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PLANAR PATCH ANTENNAS WITH FRACTAL BOUNDARY

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Abstract:

This paper describes an innovative approach for the simulation of the microstrip patch antennas with fractal boundary. First, L-System based fractal generator with the possibility of exporting the desired geometry into various EM simulators has been developed. Before the full-wave simulation is performed, modal analysis to predict the resonant frequencies, field distribution and radiation pattern is employed.

Introduction

Full wave simulation of complex structures presented by fractal microstrip patch antennas is very difficult and time-consuming indeed. Cavity model has been found to be very useful and fast in obtaining basic parameters of such structures like resonant frequencies, field distribution and radiation pattern. Based on derived results, modes with remarkable field distribution regarding mainly the radiation pattern can be determined. This approach saves considerable time when finally full-wave simulation is performed. Moreover, comparison with full-wave simulation and also measurement gives reliable agreement. In addition, powerful and widely configurable L-System based fractal generator was used to create structures for EM simulators.

L-System based fractal generator

L-System (Lindenmayer System) is a recursive, text substitution scheme, followed by geometric interpretation. It has been found very useful for generating various fractal structures for EM purposes where the main advantage is its effectiveness and wide variability covering most of deterministic fractal structures. Matlab has been used for both implementing L-systems algorithm in 2D plane and creating appropriate files for widely used EM simulators like IE3D, CST-Microwave Studio and FEMLAB.

A simple L-System starts with a seed, let's say the letter F, and has one rule to replace the existing seed. A simple replacement rule might be for example: **F-F++F-F** (see explanation below). This rule would be applied according to iteration level to produce a series of strings of increasing complexity. To produce fractals, strings generated by L-Systems must contain the necessary information about figure geometry. A graphic interpretation of strings is based on assigning the language to the motion of the imaginary turtle.

L-system language

Let us now describe specific language used in our simple L-System Matlab script.

F move forward a step of length f

+ turn left by a specified angle θ

- turn right by a specified angle θ

Beyond to the above, there is also more letters defined. We will not discuss it here since they are not very important when considering planar antennas with fractal boundary only. Useful feature of the described generator is the possibility to control the area of designed shapes over all the iterations.

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Examples of L-System language

Let's now see some examples of creating planar fractal antennas to show the simplicity and variability of the proposed generator:
Koch Snowflake: seed **F+F+F**, rule **F-F++F-F**, θ=60°
Quadratic Koch Island (QKA): seed **F+F+F+F**, rule **F-F+F+FFF-F-F+F**, θ=90°, see Fig.1
Koch Coastline (KC): seed: **F+F+F+F**, rule **FFFF+F+F-F-FF-FF+F**, θ=90°, see Fig.1

Simulation of fractal plannar antennas

Simulation methods based on MoM or FDTD are in general very time consuming even when considering such complex structures as presented by fractal antennas. We have successfully used so-called modal analysis (represented by cavity model here) to predict the antenna's behaviour.

Cavity model

Cavity model [1] reduces microstrip planar antenna to planar resonator with perfect magnetic conductor at its boundary. Having an antenna lying in XY plane for example, we consider only E_z component under the patch and no fringing fields. Using this simplification, Helmholz wave equation with Neumann boundary condition have the following form

$$(\Delta + k_d^2)E_z(x,y) = 0, \ \frac{\partial E_z(x,y)}{\partial n} = 0 \ . \tag{1}$$

For sets with fractal boundary, the solution haven't been found in closed form yet so we turned to use finite element method implemented in Femlab software.
The solution of (1) consists of eigenvalues $\{k^2\}_n$ and corresponding eigenfunctions $\{E_z\}_n$. Eigenvalues are related to resonant frequency

$$f_n = \frac{c_0 \cdot k_n}{2\pi\sqrt{\epsilon_r}} \ , c_0 = 3 \cdot 10^8 \mathrm{ms}^{-1} \tag{2}$$

and eigenfunctions are related to modal fields, namely E_z inside the cavity (under the patch). Fig. 1 shows the spectra (eigenvalues k_n^2 versus it's count) of QKA 0-2 and QC 0-2 patch antennas with constant area 1000mm^2 over two iterations. From results, the influence of fractal[1] boundary with given iteration on the resonant frequencies can be seen.

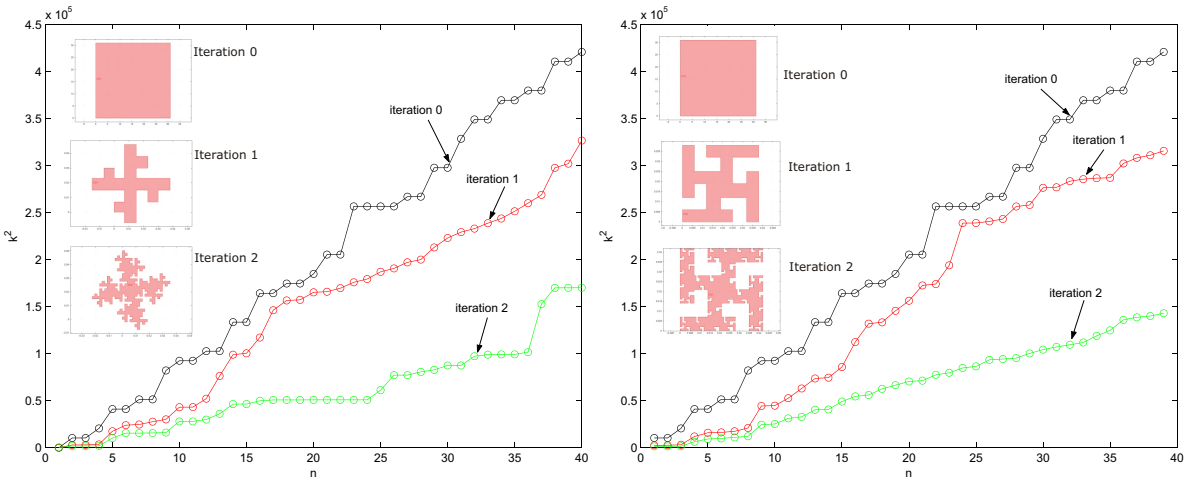

Fig. 1: The spectra of Quadratic Koch island and Koch square island for iterations 0-2

Although the cavity model is very simple, predicted resonant frequencies are very close to those obtained by IE3D full-wave simulation. In [2] we have analysed Koch Snoflake (iteration 3), the results are shown in Tab. 1. Measured frequencies aren't shown in table as they are very close to IE3D's one. As a conclusion, we have found that "high directivity" mode is even mode number 13+14 (degenerated) opposite to mode 6 Borja [3].

[1] We should rather use the word prefractal since the iteration number is finite

mode nr.	f_{IE3D}[GHz]	f_{FEM}[GHz]	rel. error %
1+2	0.7925	0.8016	1.14
3+4	1.108	1.125	1.5
5	1.218	1.23	0.95
6	1.645	1.6768	1.9
7+8	2.124	2.148	1.11
9	2.406	2.4775	2.88
11	2.448	2.5707	4.77
13+14	**2.756**	**2.8193**	**2.25**
15+16(+17)	2.867	2.9148	1.6
18	3.157	3.1808	0.75
20+21	3.369	3.4145	1.33
23(+22)	3.547	3.5444	0.07

Tab. 1: Resonant frequencies of Koch Snowflake-3 obtained by cavity model and using IE3D

Surface currents

Once knowing electric field distribution E_z under the patch, calculating the surface electric current $\mathbf{J}$ on the desired resonant mode is straightforward [1]

$$\mathbf{J} = \widehat{\mathbf{n}} \times \mathbf{H} \quad \text{where } \mathbf{H} = \frac{1}{j\omega\mu_0} \times \nabla E_z \ . \tag{3}$$

Obtained surface currents are very important for determining the resonant mode comparing with full-wave simulation results and also for radiation pattern estimation as well. Fig. 2 shows electric field E_z together with electric current density $|\mathbf{J}|$ of QKA-2 at mode 9 calculated by cavity model and full-wave simulator IE3D. It can be easily seen that the results are quite similar even if we recall the fact that the cavity model doesn't account influence of the feeder and fringing fields into calculation. Modal analysis is thus very important for quick identification of excited mode(s) when one looks at the results of full-wave simulation and compare current distributions. Additional care must be taken outside the resonant frequencies indeed. Thanks to the nature of the problem, an superposition of several modes will occur depending on feeder's type and position and could make identification slightly problematic.

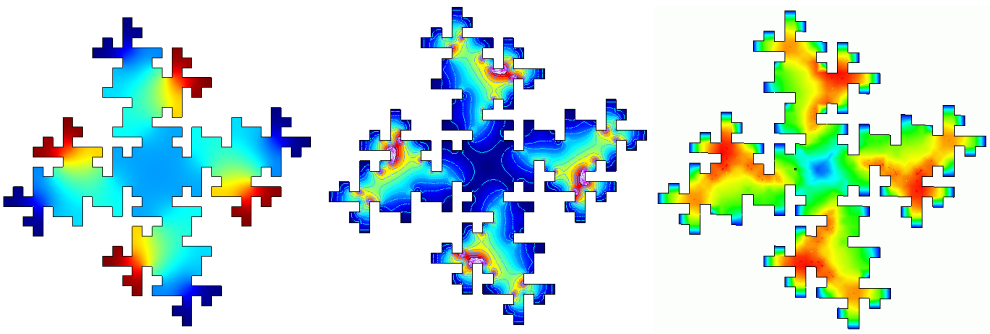

Fig. 2: QKA-2: electric field E_z from cavity model (left), electric current density $|\mathbf{J}|$ on mode 9, cavity model (middle) and full wave solution (right)

Radiation pattern

Radiation pattern can be obtained considering magnetic or electric current density respectively. It has been found out that using electric currents at the patch surface is more comfortable regarding the algorithm complexity - we don't need to know the outer normal to the antenna boundary needed for magnetic current calculation. Once knowing the electric currents densities J_x and J_y obtained by (3) on the desired mode, we are able to calculate the radiation vector $\mathbf{N}$

$$N_x(\theta,\phi) = \sum_{x'_i,y'_i} J_x(x'_i,y'_i)\exp\left\{jk(x'_i\sin\theta\cos\phi + y'_i\sin\theta\sin\phi)\right\}\Delta x'\Delta y' \tag{4}$$

$$N_y(\theta,\phi) = \sum_{x'_i,y'_i} J_y(x'_i,y'_i)\exp\left\{jk(x'_i\sin\theta\cos\phi + y'_i\sin\theta\sin\phi)\right\}\Delta x'\Delta y' \ , \tag{5}$$

where x'_i and y'_i are the source points where the current density is considered. $\Delta x'$ and $\Delta y'$ are discretization steps determining the accuracy of radiation pattern calculation. The sumation in eqs (4) and (5) is performed over all the patch area. See Fig. 3 for an example of the discretization (4524 cells) used for radiation pattern calculation, J_x and J_y components at mode 9 are shown. Calculated cut $\phi = 0$ is at Fig. 4 with full-wave result for comparison. The effects of ground plane and dielectric substrate haven't been included into calculation for simplicity.

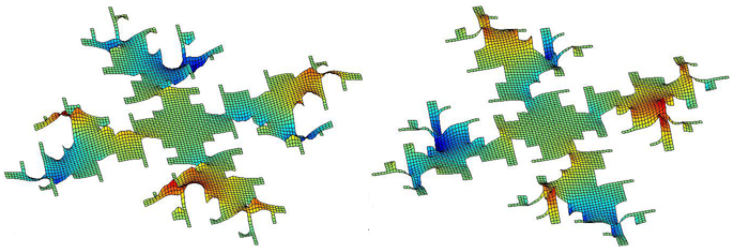

Fig. 3: QKA-2 current density J_x and J_y at mode 9 used for radiation pattern calculation

Using radiation vectors (4), (5), far-field pattern is then obtained in spherical coordinates

$$F_\theta(\theta,\phi) = (N_x\sin\phi - N_y\cos\phi) \tag{6}$$

$$F_\phi(\theta,\phi) = (N_x\cos\theta\cos\phi + N_y\cos\theta\sin\phi) \ . \tag{7}$$

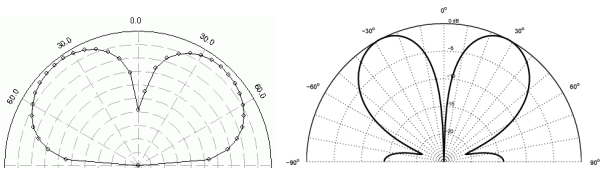

Fig. 4: Farfield pattern at mode 9, $\phi = 0$, obtained by full-wave and quasistatic simulation

Conclusions

We presented very general and widely configurable generator of fractal antennas based on L-Systems. This generator has been used here for creating selected microstrip patch antennas with fractal boundary which have been studied using cavity model. The results shows that fast modal analysis gives reliable estimation of basic antenna parameters and finally reduces time needed for full-wave analysis. In addition, knowledge of the spectra of fractal resonators is valuable for studying exciting problems like electromagnetic chaos and some quantum theory opened problems.

Acknowledgement

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