B-7-2 OPTIMUM DESIGN OF A BEAM COMPRESSION ARRAY BY MULTIPLICATIVELY PROCESSING

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Introduction

Multiplicative arrays of two different types are shown in Figure 1. The configuration of Array 1 gives good sidelobe level but poor output SNR because the reference elements are small. On the other hand, the configuration of Array 2 gives good output SNR but poor sidelobe level. Fig.2 shows the typical pattern of the multiplicative array. The object of this paper is to find the optimum configuration of array for shipborne radar which compromises both sidelobe level and output SNR by choosing appropriate ratio of the length of two elements under the condition that the full length of array L, is constant as shown in Figure 3.

Output SNR of a multiplicative array with matched filter

It is assumed that the input pulses of multiplier M, are

$$s_i(t) = \begin{cases} A_i \cos(\omega_0 t + \phi_i) & 0 \le t \le T \\ 0 & \text{elsewhere} \end{cases} \qquad i = 1, 2 \tag{1}$$

and that the input noises of multiplier are white gaussian (with zero mean) with power spectral density N_1, N_2 respectively.

The spectral power of the output signal of matched filter $S_{out}(\omega)$ and the spectral power of the output noise of matched filter $N_{out}(\omega)$ are, respectively,

$$S_{out}(\omega) = |G(\omega)|^2 \cdot S_{in}(\omega) \tag{2}$$

$$N_{out}(\omega) = \frac{1}{2\pi} |G(\omega)|^2 \cdot \left(N_2 \int_{-\infty}^{\infty} s_1(\omega) d\omega + N_1 \int_{-\infty}^{\infty} s_2(\omega) + N_1 N_2 \right) \tag{3}$$

where $G(\omega)$ is the transfer function of matched filter and $S_1(\omega)$, $S_2(\omega)$ are the spectral power of the input signals of multiplier respectively and $S_{in}(\omega)$ is the spectral power of the input signal of matched filter. They are given as follows:

$$|G(\omega)|^2 = T^2 \frac{\sin^2(\frac{\omega T}{2})}{(\frac{\omega T}{2})^2} \tag{4}$$

$$S_{in}(\omega) \doteqdot \frac{A_1^2 A_2^2 T^2}{4} \left\{ \cos^2(\phi_1 - \phi_2) \cdot \frac{\sin^2(\frac{\omega T}{2})}{(\frac{\omega T}{2})^2} + \frac{1}{4} \left\{ \frac{\sin^2(\frac{\omega + 2\omega_0}{2} T)}{(\frac{\omega + 2\omega_0}{2} T)^2} + \frac{\sin^2(\frac{\omega - 2\omega_0}{2} T)}{(\frac{\omega - 2\omega_0}{2} T)^2} \right\} \right. \tag{5}$$

$$s_i(\omega) \doteqdot \frac{A_i^2 T^2}{4} \left\{ \frac{\sin^2(\frac{\omega + \omega_0}{2} T)}{(\frac{\omega + \omega_0}{2} T)^2} + \frac{\sin^2(\frac{\omega - \omega_0}{2} T)}{(\frac{\omega - \omega_0}{2} T)^2} \right\} \qquad i = 1, 2 \tag{6}$$

Substituting (4), (5), (6) into (2), (3), the output SNR of matched filter becomes

$$SNR_{out} = \frac{\frac{1}{2\pi} \int_{-\infty}^{\infty} S_{out}(\omega) d\omega}{\frac{1}{2\pi} \int_{-\infty}^{\infty} N_{out}(\omega) d\omega} \doteqdot \frac{\cos^2(\phi_1 - \phi_2)}{3\pi} \frac{(\pi T A_1^2) \cdot (\pi T A_2^2)}{(\pi T A_1^2) N_2 + (\pi T A_2^2) N_1 + N_1 N_2} \tag{7}$$

Letting the input SNR be $(\frac{S}{N})_1$ $(=\pi TA_1^2/N_1)$, $(\frac{S}{N})_2$ $(=\pi TA_2^2/N_2)$, respectively, (7) yields

$$SNR_{out} \doteqdot \frac{\cos^2(\phi_1-\phi_2)}{3\pi}\cdot\frac{(\frac{S}{N})_1\cdot(\frac{S}{N})_2}{(\frac{S}{N})_1+(\frac{S}{N})_2+1} \qquad (8)$$

This shows that the output SNR is a function of the input SNR and the direction of incident wave.

Considering broadside direction, $\phi_1=\phi_2$, then $\cos^2(\phi_1-\phi_2)=1$. The output SNR becomes

$$SNR_{out} \doteqdot \frac{1}{3\pi}\cdot\frac{(\frac{S}{N})_1\cdot(\frac{S}{N})_2}{(\frac{S}{N})_1+(\frac{S}{N})_2+1} \qquad (9)$$

If both $(\frac{S}{N})_1$ and $(\frac{S}{N})_2$ are small, SNR_{out} becomes

$$SNR_{out} \doteqdot \frac{1}{3\pi}(\frac{S}{N})_1\cdot(\frac{S}{N})_2 \qquad (10)$$

Now consider the case that the ratio of the element length varies keeping the full length of array is constant. It is assumed that $N_1 = N_2$, $(\frac{S}{N})_1 \propto A_1$ and $(\frac{S}{N})_2 \propto A_2$ where A_1 and A_2 are the effective aperture of elements respectively. Furthermore $A_1 \propto L-D$ and $A_2 \propto D$. Letting $D/(L-D)=\gamma$, $(\frac{S}{N})_1=\alpha(L-D)$ and $(\frac{S}{N})_2=\alpha D$ (α=const.), (9) yields

$$SNR_{out} \doteqdot \frac{1}{3\pi}\cdot\frac{\alpha^2L^2}{\alpha L+1}\,\frac{\gamma}{(1+\gamma)^2} \qquad (11)$$

From equation (11), the relation of normalized SNR_{out} to the ratio of the element length is shown in Figure 4.

Sidelobe level

In Fig.3 the output pattern of multiplier $D_{12}(\theta)$ is

$$D_{12}(\theta) = D_1(\theta)\cdot D_2(\theta)\cdot \cos(\beta L \sin\theta) \qquad (12)$$

where $D_1(\theta)$ and $D_2(\theta)$ are element pattern respectively. $D_{12}(\theta)$ is a function of both current distribution on elements and element spacing. For case a) uniform distribution and b) cosine distribution as typical current distributions, their first negative sidelobe level which may mask a small target near a large target can be calculated by using (12). The relations between the ratio of the element length and the first negative sidelobe level are shown in Fig.4 for the configuration in Figure 3.

Beamwidth

Similarly, the relations between the ratio of the element length and the beamwidth can be calculated. The results obtained for both uniform distribution and cosine distribution are shown in Figure 4. Slight change of beamwidth can be seen.

Conclusion

From Fig.4, the optimum configuration of array which compromises the first negative sidelobe level,the output SNR and the beam-width can be found. That is,in order to limit the decrease in the output SNR to less than 3 dB and the increase in the first negative sidelobe level to less than 5 dB,the value of D/(L-D) must lie between 0.17 and 0.35. Therefore,D:L-D=1:3 or 1:4. It is interesting that the first negative sidelobe level for cosine distribution is less than that for uniform distribution.

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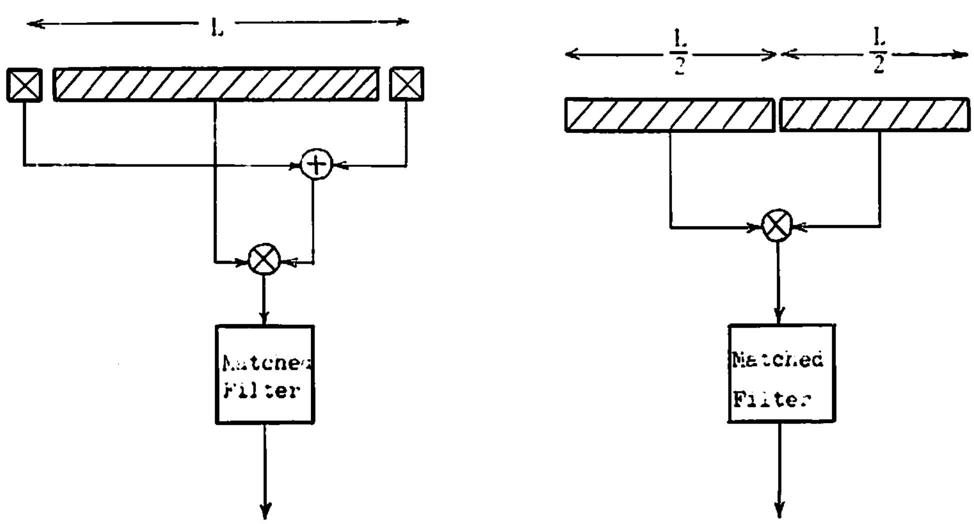
References

1. R.Blommendaal,"A note on multiplicative receiving system for radar," Radio Electronic Engineer,November 1964.
2. A.Ksiensky,"Signal processing antenna systems," Ohio State Univ. summer short course,1965.
3. R.E.Collin and F.J.Zucker(eds.),"Antenna Theory,part II ," McGraw-Hill Book Co.,1969.
4. T.Suzuki,"On a beam compression radar," Report of Technical Meeting,IECE of Japan,SANE 75-32,1976.
5. T.Suzuki,"Compression of radar beam by signal processing," National Convention Record of IECE,Japan,S7-4,1977.



Array 1 Array 2

Fig.1 Typical configurations of multiplicative arrays

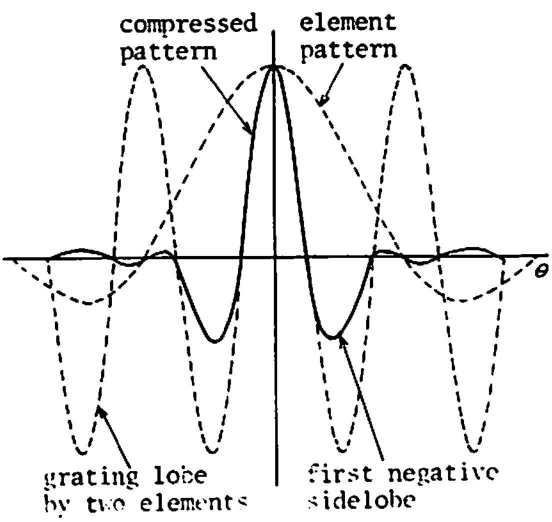


Fig.2 Typical pattern of a multiplicative array

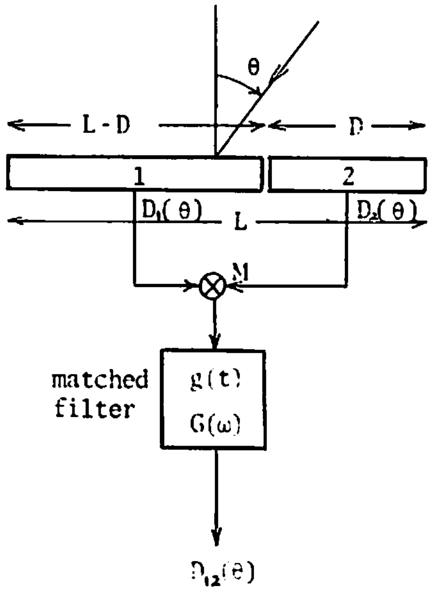


Fig.3 Configulation of the multiplicative array

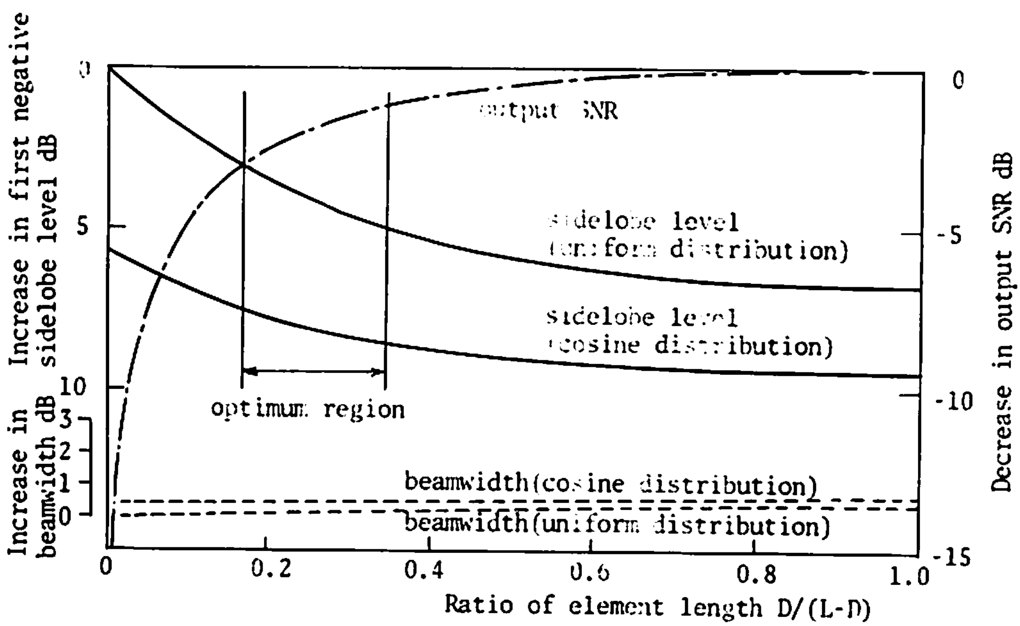


Fig.4 First negative sidelobe level,output SNR and beamwidth versus ratio of element length