RADAR REMOTE SENSING OF SURFACES CHARACTERIZED BY 1-D PROFILES

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1. Introduction

For solving the remote sensing problems, it is important to know the polarization patterns of radio waves reflected from underlying surfaces. Information is extracted from the elements of the scattering matrix when illuminating areas on the surface. It is clear that variation in electrical-and-physical properties (salinity, moisture, soil composition, etc.) of such areas will cause changes in the main electrodynamics characteristic of the surface - its complex permittivity ***e***. Changes in the complex permittivity variation result into variation in reflecting characteristics of the underlying surface (i.e. characteristics of its elements of the scattering matrix). Basic results are presented for different models: exponential and quadratic profile. Attention is paid to vertical probing for investigating polynomial, linear, quadratic, matching and intermediate layered profiles.

2. General relations

In Radar Remote Sensing the scanning of underlying surfaces is carried out from above (platform on–board aircraft, satellite, stationary tower, etc.) The dimensions of the illuminated area of the underlying surface are determined by the height (*H*) from which scanning is carried out, the scanning angle (*q*) and the antenna beam width (in two mutually perpendicular planes *Da* and *Db*); see Fig. 1.

The wave radiated by the antenna induces (at the surface *D*) currents which are the sources of a scattered field. In the general case, the energy is scattered omnidirectionally and also in the direction of the antenna

We shall consider a medium which fills the half-space $z<0$. Electrical and physical properties of this medium depend only on the depth inside the medium surface, meaning that the permittivity profile $e = e(z)$ is just a function of z.

The plane electromagnetic wave is incident on the boundary (i.e. medium surface) at the angle ***q***. The electric field vector of this wave is perpendicular to the x-z plane of incidence (i.e. horizontal polarization parallel to the *y–axis*). The total field in the upper half-space consists of the incident and reflected waves. The electric field vector of the wave inside this medium wave should meet the differential equation: $\Delta E(x,z)+k^2 e(z)E(x,z)=0$. After substitution of $E(x,z)=g(z)\exp\{ikx\sin q\}$ this equation will be reduced to the differential equation:

$$g''_{zz}+k^2\left[e(z)-\sin^2 q\right]g(z)=0\,. \qquad (1)$$

Using standard conditions of continuity of tangential components of the vectors $\vec{E}$ and $\vec{H}$, it is possible to derive the formula for the reflection coefficient of horizontal polarization (HP): $R_{HP}(q)=\left[ikg(0)\cos q+g'_z(0)\right]/\left[ikg(0)\cos q-g'_z(0)\right]$.

For a vertical polarization, the magnetic field vector inside the medium may be represented by: $H(x,z)=n(z)v(z)\exp(-ikx\sin q)$, where $n(z)=\sqrt{e(z)}$ and the function $v(z)$ is the solution of the differential equation: $\left[n(z)n(z)\right]''_{zz}+k^2\left[n^2(z)-\sin^2 q\right]n(z)v(z)=0$. Using similar conditions of continuity it is possible to derive the formula for the reflection coefficient: $R_{VP}=\left[g'_z(0)+ikg(0)e(0)\cos q\right]/\left[g'_z(0)-ikg(0)e(0)\cos q\right]$.

3. Exponential layer

The complex permittivity varies according an exponential law: $e(z) = b \cdot \exp\{2a \cdot z\} = (b_1 + ib_2)\exp\{2a \cdot z\}$; $\operatorname{Im} a = 0$. In this case we find for the function $g(z)$ as solution of equation (1): $g(z) = H_x^{(2)}\left(\frac{k}{a}\sqrt{b} \cdot \exp\{a \cdot z\}\right)$, where $H_x^{(2)}$ is the Hankel function of the second kind. The order of this function is $x = \frac{k}{a}\sin q$.

For the reflection coefficient R_{HP} we may derive (after rewriting the derivative of the Hankel function):

$$R_{HP} = \left[iH_x^{(2)}\left(\frac{k}{a}\sqrt{b}\right)e^{-iq} - \sqrt{b}\,H_{x+1}^{(2)}\left(\frac{k}{a}\sqrt{b}\right)\right] \Big/ \left[iH_x^{(2)}\left(\frac{k}{a}\sqrt{b}\right)e^{-iq} + \sqrt{b}\,H_{x+1}^{(2)}\left(\frac{k}{a}\sqrt{b}\right)\right].$$

4. Quadratic layer

The complex permittivity in a quadratic layer varies according: $e(z) = (az + b)^2$. For such a medium it is impossible to find the exact solution of (1) with the use of known functions. With small α, it may be derived: $g(z) = \frac{1}{\sqrt[4]{(az+b)^2 - \sin^2 q}} \exp\left\{-ik\int_0^z \sqrt{(az+b)^2 - \sin^2 z}\,dz\right\}$.

5. Vertical scanning

Vertical scanning is quite often used as technique for solving the remote-sensing problems probing surfaces. In this case there is no difference between vertical and horizontal polarizations and the scattering matrix becomes the identity matrix. However, the reflected-wave power depends to a great extent on the behavior of dielectric qualities of the investigated surfaces. Let us consider the reflection for different analytical permittivity profiles in which the complex permittivity changes with depth.

5.1 Polynomial layer

The complex permittivity change is described by the relation: $e(z) = (az + b)^m$.

Substitution in equation (1) (with θ=0) leads to:

$$g(z) = \sqrt{az+b} \cdot J_{\frac{1}{m+2}}\left[\frac{2}{m+2}\left(\frac{k}{a}\right)^{\frac{m+2}{4}}(az+b)^{\frac{m+2}{2}}\right].$$

This solution satisfies the radiation condition at infinity.

5.2 Linear layer

The complex permittivity change is now described by the relation: $e(z) = az + b$. The reflection coefficient becomes: $R = \left[H_{1/3}^{(2)}\left(\frac{2k}{3a}\right) - i\sqrt{b} \cdot H_{-2/3}^{(2)}\left(\frac{2k}{3a}\right)\right] \Big/ \left[H_{1/3}^{(2)}\left(\frac{2k}{3a}\right) + i\sqrt{b} \cdot H_{-2/3}^{(2)}\left(\frac{2k}{3a}\right)\right]$.

Analysis of this expression with arbitrary complex values a and b is difficult. We therefore consider only extreme cases of small and large $|a|$ and give the relations between $|R|$ and $|a|$ for several (typical and practical) important cases.

With small $|a|$ (when the complex permittivity modulus slowly varies with depth), we derive: $R = \left[1 - b^{0,5} + ib^{-1,5}\frac{a}{k}\left(0{,}104 + 0{,}146b^{0,5}\right)\right] \Big/ \left[1 + b^{0,5} + ib^{-1,5}\frac{a}{k}\left(0{,}104 - 0{,}146b^{0,5}\right)\right]$. When a sharp boundary is absence (i.e. b=1), we have: $R = 0{,}004|a|^2\ l^2$. In this case the reflection is nearly absent.

Another extreme is the case in which the complex permittivity increases rapidly with depth, i.e. $|a|$ is large. In this case we derive: $R = -1 + 1{,}58\left(\frac{2kb\sqrt{b}}{3a}\right)^{1/3}(1 - i \cdot 1{,}73)$. This formula shows that for large values of $|a|$ the reflection coefficient module is close to 1. With a further increase of $|a|$ and λ it tends to 1.

A similar behavior of R can be found for the polynomial layered model. The dependence of $|R|^2$ (power reflection coefficient) on the parameter $|a|$ for the linear layer is shown in Fig.2.

5.3 Parabolic layer

The parabolic layer is described as quadratic layer: $e(z) = (az + b)^2 = [(a_1 + ia_2)z + (b_1 + b_2)]^2$. In this case the reflection coefficient is given by the expression:

$R = \left[H^{(2)}_{0,25}(w) - ibH^{(2)}_{-0,75}(w)\right] / \left[H^{(2)}_{0,25}(w) + ibH^{(2)}_{-0,75}(w)\right]$ with $w = \frac{kb^2}{2a}$. When $|a|\ l \ll b$ we obtain: $R = [(1 - b)kw + i(0{,}0094 + 0{,}156b)] / [(1 + b)kw + i(0{,}0094 - 0{,}156b)]$. When a sharp boundary is absent (b=1) we get: $R = 0{,}016|a|^2\ l^2$. In another extreme case, when $|a|\ l \gg b$ we may find: $R = -1 + \frac{5{,}1(1 - i)}{\sqrt{|a|\ l}}$.

5.4 Matching layer

In a number of extreme cases a thin intermediate layer is formed at the medium-air boundary. In this layer, the complex permittivity smoothly varies from 1 (atmosphere) to its final permittivity value after some depth. The presence of such a layer results in a number of cases into a substantial decrease of the reflection coefficient due to the decrease in reflections that takes place at the boundary.

In order to describe this situation we consider the reflection from the following structure: the region $z \le -h$ is filled with a medium with the complex permittivity e_k. The “matching” layer is located within $-h \le z \le 0$. The complex permittivity e_s of this layer varies according a cosinusoidal law: $e_s = \frac{1 + e_k}{2} + \frac{1 - e_k}{2}\cos p\frac{z}{h}$. This relation shows that when z=0, the complex permittivity $e_s = 1$ and when $z=-h \rightarrow e_s = e_k$ and $e'_s(0) = e'_s(h) = 0$. This profile is chosen so that matching (including the derivatives) takes place at the boundaries of the layer. The wave equation for the matching layer is simplified by means of the substitution of an independent variable into the Mathieu equation.

The results of the calculation of the reflection coefficient with different values of the complex permittivity are shown in Fig.3. This figure shows that the reflection coefficient varies from $\left|\left(1 - \sqrt{e}\right) / \left(1 + \sqrt{e}\right)\right|^2$ to zero. Similar curves for a matching layer with a linear dependence (without the matching for the derivatives) are shown in the same figure for comparison. The figure shows that a decrease in the reflection coefficient for the linear layer is faster than for the matching “cosimusoidal” layer. For a thick layer ($h \to \infty$) we find $R = 0{,}002|1 - e_k|^2\left(\frac{l}{h}\right)^2$ and for a thin layer ($h \to 0$) $R = \frac{20{,}4}{\sqrt{|1 - e_k|}}\sqrt{\frac{h}{l}}$.

5.5 Intermediate layer

The exponential permittivity profile as given in Section 3, may correspond to an intermediate layer. In this case the formulae in Section 3 remain valid. For a medium with small α the reflection becomes:

$$R = \left[1 - \sqrt{|b|}e^{i\,d/2} + 0{,}125\frac{i}{r_0}\left(1 + 3\sqrt{|b|}e^{-i\,d/2}\right)\right] / \left[1 + \sqrt{|b|}e^{-i\,d/2} + 0{,}125\frac{i}{r_0}\left(1 - 3\sqrt{|b|}e^{-i\,d/2}\right)\right],$$ where $r_0 = k\frac{\sqrt{|b|}}{a}e^{-i\,d/2}$, and d is the angle describing the losses. This formula shows that the reflection coefficient is mainly determined by the complex permittivity on the boundary. In particular, for a medium with a low permittivity (sweet water) we derive: $R = \frac{1-\sqrt{|b|}}{1+\sqrt{|b|}}\left(1 - \frac{0{,}14a\ l}{|b|^2 - 1}\right)$. In the other extreme in which α is large we get: $R \approx -1 + 2N_0(r_0)/\left[N_0\left(r_0 + 2i\sqrt{b}\,e^{i\,d/2}/p\,r_0\right)\right]$. Some dependencies of the reflection coefficient upon α with different values of β and δ are shown in Fig.4.

6. Conclusion

In this paper we show that 1-D models characterizing the 2-medium problem may give interesting insights in the e.m. properties of the soil. The models are based on 1-D permittivity profiles with depth. Vertical probing results have been shown using expressions for the power reflection coefficient. Probing under certain scan angles allow a polarimatric radar approach and may give even more information on the underlying medium.

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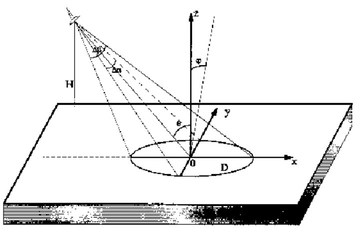

Fig.1. Coordinate System

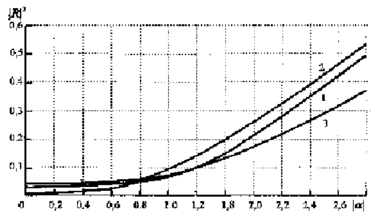

Fig.2. Dependence $|R|^2$ upon $|a|$ cm^{-1} for a linear layer. l =3cm.

$1.e(z) = 2 + i|a|z;\quad 2.e(z) = 1 + i|a|z;\quad 3.e(z) = 2 + 0{,}5\sqrt{2}(1+i)|a|\,z$

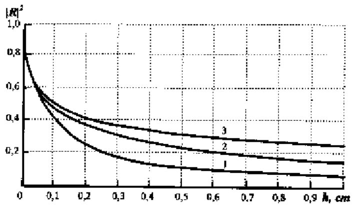

Fig.3.Dependence $e(z)$ upon the thickness h of an intermediate layer. In the intermediate layer $e(z)$ changes from $e = 1$ up to $e = 65 - 40i$ according to a linear (1), harmonic (2) and exponential (3) profile. l =3cm.

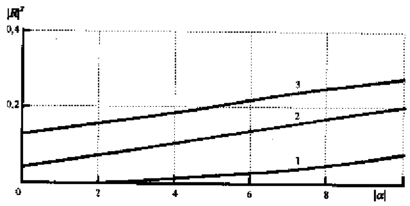

Fig.4.Dependence $|R|^2$ upon $|a|$ cm^{-1} for an exponential profile.
1. e(z) = exp(1+i0,1az); 2. e(z) = (1+i) exp(az);
3. e(z) = (3,5+2i) exp(az)