautical mobile-satellite systems can be done.

Since the model developed here is applicable not only to frequency correlation analysis but also to spatial correlation analysis, the latter analysis is expected to be carried out as a next step.

REFERENCES

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