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Comparison of various methods for Poles' Extraction in Microstrip Problem

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I. Introduction

The precise poles' locations of the Green's function are very important for the accurate evaluation of the closed-form spatial-domain Green's function for microstrip problem [1]. Most of the proposed techniques of poles' extraction to date have been confined to single-layered microstrip geometry. For simplicity, we will explain the TM cases in this paper. Our earlier success in deriving fast and efficient poles' extraction for single- [2] and double-layered [3] dielectric microstrip problem have provided a good hint on deriving a generalized approach for multilayered microstrip poles' extraction. In this paper we will illustrate how these functional expressions and good initial values in [2] & [3], can be extended to multilayered microstrip problem.

II Extraction of Surface Wave Poles for N-layered Microstrip Geometry

It is assumed that there's a ground layer on 0th-layer and metal layers are infinitely thin (See Fig. 1).

(i) Functional Expression Derivation

The TM mode characteristic equation is $\sqrt{\Delta_o^2 - x^2} - \frac{x}{\varepsilon_{r(n)}}\tan\left(x - jA'_{(n-1)}\right) = 0,$ (1)

where $\tanh\left(A'_{(n-1)}\right) = \frac{y_{(n-1)}}{y_{(n)}}\frac{\varepsilon_{r(n)}h_{(n)}}{\varepsilon_{r(n-1)}h_{(n-1)}}\tanh\left(A'_{(n-2)} + jy_{(n-1)}\right)$, $\tanh\left(A'_1\right) = \frac{y_1}{y_2}\frac{\varepsilon_{r2}h_2}{\varepsilon_{r1}h_1}\tanh\left(jy_1\right)$,

$y_i = h_i\sqrt{x^2 - \Delta_i^2}\big/h_N$, $\forall i = 1,2,\ldots,N.$, $K_{z(n)} = \sqrt{\varepsilon_{r(n)}K_o^2 - K_\rho^2}$, $x = h_{(N)}\sqrt{\varepsilon_{r(N)}k_o^2 - k_\rho^2} = y_n$,

$\Delta_i = k_o h_{(N)}\sqrt{\varepsilon_{r(N)} - \varepsilon_{ri}}$, $\forall i = 0,1,2,\ldots,N-1$, and $\varepsilon_{r0} = 1$.

The proposed functional expressions for the efficient extraction of the surface wave poles are given as

$$D_{TM} = \begin{cases} \Delta_o\varepsilon_{r(N)}\Big/\sqrt{\varepsilon_{r(N)}^2 + \tan^2\left(x - jA'_{(n-1)}\right)} - x, & if \quad k = (N-1)\,\&\,k\pi < \hat{\Delta}_{TM} < (2k+1)\pi/2, \\ \tan^{-1}\left[\dfrac{\varepsilon_{r(N)}\sqrt{\left(\Delta_o/x\right)^2 - 1} - \tan\left(-jA'_{(n-1)}\right)}{1 + \varepsilon_{r(N)}\tan\left(-jA'_{(n-1)}\right)\sqrt{\left(\Delta_o/x\right)^2 - 1}}\right] + k\pi - x, & elsewhere \quad for \quad k = 0,1,2,\ldots,N, \end{cases} \quad (2)$$

where $\hat{\Delta}_{TM} = \left(x - jA'_{(n-1)}\right)\Big|_{x=\Delta_o}$. Similar expression for TE mode can also be derived.

(ii) Initial Guess

For TM mode, the locations of the zeros and infinities of $x\tan\left(x - jA'_{(n)}\right)$ equation (8) can be approximated through the locations of zeros and infinities of the function $\tan\left(\sum_{i=1}^{N} Y_i\right)$,

$$\text{where } Y_i = \begin{cases} y_i, & \left(jy_i\right) \in real, \\ -\tan^{-1}\left(\dfrac{\varepsilon_{r(i+1)}h_{(i+1)}}{\varepsilon_{r(i)}h_{(i)}}\right), & \left(jy_i\right) \in complex. \end{cases} \quad (3)$$

With the known locations of the zeros and infinities, the required initial guess is obtained through

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$$x_k = \begin{cases} b_k - (b_k - a_k)\cos^2\left[\tan^{-1}\left(\dfrac{\sqrt{\Delta_o^2 - a_k^2}}{b_k - a_k}\right)\right], if \cos^{-1}\left[\sqrt{\dfrac{\Delta_o - a_k}{b_k - a_k}}\right] > \tan^{-1}\left[\dfrac{b_k - a_k}{\sqrt{\Delta_o^2 - a_k^2}}\right], \\ \Delta_o - (\Delta_o - a_k)\cos^2\left[\tan^{-1}\left(\sqrt{\dfrac{\Delta_o + a_k}{\Delta_o - a_k}}\right)\right], if \cos^{-1}\left[\sqrt{\dfrac{\Delta_o - a_k}{b_k - a_k}}\right] < \tan^{-1}\left[\dfrac{b_k - a_k}{\sqrt{\Delta_o^2 - a_k^2}}\right] \end{cases} \quad (4)$$

where $k = 0,1,2,\ldots,N$, N is the number of D_{TM} or D_{TE} roots, a_k and b_k denote respectively the location of the zeros and infinities of either the $x\tan\left(x - jA_n^{'}\right)$ curve as shown in Fig. 2. The number of TM surface wave modes can be determined by using $N_{TM} < 1 + \hat{\Delta}_{TM}/\pi$ It is similar for TE mode.

III Extraction of Leaky Wave Poles for N-layered Microstrip Geometry

The extraction of the leaky-wave poles is given in the following section.

(i) Functional Expression Derivation

For the TM, the characteristic equation becomes $-\sqrt{\Delta_o^2 - x^2} - \dfrac{x}{\varepsilon_{r(n)}}\tan\left(x - jA_{(n-1)}^{'}\right) = 0,$ (5)

The required functional expression for the TM leaky-wave mode is thus expressed as

$$\sqrt{\Delta_o^2 - x^2}\left(1 + \tan(x)\tan\left(jA_{(n-1)}^{'}\right)\right) + \frac{x}{\varepsilon_{r(n)}}\left(\tan(x) - \tan\left(jA_{(n-1)}^{'}\right)\right) = 0, \quad (6)$$

(ii) Initial Guess

We notice that when the semicircle is tangent to the "tan" function, there is an intersection point B_o. Step 1: Assume two leaky poles in the last quadrant, where x_{left} is on the left side of B_o, and x_{right} is on the right side of B_o. Use similar method from section II to obtain the initial guess for all the roots on the left side of B_o, x_1, x_2, x_{n-1}, and x_{left}.Step 2: The last root is obtained using $x_{right} = x_{left} + b_n - a_n$. Step 3: then apply Halley's method [5] to the functional expressions. There are three possibilities in the last quadrant of tangent curve, no root, 1 root and 2 roots. If $x_{left} \neq x_{right}$ and $\varepsilon < 1e-10$, the last quadrant has two roots. If $x_{left} = x_{right}$ and $\varepsilon < 1e-10$, the last quadrant has one root. If the algorithm cannot find any root within 100 iterations, this implies that there is no root in the last quadrant.

IV Extraction of Lossy Improper Poles for N-layered Microstrip Geometry

The extraction of the lossy improper poles is described in the following section.

(i) Functional Expression Derivation

The expression for TM mode is $\sqrt{\Delta_o^2 - x^2}\left(1 + \tan(x)\tan\left(jA_{(n-1)}^{'}\right)\right) - \dfrac{x}{\varepsilon_{r(n)}}\left(\tan(x) - \tan\left(jA_{(n-1)}^{'}\right)\right) = 0.$

(ii) Initial Guess

The initial guess for this case is obtained from the following steps. Step 1: Assume initially that the substrate is lossless and let $\varepsilon_r = \varepsilon_{rr}$ and $\varepsilon_{ri} = 0$. Step 2: Use the exact same method as outline in section II to obtain the initial guess of x_{ini_lossy}.Step 3: Using the initial guess x_{ini_lossy}, we apply the well-known Halley's, and Traub's method to the functional expressions to extract the roots.

V Extraction Algorithm

With all the functional expressions and initial guesses for the surface wave modes, leaky wave modes and lossy improper modes been derived, we next apply the well-known Halley's method [5] and Traub's method [10] for the poles' extraction.

VI Numerical Results and Discussions

(i) Surface Wave Poles

Our algorithm is compared with some classical approaches, Brent method, bisection method and Davidenko's method [6] and Muller's method [7] as shown in Figs. 3. With the exception of the functional expression, the results obtained in Fig. 3 are subject to the same constraints, namely, (a) the function variation, $df = \left|1 - \frac{f_{n+1}}{f_n}\right| < 10^{-9}$, (b) the step size variation, $dx = \left|1 - \frac{x_{n+1}}{x_n}\right| < 10^{-9}$, and (c) the same initial guesses are used. Our proposed algorithm outperforms all the various classical methods. On average, an accuracy of 10^{-16} and an average of 6 iterations are required for the determination of all of roots under our proposed new method.

(ii) Leaky Wave Poles & lossy Improper Poles

The results of using Halley's and Traub's method with the new functional expressions for leaky wave poles and lossy improper poles extraction and Davidenko's method with classical equations [5] are shown in Table 1 and 2.

VII Conclusion

A unified approach for the fast evaluation of the locations of the surface wave poles, leaky wave poles and lossy improper poles in multi-layered microstrip technology has been presented for the first time. Using a simple re-organization of the classical characteristic equations, together with the Halley's method, a third order convergence is observed for the multi-layered microstrip poles' extraction. For the first time, good initial values have also been proposed.

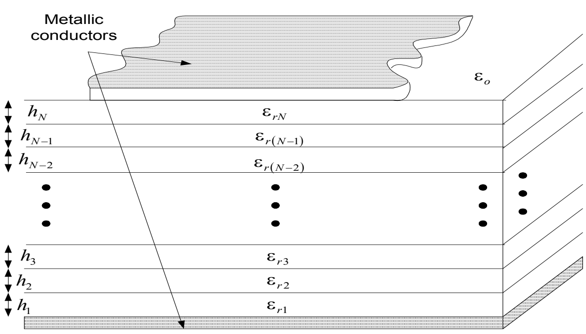


Fig. 1 N-layered microstrip topology

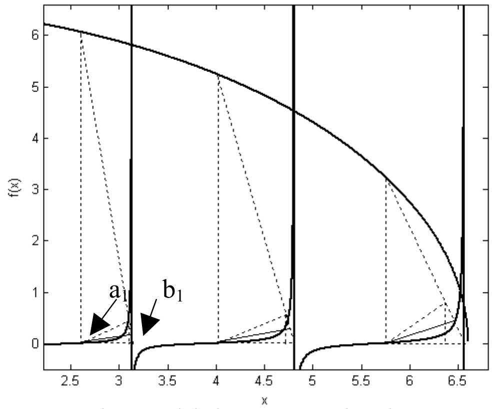
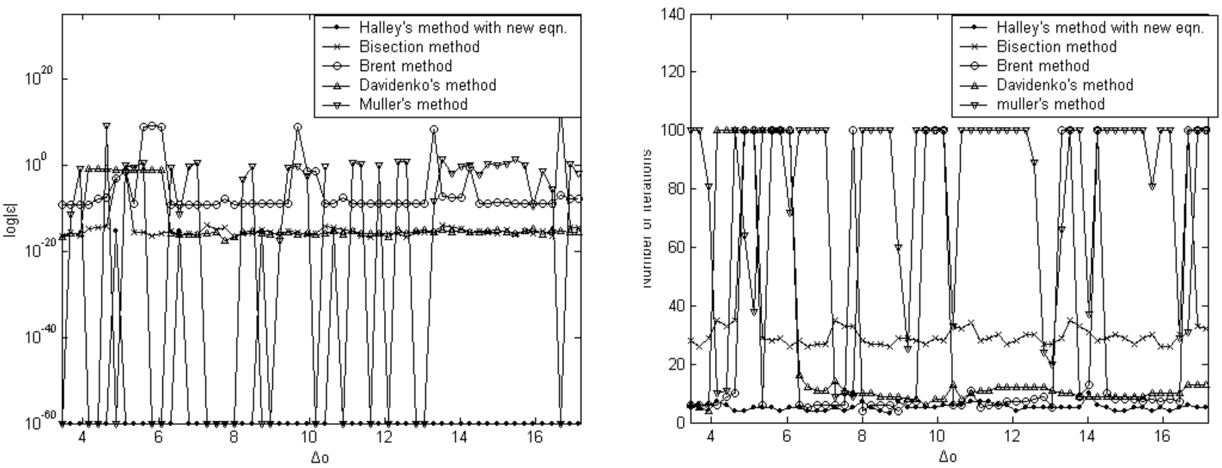


Fig.2 Initial guess evaluation.



(a) (b)

Fig.4 A numerical comparisons of the various classical methods in terms of the residue of the function and the number of iterations. (a)-(b) For the first TM root of the three-layered microstrip geometry under the case of $\varepsilon_{r1} < \varepsilon_{r2} < \varepsilon_{r3}$ and $k_o \le k_\rho \le k_o\sqrt{\varepsilon_{r1}}$

Leaky Poles	Traub's method(new eqn)	Halley's method (new eq.)	Davidenko's method (old eq.)
No. of Iterations	5	6	100
1st root	1.583226666	1.583226666	-1.424774824
df	10^{-32}	10^{-32}	-0.948361489
dx	10^{-32}	10^{-32}	481.9477339
2nd root	3.575614406	3.575614406	3.575614406
df	10^{-32}	10^{-32}	10^{-32}
dx	10^{-32}	10^{-32}	10^{-32}

Table 2 Comparison of the proposed approach and the Davidenko's method [11] for leaky-wave poles extraction. The parameters adopted are $\varepsilon_{r1} = 15, \varepsilon_{r2} = 59$, $\varepsilon_{r3} = 37.5$, $h_1 K_o = 0.8$, $h_2 K_o = 0.9$, $h_3 K_o = 0.95$, and $\sqrt{\varepsilon_{r1}} K_o < K_\rho < \sqrt{\varepsilon_{r3}} K_o$.

Lossy poles	Traub's method (new eq.)	Halley's method (new eq.)	Davidenko's method (old eq.)
No. of Iterations	12	14	100
1st root	1.01182227081261 - 0.00043924163301i	1.01182227081261 - 0.00043924163301i	1.23730715979912 + 0.31064812216076i
df	10^{-32}	10^{-32}	-3.23468912
dx	10^{-32}	10^{-32}	5.752047286
2nd root	2.97282693490741 + 0.01579544532283i	2.97282693490741 + 0.01579544532283i	2.97282693490741 + 0.01579544532283i
df	10^{-32}	10^{-32}	10^{-32}
dx	10^{-32}	10^{-32}	10^{-32}
3rd root	6.38008671592520 + 0.00807064377279i	6.38008671592520 + 0.00807064377279i	6.13059487401265 + 0.01884096006964i
df	10^{-32}	10^{-32}	2.309324782
dx	10^{-32}	10^{-32}	2.623262689

Table 3 Comparison of the proposed approach and the Davidenko's method [11] for lossy improper poles extraction. The parameters adopted are $\varepsilon_{r1} = 35.5 + i0.1, \varepsilon_{r2} = 37.5 + i0.1$, $\varepsilon_{r3} = 59 + i0.1$, $h_1 K_o = 0.4$, $h_2 K_o = 0.4$, and $h_3 K_o = 0.5$.

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