

THE REMOTE SENSING OF ORIENTED VOLUME SCATTERING USING  
POLARIMETRIC RADAR INTERFEROMETRY

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1. INTRODUCTION AND BACKGROUND

Scattering polarimetry based on the Mueller or coherency matrix is sensitive to the shape, orientation and dielectric constant variations inside a volume [1]. However, it is insensitive to the spatial distribution of scatterers and so polarimetry alone is unable to extract full information about volume scattering. Such a situation arises in radar remote sensing of the Earth's land surfaces, where vegetation cover generates anisotropic volume scattering. Hence in order to determine the physical structure of vegetation a new type of sensor is required.

Radar Interferometry is sensitive to the position of a scattering element through the phase difference between signals at either end of a baseline [2]. Also of importance is the interferometric coherence, a measure of the local fluctuations in phase based on the correlation between the two signals. This coherence is related to the spatial extent of the scatterer [3].

Polarimetric Interferometry [4] coherently combines these techniques to provide a new type of sensor that is sensitive both to the vertical distribution and anisotropy of the scattering elements. In Polarimetric Interferometry, two measurement positions 1 and 2 are separated by an effective normal baseline  $B_n$ . Radar measurements of the  $2 \times 2$  coherent scattering matrix [S] are made for wavelength  $\lambda$  at each position in the range/cross range co-ordinate system. By transforming the data into the surface co-ordinates and overlapping the data in the spectral domain (through a process called range filtering [1]) the sensor can be used to locate the  $z$  co-ordinate of a scattering point. To study decorrelation in the 'z' direction, we then define an effective propagation constant from the range  $R$  and angle of incidence  $\theta_0$  as

$$k_z = \frac{4\pi B_n}{\lambda R \sin \theta_0} \quad - 1)$$

If there is a vertical distribution of scattering elements (a forest volume for example) then the resultant radar return is a coherent average over the volume. The polarimetric and interferometric information is then contained in a  $6 \times 6$  hermitian matrix [P] defined from an average of the outer product of scattering vectors  $\underline{k}_1$  and  $\underline{k}_2$  as

$$[P] = \left\langle \begin{bmatrix} \underline{k}_1 \\ \underline{k}_2 \end{bmatrix} \begin{bmatrix} \underline{k}_1^{*T} & \underline{k}_2^{*T} \end{bmatrix} \right\rangle = \begin{bmatrix} [T_{11}] & [\Omega_{12}] \\ [\Omega_{12}]^{*T} & [T_{22}] \end{bmatrix} \quad - 2)$$

In polarimetric interferometry the interferometric correlation is generalised to account for arbitrary choice of scattering mechanisms  $\underline{w}_1$  and  $\underline{w}_2$  at either end of the baseline [1,4]. In terms of the three sub matrices of [P] defined in equation 2, it can be written as

$$\tilde{\gamma} = \gamma e^{i\phi} = \frac{\underline{w}_1^{*T} [\Omega_{12}] \underline{w}_2}{\sqrt{\underline{w}_1^{*T} [T_{11}] \underline{w}_1 \underline{w}_2^{*T} [T_{22}] \underline{w}_2}} \quad - 3)$$

Our objective is to relate the amplitude and phase of this complex scalar to physical parameters of the scattering volume in the scene. To do this we use a coherence optimisation procedure [4,5]. The maximum values of coherence  $\gamma$  are given as the square root of the eigenvalues of a matrix  $[K]$  defined as

$$[K] = [T_{11}^{-1}] [\Omega_{12}] [T_{22}^{-1}] [\Omega_{12}^{*T}] \quad - 4)$$

This paper concerns a study of the physical interpretation of the matrix  $[K]$  for the special case of volume scattering by a cloud of partially oriented scatterers. We show that by using this optimiser as a pre-processing step, significant simplifications accrue for data inversion and hence for quantitative studies in microwave remote sensing.

## 2. ORIENTED VOLUME SCATTERING

In many vegetation problems the scatterers in a volume may have some residual orientation correlation due to the natural structure (branches in a tree canopy for example) or due to agriculture (oriented corn stalks for example). The propagation of radar signals through such a volume can no longer be assumed to be scalar. In this case the volume has two eigenpolarisations a and b (which will generally be orthogonal). Only along these eigenpolarisations is the propagation simple, in this sense that the polarisation state does not change with depth into the volume. If there is some mismatch between the radar co-ordinates and the medium's eigenstates then a very complicated situation arises where the polarisation of the incident field changes as a function of distance into the volume. In this paper we show how the optimiser of equation 4 always obtains a matched solution and hence is useful in the application of inversion schemes for oriented volume scattering problems.

By assuming that the medium has reflection symmetry about the (unknown) axis of its eigenpolarisations, then we obtain a polarimetric coherency matrix  $[T]$  and covariance matrix  $[C]$  for backscatter from the volume as shown in equation 5 [1]

$$[T] = \begin{bmatrix} t_{11} & t_{12} & 0 \\ t_{12}^* & t_{22} & 0 \\ 0 & 0 & t_{33} \end{bmatrix} \Leftrightarrow [C] = \begin{bmatrix} c_{11} & 0 & c_{13} \\ 0 & c_{22} & 0 \\ c_{13}^* & 0 & c_{33} \end{bmatrix} \quad - 5)$$

We can now obtain an expression for the matrices  $[T_{11}]$  and  $[\Omega_{12}]$  for an oriented volume extending from  $z = z_0$  to  $z = z_0 + h_v$  as vector volume integrals as shown in equations 6 and 7

$$[\Omega_{12}] = e^{i\phi(z_0)} R(2\beta) \left\{ \int_0^{h_v} e^{ik_z z'} e^{\frac{(\sigma_a + \sigma_b)z'}{\cos \theta_o}} P(\tau) T P(\tau^*) dz' \right\} R(-2\beta) \quad - 6)$$

$$[T_{11}] = R(2\beta) \left\{ \int_0^{h_v} e^{\frac{(\sigma_a + \sigma_b)z'}{\cos \theta_o}} P(\tau) T P(\tau^*) dz' \right\} R(-2\beta) \quad - 7)$$

where for clarity we have dropped the brackets around matrices inside the integrals and defined

$$R(\beta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \beta & \sin \beta \\ 0 & -\sin \beta & \cos \beta \end{bmatrix} \quad - 8)$$

$$P(\tau) T P(\tau^*) = \begin{bmatrix} \cosh \tau & \sinh \tau & 0 \\ \sinh \tau & \cosh \tau & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} t_{11} & t_{12} & 0 \\ t_{12}^* & t_{22} & 0 \\ 0 & 0 & t_{33} \end{bmatrix} \begin{bmatrix} \cosh \tau^* & \sinh \tau^* & 0 \\ \sinh \tau^* & \cosh \tau^* & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad - 9)$$

$$\tau = v z = \left( \frac{\sigma_a - \sigma_b}{2} + ik(\chi_a - \chi_b) \right) \frac{z'}{\cos \theta_o} \quad - 10)$$

where  $\sigma$  are the extinction coefficients of the volume and  $\chi$  the refractivities (index of refraction – 1). Note that if we cannot align the radar co-ordinates with the volume then the matrix term  $R(2\beta)$ , which multiplies the whole matrix integral expression inside the brackets, causes a coherent mixing of terms which is very difficult to interpret. We will show that the polarimetric optimiser automatically aligns the radar to the oriented volume. This result follows from knowledge of the explicit form of the matrix  $K$ , which for this problem enables direct calculation of its eigenvalues and eigenvectors and hence optimisation parameters in closed form.

### 3. OPTIMUM COHERENCE VALUES FOR ORIENTED VOLUME SCATTERING

To account for the effects of propagation on the polarimetric response of an oriented volume it is simpler to employ the covariance matrix  $[C]$  rather than the coherency matrix  $[T]$ . For a general oriented volume we then have

$$C_{11} = \begin{bmatrix} c_{11}I_1 & 0 & c_{13}I_2 \\ 0 & c_{22}I_3 & 0 \\ c_{13}^*I_2^* & 0 & c_{33}I_4 \end{bmatrix} \Rightarrow C_{11}^{-1} = \frac{1}{f} \begin{bmatrix} c_{33}I_4 & 0 & -c_{13}I_2 \\ 0 & \frac{f}{c_{22}I_3} & 0 \\ -c_{13}^*I_2^* & 0 & c_{11}I_1 \end{bmatrix} \quad - 11)$$

where  $f = (c_{11}c_{33} - c_{13}c_{13}^*)I_1I_4$  and similarly for the polarimetric interferometry we can write

$$\Omega_{12} = e^{i\phi(z_o)} \begin{bmatrix} c_{11}I_5 & 0 & c_{13}I_6 \\ 0 & c_{22}I_7 & 0 \\ c_{13}^*I_8 & 0 & c_{33}I_9 \end{bmatrix} \quad - 12)$$

Note that  $\Omega_{12}$  is neither symmetric nor Hermitian. The integrals  $I_1 - I_9$  are defined as

$$I_1 = \int_0^{h_v} e^{2\sigma_a z} dz \quad I_2 = \int_0^{h_v} e^{2(k_a+k_b^*)z} dz \quad I_3 = \int_0^{h_v} e^{(\sigma_a+\sigma_b)z} dz \quad I_4 = \int_0^{h_v} e^{2\sigma_b z} dz \quad I_5 = \int_0^{h_v} e^{ik_z} e^{2\sigma_a z} dz \\ I_6 = \int_0^{h_v} e^{ik_z} e^{2(k_a+k_b^*)z} dz \quad I_7 = \int_0^{h_v} e^{ik_z} e^{(\sigma_a+\sigma_b)z} dz \quad I_8 = \int_0^{h_v} e^{ik_z} e^{2(k_b+k_a^*)z} dz \quad I_9 = \int_0^{h_v} e^{ik_z} e^{2\sigma_b z} dz$$

and  $k_z$  is defined in equation 1,  $k_a, k_b$  are the complex propagation constants of the eigenstates. Hence the first part of the matrix  $[K]$  (equation 4) has the form

$$C_{11}^{-1}\Omega_{12} = \frac{e^{i\phi(z_o)}}{f} \begin{bmatrix} c_{33}I_4 & 0 & -c_{13}I_2 \\ 0 & \frac{f}{c_{22}I_3} & 0 \\ -c_{13}^*I_2^* & 0 & c_{11}I_1 \end{bmatrix} \begin{bmatrix} c_{11}I_5 & 0 & c_{13}I_6 \\ 0 & c_{22}I_7 & 0 \\ c_{13}^*I_8 & 0 & c_{33}I_9 \end{bmatrix} \quad - 13)$$

which is diagonal if  $I_4I_6 - I_2I_9 = I_8I_1 - I_2^*I_5 = 0$ . From equation 13 we can easily show that both equations are satisfied for arbitrary medium parameters as we have

$$I_4I_6 = \int_0^{h_v} e^{2\sigma_b z} e^{ik_z} e^{2(k_a+k_b^*)z} dz = I_2I_9 \quad I_8I_1 = \int_0^{h_v} e^{2\sigma_a z} e^{ik_z} e^{2(k_b+k_a^*)z} dz = I_2^*I_5 \quad - 14)$$

Hence the product  $C_{11}^{-1}\Omega_{12}$  is diagonal. We can also find the complex diagonal values as

$$\begin{aligned}
\tilde{\gamma}_1 &= \frac{(c_{11}c_{33}I_4I_5 - c_{13}c_{13}^*I_2I_8)}{(c_{11}c_{33} - c_{13}c_{13}^*)I_1I_4} = \frac{I_5}{I_1} = f(\sigma_a) = \frac{2\sigma_a e^{i\phi(z_o)}}{\cos\theta_o (e^{2\sigma_a h_v/\cos\theta_o} - 1)} \int_0^{h_v} e^{ik_z z'} e^{\frac{2\sigma_a z'}{\cos\theta_o}} dz' \\
\tilde{\gamma}_2 &= \frac{I_7}{I_3} = f(\sigma_a, \sigma_b) = \frac{(\sigma_a + \sigma_b) e^{i\phi(z_o)}}{\cos\theta_o (e^{(\sigma_a + \sigma_b) h_v/\cos\theta_o} - 1)} \int_0^{h_v} e^{ik_z z'} e^{\frac{(\sigma_a + \sigma_b) z'}{\cos\theta_o}} dz' \quad - 15) \\
\tilde{\gamma}_3 &= \frac{(c_{11}c_{33}I_1I_9 - c_{13}c_{13}^*I_6I_2^*)}{(c_{11}c_{33} - c_{13}c_{13}^*)I_1I_4} = \frac{I_9}{I_4} = f(\sigma_b) = \frac{2\sigma_b e^{i\phi(z_o)}}{\cos\theta_o (e^{2\sigma_b h_v/\cos\theta_o} - 1)} \int_0^{h_v} e^{ik_z z'} e^{\frac{2\sigma_b z'}{\cos\theta_o}} dz'
\end{aligned}$$

It follows from the above that  $\mathbf{K}_c = \mathbf{C}_{11}^{-1} \mathbf{\Omega}_{12} \mathbf{C}_{11}^{-1} \mathbf{\Omega}_{12}^{\dagger}$  is also diagonal and hence by using the relationship between [T] and [C] we can show from equation 6 that the eigenvectors of  $\mathbf{K} = \mathbf{T}_{11}^{-1} \mathbf{\Omega}_{12} \mathbf{T}_{11}^{-1} \mathbf{\Omega}_{12}^{\dagger}$  are functions of  $\beta$  as

$$\underline{w}_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -\cos 2\beta \\ \sin 2\beta \end{bmatrix} \quad \underline{w}_2 = \begin{bmatrix} 0 \\ \sin 2\beta \\ \cos 2\beta \end{bmatrix} \quad \underline{w}_3 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ \cos 2\beta \\ -\sin 2\beta \end{bmatrix} \quad - 16)$$

We see that the eigenvectors of [K] tell us about the orientation of the medium eigenpolarisations and the eigenvalues are the coherences for the corresponding wave extinctions. As expected from physical arguments, the highest (lowest) coherence is obtained for the polarisation with the highest (lowest) extinction. This is the physical basis for the successful operation of the optimiser.

In the oral presentation, L-Band Radar data from the DLR E-SAR sensor will be used to illustrate the application of this algorithm to forested and agricultural terrain.

#### 4. CONCLUSIONS

In this paper we have shown that the coherence optimiser algorithm provides a full solution to the oriented volume scattering problem in that the optimiser automatically corrects for any misalignment between the radar and volume co-ordinates. As a bonus, the optimum values of coherence then relate directly to the depth and extinction coefficient of the volume. This result provides the basis for an algorithm for the remote sensing of oriented vegetation structure using polarimetric interferometry and has applications in agriculture and low frequency forest remote sensing problems.

#### 5. REFERENCES

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