

IMAGE RECONSTRUCTION FROM FREQUENCY-OFFSET FOURIER DATA OF OBJECT WITH FRACTAL STRUCTURE

Tah-Hsiung Chu and Cheng-Cheh Yu

Electrical Engineering Department
National Taiwan University
Taipei, Taiwan, Republic of China

I. Introduction

In radar imaging system or optical holography, the scattering object under coherent illumination is usually modeled as a complex spatial function to represent the object surface reflectivity. Its magnitude is proportional to the reflected energy from the object, whereas the phase is related to the electrical and geometrical properties of the object surface. It was known that the object scattered far field is the two-dimensional Fourier transformation of the object reflectivity. Therefore image of the scattering object (defined as the magnitude value of the object reflectivity) can be reconstructed from the recorded Fourier data through a two-dimensional Fourier inversion then taking the absolute value of the result.

In a practical coherent imaging system, one can only gather frequency-offset Fourier data of the scattering object due to the limitation of bandwidth and viewing angle used [1,2]. The reconstructed image then becomes distorted. However, unlike the noncoherent imaging system such as the X-ray tomography, the reconstructed image from these incomplete Fourier data is shown with good quality, for example, the reconstructed terrain map in spotlight synthetic aperture radar system. This is mainly due to the "randomness" of the scattering object phase value caused by the "roughness" of the object surface. The Fourier information of the scattering object then "uniformly" distributed over the entire Fourier space rather than concentrated to the origin of the Fourier space as in the case of a noncoherent imaging system. Therefore "good quality" image can be reconstructed from a finite area of frequency-offset Fourier data in a coherent imaging system.

Fractal geometry, introduced by Mandelbrot [3], has been used in computer graphics to model surfaces or scenes that closely resemble the natural objects such as mountains, coasts and fields. In this paper we will present studies of the effect of the surface "roughness" to the reconstructed image quality by using fractal geometry with different fractal dimension to simulate the phase value of the scattering object. Discussion of the approach to reduce the speckle noise encountered in the reconstructed image is also given.

II. Computer Simulation

In this section, numerical simulation of image reconstruction from frequency-offset Fourier data is described.

The scattering object reflectivity is denoted as

$$O(x,y) = A(x,y)e^{j\phi(x,y)} \quad (1)$$

where $A(x,y)$ and $\phi(x,y)$ are the magnitude and phase. Its Fourier data are recorded in polar format as

$$F(f, \theta) = \iint A(x,y) e^{j\phi(x,y)} e^{-j2\pi f(x\cos\theta + y\sin\theta)} dx dy \quad (2)$$

in which θ is the observation angle and radial distance f represents the frequency used.

In this study, only an annular portion of Fourier data is available (i.e., $f_1 < f < f_2$ as shown in Fig.1) for image reconstruction. The object phase $\phi(x,y)$ is calculated by

$$\phi(x,y) = \phi(x)\phi(y) \quad (3)$$

where $\phi(x)$ and $\phi(y)$ are the bandlimited Weierstrass function given as [3]

$$\phi(x) = \frac{\eta}{[1-b^{(2D-4)}]^{1/2}} \sum_{n=N_1}^{N_2} b^{(D-2)n} \cos(2\pi b^n x + \varphi_n) \quad (4)$$

where η is a constant determined from the variance of ϕ , b is a positive real number representing the fundamental tone, D is the fractal dimension ($1 < D < 2$), φ is a random phase term, and $N=N_2-N_1+1$ is the number of tones used.

It was known that the bandlimited Weierstrass function $\phi(x)$ can be represented as a physical fractal since it possesses a very large derivative and is self similar over scale lengths ranging from $1/b^{N_1}$ to $1/b^{N_2}$. Figures 2(a) and (b) are plots of $\phi(x)$ with fractal dimension $D=1.01$ and $D=1.99$, and 40 tones are used for calculation. The simulated object phase is shown smooth as $D=1.01$, and is close to "randomness" as $D=1.99$.

The object magnitude $A(x,y)$ used in the simulation is a 9×9 square with unity in a 64×64 frame as shown in Fig.3. Two different fractal phases $\phi(x,y)$ calculated from Weierstrass function given in eq.(4) with fractal dimension $D=1.01$ and $D=1.99$ are used to simulate the scattering object. Frequency-offset Fourier data are then calculated according to eq.(2) for 100 views over 360° and $11 < f < 32$. According to the slice-projection theorem [4], each radial line data in the Fourier space gives a projection image of the object, therefore the reconstructed image can be calculated using the backprojection algorithm [4] instead of two-dimensional Fourier inversion. However one should note the projection images obtained in this study are complex valued data due to the original complex object reflectivity. Therefore the summation operation of those projection images used in the backprojection procedure is a coherent superposition (containing real and imaginary values) instead of noncoherent superposition (containing only real values) used in X-ray tomography.

Figures 4(b) and (c) are results of the reconstructed images of the simulated objects with fractal phase $\phi(x,y)$ using $D=1.01$ and 1.99 . As comparing with the reconstructed image of a real valued object given in Fig.4(a), image shown in Fig.4(c) gives the closest resemblance to the original object image. Due to only the frequency-offset Fourier data available, image given in Fig.4(a) shows only the edge information of the scattering object. As fractal dimension becomes larger, the phase $\phi(x,y)$ becomes more "random" and seems to translate the illuminated energy to the entire Fourier space. Therefore "good quality" image can be reconstructed from frequency-offset Fourier data as the fractal dimension of the scattering object is large.

The reconstructed image shown in Fig.4(b) contains speckle noise arising from the random version of the destructive and constructive interference caused by fractal phase. Figure 4(d) is the result obtained by using the noncoherent superposition of backprojected images to reduce the effect of speckle noise by averaging, but the image background becomes more emergent and some image detail may be lost. Further development to reduce the speckle noise is under study.

References

- [1] J.L.Walker, "Range-Doppler imaging of rotating objects", IEEE trans. Aerosp. Electron. Syst. vol.AES-16, pp.23-52, Jan. 1980.
- [2] D.C.Munson, Jr, and J.L.C.Sanz, "Image reconstruction from frequency-offset Fourier data, Proc. of IEEE, vol.72, pp.661-669, June 1984.
- [3] B.B.Mandelbrot, The Fractal Geometry of Nature, Freeman, 1983.
- [4] G.E.Herman ed., Image Reconstruction from Projections-Implementation and Applications, Springer-Verlag, 1979.

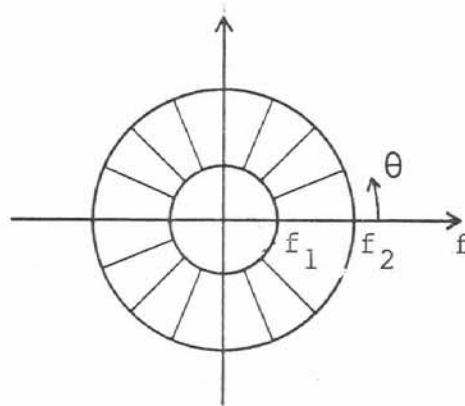


Fig.1 Polar format of the recorded frequency-offset Fourier data.

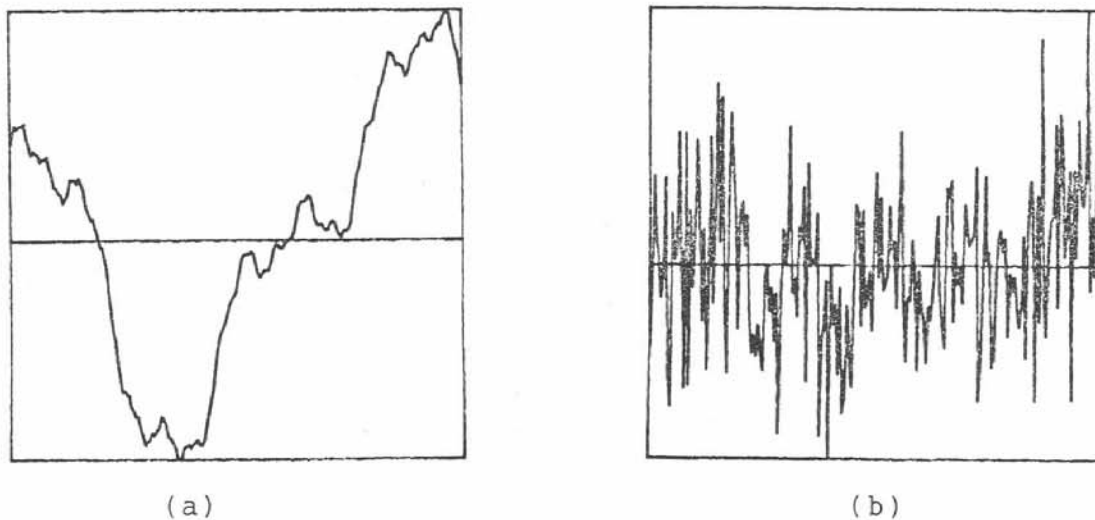


Fig.2 Plots of fractal phase with (a) $D=1.01$ and (b) $D=1.99$.

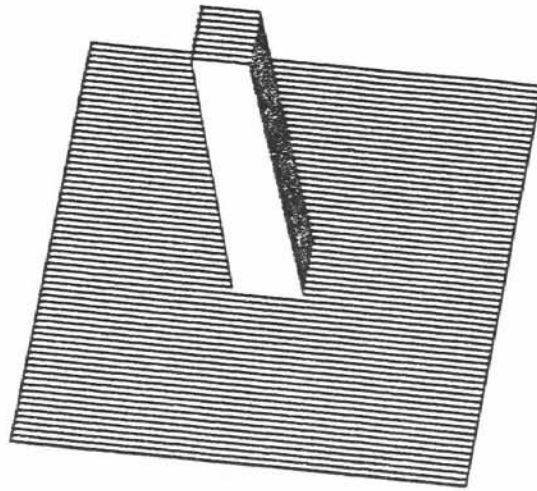


Fig.3 Original object image (only magnitude shown).

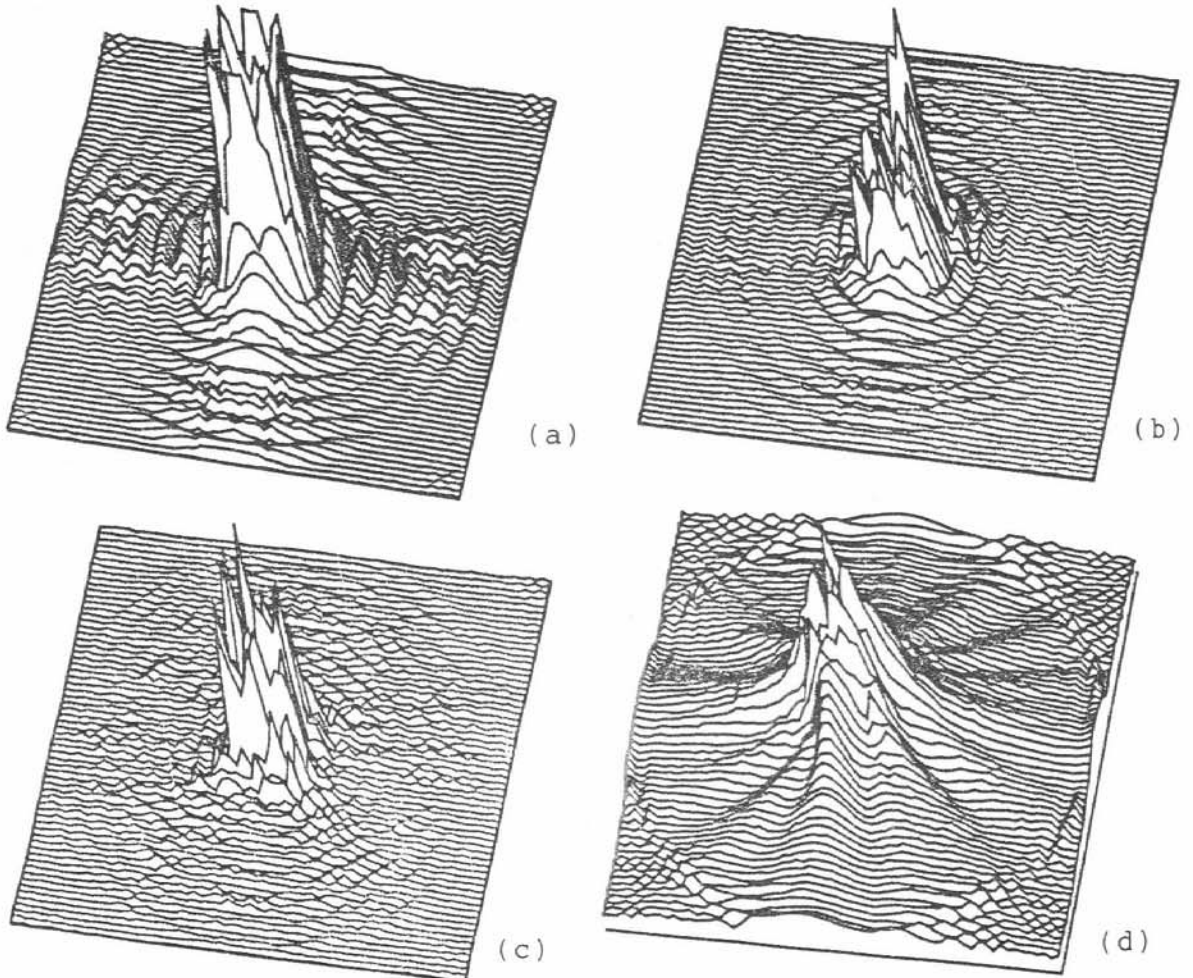


Fig.4 Reconstructed images of (a) real-valued object (b) complex-valued object with fractal phase with $D=1.01$ (c) complex-valued object with fractal phase with $D=1.99$ using coherent backprojection algorithm and (d) complex-valued object with fractal phase with $D=1.99$ using noncoherent backprojection algorithm.