Dependence of backscatter radar cross section on wavelengths of ocean waves in the presence of varying currents

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1 Introduction

Understanding of microwave backscatter from sea surface is an important topic not only as a fundamental study but also for interpreting the radar data of various oceanic features. The general theoretical approach is to compute, using an appropriate scattering model, the RCS from ocean waves of entire spectrum within an illuminating area or a resolution cell. The resultant RCS is therefore an integral or sum of all RCS contributions from waves of different wavenumbers; however, little is known on the relative RCS contribution from waves of individual wavenumber. In the present article, we present a new and simple theory [3] to illustrate the dependence of backscatter RCS on the ocean waves of individual wavenumbers. The theory is based on the Kirchhoff scattering (Physical Optics) model [2] to compute the distribution function of RCS in terms of individual ocean wavenumbers by differentiating the cumulative RCS. The principal theory is first described and the C- and L-band RCS distribution function is evaluated for the sea surface in the presence of varying surface currents, and the consequence of the results is discussed in interpreting the HF radar and along-track interferometric synthetic aperture radar (InSAR) data [1] used to estimate the ocean currents.

2 Formulation

We wish to determine the RCS, $\delta\sigma(\mathbf{k})$, from waves of a single wave vector $\mathbf{k}$. Let $\sigma_c(\mathbf{k})$ is the cumulative RCS from waves of wave vector between 0 and $\mathbf{k}$, then we may put

$$\sigma_c(\mathbf{k}) = \int_0^{\mathbf{k}} \delta\sigma(\mathbf{k}')\, d\mathbf{k}' \tag{1}$$

The limit of integral $\mathbf{k}$ implies that the integral is taken over the angles from 0 to 2π and the wavenumbers from 0 to k. Differentiating (1) yields

$$\frac{\partial}{\partial \mathbf{k}}\sigma_c(\mathbf{k}) = \frac{\partial}{\partial \mathbf{k}} \int_0^{\mathbf{k}} \delta\sigma(\mathbf{k}')\, d\mathbf{k}' = \delta\sigma(\mathbf{k}) \tag{2}$$

Thus, if the cumulative RCS, $\sigma_c(\mathbf{k})$, is known as a function of $\mathbf{k}$, the RCS contribution from a particular wavenumber can be calculated by differentiating the cumulative RCS. This is the basis of computing the RCS distribution function. The cumulative RCS can be calculated as follows.

According to the Kirchhoff model, the mean RCS, $\sigma \equiv \langle\sigma\rangle$, from the ocean surface is given by

$$\sigma = \frac{k_r^4}{\pi k_z^2} \int_{-\infty}^{\infty} \exp\left(-4k_z^2\left(\langle H^2(0)\rangle - \langle H(0)H(\mathbf{r})\rangle\right)\right) \cdot \exp\left(-i2\mathbf{k}_H \cdot \mathbf{r}\right)\, d\mathbf{r} \tag{3}$$

where the surface is assumed to be perfectly conducting, H is the surface height at the position $\mathbf{r}$, $k_z = k_r \cos(\theta_i)$, $\mathbf{k}_H = \mathbf{k}_r \sin(\theta_i)$, $\mathbf{k}_r$ is the radar wave vector, and θ_i is the incidence angle. The surface height is assumed to obey a Gaussian random process, and the angular brackets $\langle\rangle$ indicate the ensemble average. $\langle H^2(0)\rangle$ and $\langle H(0)H(\mathbf{r})\rangle$ in (3) are the variance and the autocorrelation function (ACF) of surface height respectively. The cumulative RCS can be computed from (3) by substituting the ACF of waveheight

$$\langle H(0)H(\mathbf{r})\rangle = \int_0^{\mathbf{k}} \Psi(\mathbf{k}') \cos(\mathbf{k}' \cdot \mathbf{r})\, d\mathbf{k}' \tag{4}$$

where the $\Psi(\mathbf{k}')$ is the waveheight spectrum, and the integral is taken over all angles and the wavenumber from 0 to k. We consider the equilibrium waveheight spectrum given by the Pierson and Moskowitz spectrum with $\cos^2((\theta-\theta_W)/2)$ wind directional dependence, where θ is the angle of $\mathbf{k}'$ in the polar co-ordinate, and θ_W is the wind direction.

In the presence of a varying current induced by internal waves or current interaction with bottom topography, however, it is necessary to compute the waveheight spectrum perturbed from its equilibrium state. To describe the hydrodynamic interaction between ocean waves and a slowly-varying surface current we use the following action balance equation [2].

$$\left(\frac{\partial}{\partial t}+\frac{\partial \omega}{\partial \mathbf{k}}\frac{\partial}{\partial \mathbf{r}}-\frac{\partial \omega}{\partial \mathbf{r}}\frac{\partial}{\partial \mathbf{k}}\right)N=S \tag{5}$$

where t is a time variable, $\omega(\mathbf{k},\mathbf{r},t)=\omega'+\mathbf{k}\cdot\mathbf{U}$ is the apparent wave frequency in the frame moving with the current field $\mathbf{U}(\mathbf{r},t)$, $\omega'(\mathbf{k})$ is the wave intrinsic frequency in the local rest frame, given by $\omega'^2=gk+\varepsilon k^3$, ε is the ratio of the surface tension to the water density, and $N(\mathbf{k},\mathbf{r},t)$ is the action spectrum related to the surface height spectrum $\Psi(\mathbf{k},\mathbf{r},t)$ by $N=(\omega'/k)\Psi$. The first term on the left hand side of (5) describes the change in the action spectrum caused by the intrinsic local time change, $\partial\omega/\partial\mathbf{k}$ is the wave group velocity in the moving frame, and $\partial\omega/\partial\mathbf{r}$ is the rate of change in the wavenumber due to wave refraction. Thus, the left hand side describes the energy transport and refraction, and the interaction with a varying surface current. This total rate of change in the action spectrum is balanced with the source function S on the right hand side of (5). The source function describes the rate of change in N due, for the present interest of the wave-current interaction, to the energy input by wind and the loss of energy by wave dissipation such as wave breaking or white capping. For the present analysis, we consider the source function given by $S=\beta N(1-N/N_0)$, where the first and second terms represent the exponential wave growth and decay respectively, $N_0(\mathbf{k})$ is the equilibrium action spectrum and $\beta(\mathbf{k})$ is the spectral relaxation rate The action balance equation of (5) can be solved by assuming $\partial Q/\partial\mathbf{k}\simeq\partial Q_0/\partial\mathbf{k}$, and putting $Q(\mathbf{k},\mathbf{r},t)=1/N(\mathbf{k},\mathbf{r},t)$ and $Q_0(\mathbf{k})=1/N_0(\mathbf{k})$. Then (5) becomes a first-order non-linear differential equation, yielding a solution

$$Q(\mathbf{k})=Q_0(\mathbf{k})+\int_{-\infty}^{t}\frac{\partial Q_0}{\partial \mathbf{k}}\mathbf{k}\cdot\frac{\partial \mathbf{U}(\mathbf{r},t';t)}{\partial \mathbf{r}}\exp\left(-\beta(t-t')\right)dt' \tag{6}$$

where $\mathbf{U}(\mathbf{r},t';t)=\mathbf{U}[\mathbf{r}-(\partial\omega'/\partial\mathbf{k}+\mathbf{U}-\mathbf{v}_p)(t-t'),t]$, $\mathbf{v}_p$ is the phase velocity vector of the current field, and $\partial\omega'/\partial\mathbf{k}=\mathbf{c}_g(\mathbf{k})$ is the wave group velocity.

For a varying current (soliton in this case), $U(\mathbf{r})=u_0\,\mathrm{sech}^2\left(\mathbf{K}\cdot(\mathbf{r}-\mathbf{v}_pt)\right)$, there is no analytic solution of (6), and hence (6) needs to be solved numerically [3]. Once (6) is computed, the perturbed waveheight spectrum is used in (4) and $\delta\sigma(\mathbf{k})$ can be calculated in the manner described earlier.

3 Results and Discussion

Fig.1(top) illustrates the surface current as a function of distance. The peak current speed is assumed to be 0.3 m/s, the soliton wavenumber of $K=2\pi/400$ rad/m, the wind speed of $W=5$ m/s, $v_p=0$ and $\theta_W=0°$. Fig.1(bottom) is the corresponding C-band (4.5 GHz) RCS modulation relative to the RCS from an ambient sea surface, where the solid, broken and solid-dot lines correspond respectively to the incidence angles $\theta_i=20°$, 35° and 45°. It can be seen that higher RCS modulation can be obtained for smaller incidence angle. We have also computed the L-band RCS modulation, and obtained similar results with higher modulation than the C-band.

The C-band RCS distribution function is computed for $\theta_i=20°$ in Fig.2, at (a) an ambient position, (b) minimum RCS modulation, (c) maximum current speed and (d) maximum RCS modulation. As can be seen from the figure, the RCS distribution function is centered at the ocean wavenumbers much smaller than the Bragg wavenumber k_B, particularly for (a) and

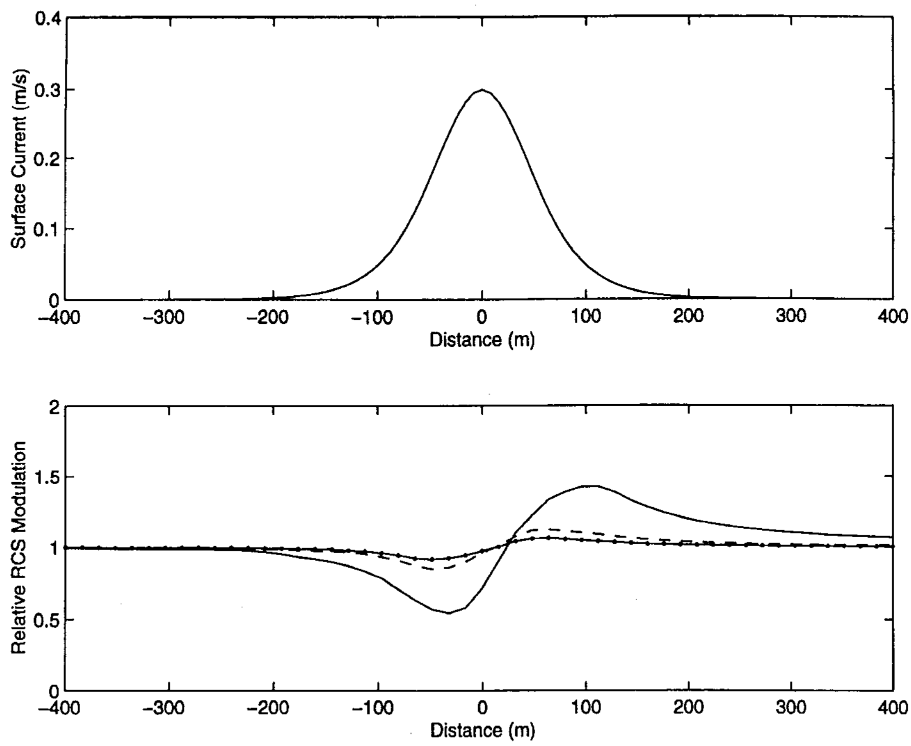

Fig.1 Illustrating the solitary surface current (upper) and C-band RCS modulation (bottom).

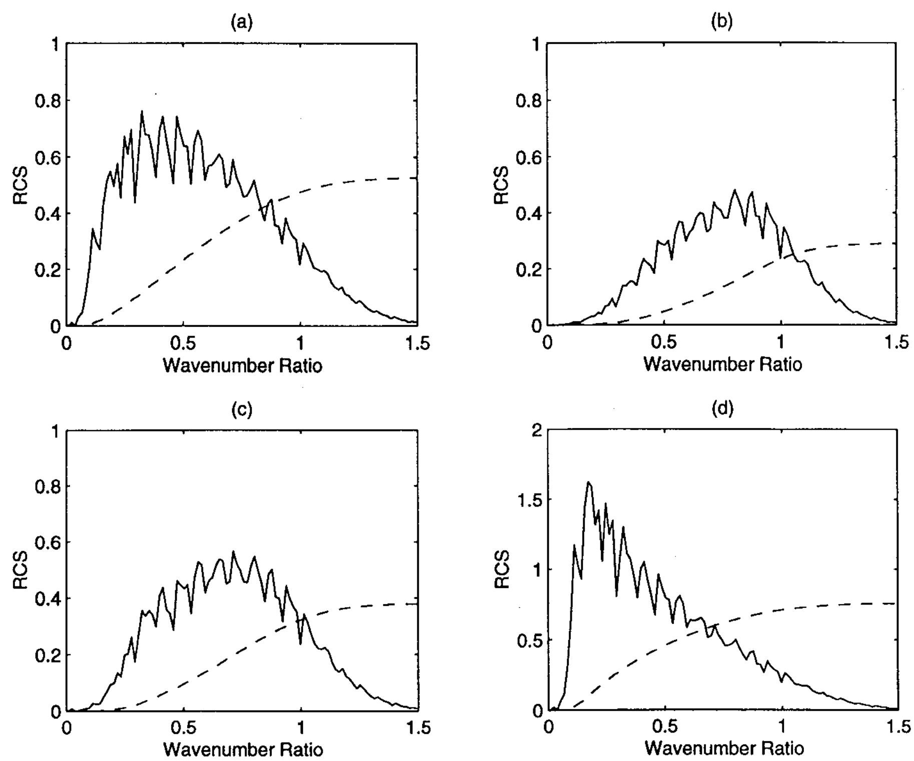

Fig.2 RCS distribution function at $\theta_i = 20°$ for different surface positions.

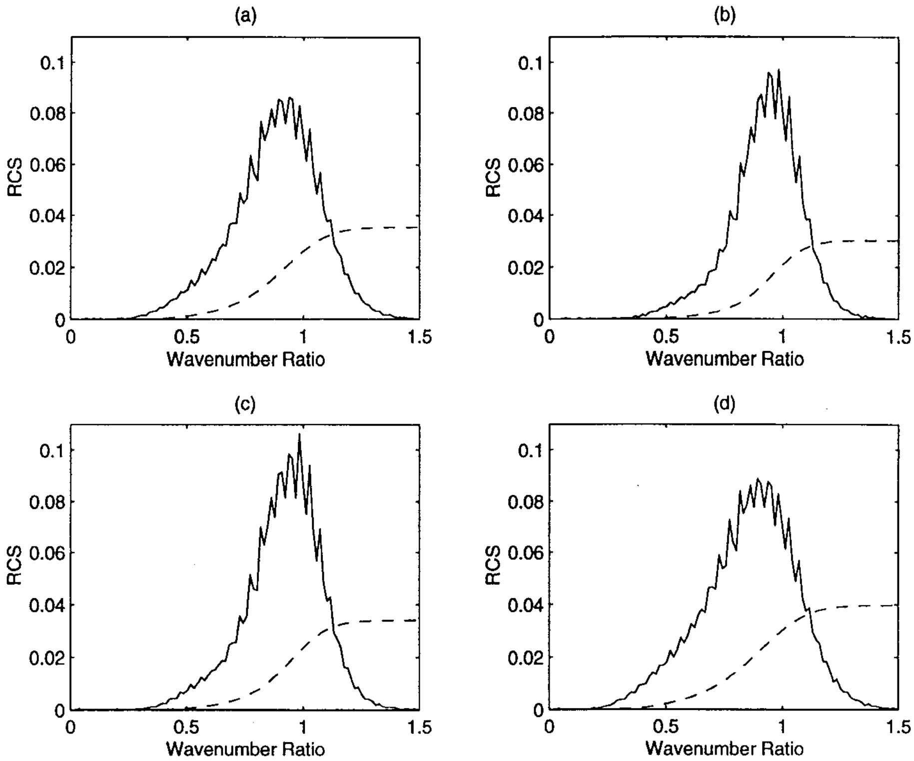

Fig.3 RCS distribution function at $\theta_i = 35°$ for different surface positions.

(d), and waves in a wide range of spectrum contribute to the total RCS. In (d) of a current convergence area, the Bragg scattering is almost negligible.

Fig.3 is the same as Fig.2 but for $\theta_i = 35°$. It can be seen that the RCS distribution function tends to be centered at ocean wavenumbers closer to the Bragg wavenumber at all surface positions in comparison with Fig.2. This feature suggests increasing Bragg wave contribution to RCS with increasing incidence angles. We have observed similar trends in L-band but the non-Bragg (quasi-specular) contribution is less significant than C-band.

4 Conclusion

A new simple way of computing the RCS distribution as a function of the wavenumber of ocean waves has been developed, and evaluated for the ocean surface having varying surface currents. In interpreting the current measurements by HF (Doppler) radars and along-track InSAR [1], the phase velocity of the Bragg waves is subtracted from the radar-derived velocity, because the Bragg scattering is assumed to be dominant. However, the present study indicates that the dominant RCS contribution depends on the positions of current divergence and convergence, and the total RCS is composed of contributions from ocean waves in a wide range of spectrum. Accordingly, the phase velocities of these waves need to be taken into consideration. A possible solution to this problem is that the RCS distribution function can be used to weight the phase velocities of ocean waves. It should yield more accurate velocity measurements than just subtracting the phase velocity of the Bragg waves.

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