

## A NEW METHOD OF TIME-DOMAIN ANALYSIS USING A SUPERRESOLUTION TECHNIQUE

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### I. INTRODUCTION

In antenna parameter measurement, to eliminate the effects of unwanted signals is one of the important problems. A recently developed network analyzer provides time-domain processing based on the fast-Fourier-transform (FFT) algorithm. When the network analyzer is employed, we can eliminate the unwanted signals easily by gating them from the time-domain presentation[1]. However, the response resolution of the FFT essentially depends on the frequency bandwidth of obtained data. Thus the difficulty arises in measuring narrow-band devices such as antennas. Furthermore, even though the peaks are resolved distinctly, we may not eliminate the unwanted signals by gating when the skirts of the responses are overlapped each other. Therefore a high-resolution time-domain estimation method whose resolution does not depend on the frequency bandwidth, has been desired.

In this paper, we present a new method of time-domain estimation with a network analyzer. The method is based on a MUSIC (Multiple Signal Classification) algorithm proposed by Schmidt[2]. The MUSIC method is one of the high-resolution eigenstructure techniques for direction-of-arrival estimation by signal processing arrays, and here we use the similarities between the outputs of a uniform linear array and those of the network analyzer. However, the incident signals to the network analyzer are continuous sinusoidal waves generated from the identical oscillator. Thus the incident signals are fully correlated. Clearly, the situation is the same as the coherent multipath environment in the direction finding problem. As a result, the MUSIC algorithm does not work correctly, unless any decorrelation preprocessing is employed. In this paper, spatial smoothing preprocessing (SSP) proposed by Shan et al.[3] and modified spatial smoothing preprocessing (MSSP) proposed by Williams et al.[4] are applied to decorrelate the incident signals.

### II. PROBLEM FORMULATION

For simplicity, we consider only reflection measurement of a device which includes  $d$  discrete reflection points. We assume, in this paper, the multiple reflections are negligibly small. Then the number of signals is equivalent to  $d$ . Following formulation can also be applied to transmission measurement.

The data obtained by the network analyzer are the reflection coefficients at sampling frequency points. Consider the  $N$  sampling frequency points ( $N > d$ ), and we represent delay times corresponding to the reflection points by  $t_1, \dots, t_d$ . We assume that the reflection coefficient of

each reflection point is constant in a narrow frequency bandwidth. Then the obtained signals by the network analyzer can be written as

$$r = A s + n \quad (1)$$

where  $r$  is the  $N \times 1$  received data vector and  $n$  is the  $N \times 1$  additive noise vector.  $s$  is the  $d \times 1$  signal vector, and  $A$  is the  $N \times d$  delay parameter matrix whose columns are the "mode vector" of each signal[2].

### III. THE MUSIC ALGORITHM AND THE DECORRELATION PREPROCESSING

The MUSIC algorithm uses the eigenstructure of the received data correlation matrix. From (1), the output correlation matrix can now be expressed as

$$R = A S A^H + \sigma^2 I \quad (2)$$

where  $S = E [s s^H]$  denotes the signal correlation matrix,  $\sigma^2$  is the variance of the additive noise,  $I$  is the identity matrix, and  $H$  denotes the complex conjugate transpose. Here we express the eigenvalues and the corresponding eigenvectors of  $R$  as  $\{\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_N\}$  and  $\{e_1, \dots, e_N\}$ , respectively. Then the following properties can hold when the signals are incoherent.

1) the minimum eigenvalue of  $R$  is equivalent to  $\sigma^2$  with multiplicity  $N-d$ :

$$\lambda_{d+1} = \lambda_{d+2} = \dots = \lambda_N = \sigma^2.$$

2) the eigenvectors corresponding to the minimum eigenvalue are orthogonal to the columns of the matrix  $A$ . Namely, they are orthogonal to the "mode vector" of the signals:

$$\{e_{d+1}, \dots, e_N\} \perp \{a(t_1), \dots, a(t_d)\}.$$

We define  $E_N$  to be the  $N \times (N-d)$  matrix whose columns are the  $(N-d)$  noise eigenvectors. Then we can estimate the position (delay time) of each reflection point by searching the peak position of the following function.

$$P_{\text{MUSIC}}(t) = a(t)^H a(t) / a(t)^H E_N E_N^H a(t) \quad (3)$$

These properties, 1) and 2), hold when the matrix  $S$  is nonsingular. However, since the incident signals are coherent in this case, the matrix  $S$  is singular[3], and the MUSIC algorithm does not work properly. Then in this paper, we apply decorrelation preprocessing to destroy the signal coherence. The SSP and MSSP techniques are examined here. Fig.1 shows  $L$  reflection coefficients measured by the network analyzer ( $L=N+M-1$ ). Here we introduce the terminology "subarray" of the direction finding problem. A subset of  $N$  serial sampling frequencies is called "subarray". Consequently, as we may see from Fig.1, we have  $M$  overlapped subarrays. We regard  $r$  in (1) as the data vector corresponding to the first subarray. Let  $r_k$  denote the vector of obtained signals at  $k$ th subarray. Following the notation of (1), we can write

$$r_k = A D^{(k-1)} + n_k \quad (4)$$

where  $D^{(k-1)}$  denotes the  $d \times d$  diagonal matrix expressed as

$$D = \text{diag}\{\exp(-j2\pi \Delta f t_1), \dots, \exp(-j2\pi \Delta f t_d)\} \quad (5)$$

Here,  $\Delta f$  is the sampling frequency separation. The correlation matrix of the  $k$ th subarray is given by

$$R_k = A D^{(k-1)} S D^{(k-1)H} A^H + \sigma^2 I \quad (6)$$

The SSP is a method which uses the following matrix  $R_{\text{SSP}}$  as the correlation matrix.

$$R_{SSP} = \frac{1}{M} \sum_{k=1}^M R_k \quad (7)$$

The MSSP is the method which uses  $R_{MSSP}$  defined as

$$R_{MSSP} = \frac{1}{2M} \sum_{k=1}^M (R_k + J R_k^* J) \quad (8)$$

where \* denotes complex conjugate and J is the  $N \times N$  exchange matrix:

$$J = \begin{bmatrix} 0 & \dots & 0 & 1 \\ \vdots & & \cdot & \vdots \\ 0 & 1 & \dots & \vdots \\ 1 & 0 & \dots & 0 \end{bmatrix} \quad (9)$$

Since these preprocessing techniques efficiently destroy the signal coherence, the MUSIC algorithm can successfully estimate the signal parameters.

#### IV. EXPERIMENTAL RESULTS

In this section, we present experimental results of the proposed technique, and compare the resolution with the inverse-FFT equipped with the network analyzer (HP8510B). As a device under test (DUT), we used a semirigid cable of about 16 cm which terminates with a dummy. Though the multiple reflection may occur in general, we assume, in this experiment, that the multiple reflection may be neglected because of the noise.

Fig.2 shows the results of the inverse-FFT. As shown in Fig.2, two reflected signals can not be detected by 1.0 GHz bandwidth data. At least 1.5 GHz bandwidth is needed to resolve their peaks. Furthermore, in a case where we eliminate one of them by gating, the responses must be completely separated each other. Then we need more than 3.0 GHz bandwidth data.

Next, we show the results of the proposed technique. Fig.3 shows the results of the MUSIC preprocessed by the SSP (MUSIC-SSP), and Fig.4 shows those by the MSSP (MUSIC-MSSP). The first frequency of the used data was 11.6 GHz, and 10 data ( $N=10$ ) with sampling frequency interval 24 MHz ( $\Delta f=24\text{MHz}$ ) were used for each subarray. Number of samples ('snapshots') was 123. The two peaks were successfully resolved with only 264 MHz ( $M=3$ ) bandwidth in Fig.3. Using the MUSIC-MSSP, the minimum required frequency bandwidth for resolving the two peaks decreases to 88 MHz ( $M=3$ ,  $\Delta f=8\text{MHz}$ ) as shown in Fig.4.

Now, we consider the resolution capability from the view point of the required frequency bandwidth. The inverse-FFT requires the bandwidth of about 1.5 GHz for detecting the two distinct peaks. On the other hand, the MUSIC-SSP requires the bandwidth of 264 MHz, almost 1/6 of the inverse-FFT, and the MUSIC-MSSP requires that of 88 MHz, 1/17 of the inverse-FFT.

From these results, it is clear that the MUSIC algorithm can be applicable for the time domain analysis, and has a great capability of resolving signals even for narrow-band measuring. Furthermore the reflection coefficient is obtained by the following equation,

$$s = (A^H A)^{-1} A^H E [r] \quad (10)$$

## V. CONCLUSIONS

In this paper, we have presented the new method of time-domain analysis of signals measured by the network analyzer. Experimental results illustrate the higher capability of resolving signals comparing with the conventional inverse-FFT. Especially, the MUSIC-MSSP reveals much better performance. These techniques are useful for the measurement of narrow-band devices such as antennas.

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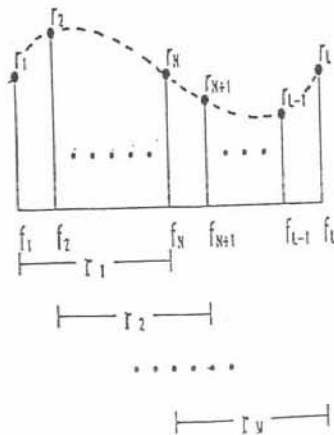


Fig.1 Arrangement of subarrays.

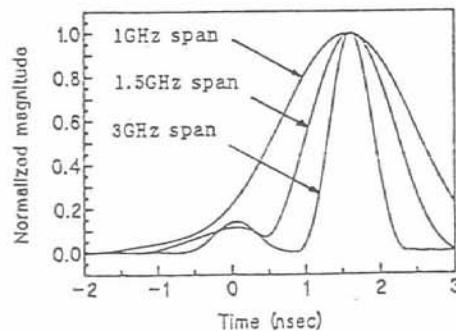


Fig.2 Time-domain analysis using inverse-FFT.  
 1GHz span : 11.5GHz-12.5GHz  
 1.5GHz span : 11.0GHz-12.5GHz  
 3.0GHz span : 10.0GHz-13.0GHz

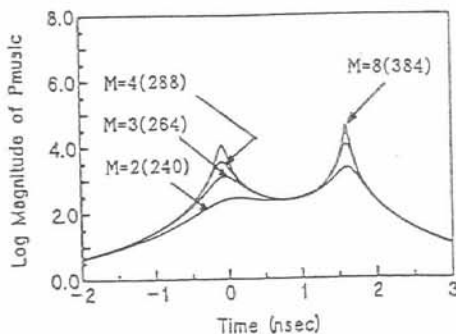


Fig.3  $P_{music}(t)$  using the SSP.  $N=10$ ,  $\Delta f=24\text{MHz}$ .  
 ( ) : required frequency bandwidth in MHz.

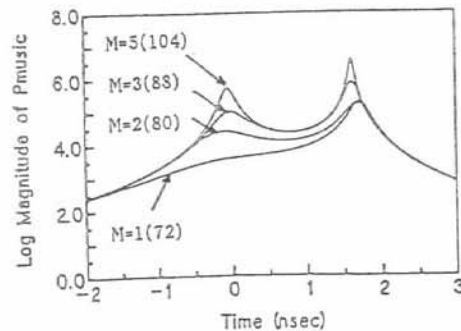


Fig.4  $P_{music}(t)$  using the MSSP.  $N=10$ ,  $\Delta f=8\text{MHz}$ .  
 ( ) : required frequency bandwidth in MHz.