Analysis of Radio Wave Propagation over Mixed-Path Including Tropospheric Ducting Effect

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1. INTRODUCTION

The problem of medium-frequency(MF) and high-frequency(HF) wave propagation in urban areas has been studied for many years. However , there exists no adequate prediction method for general applications. The measurements in London areas have shown the unexpected high attenuation in some parts of UK cities and the anomalous variation patterns of the electromagnetic field with distance[1].It has been shown theoretically that , in contrast to the homogeneous earth case , the field over a highly inductive surface impedance behaves in a peculiar manner due to interference between the usual Norton ground-wave and the trapped surface wave[2],[3]. The ground wave propagation is strongly influenced when the impedance of the propagation path changes abruptly as it occurs when the radio wave traverses a coastline[4]. Experimental measurements for frequencies between 4 and 32MHz have shown that the signal level is also influenced by the tropospheric ducting[5].

In the present study , we investigate the ground wave propagation along the inhomogeneous impedance surface. We assume that the surface duct is appeared over the mixed-path[6]. We will compare the theoretical results with the experimental data obtained by the measurements carried out in Tokyo areas in Japan.

2. FORMULATION FOR CONSTANT IMPEDANCE

A modified refractive index m(z)=n(z)+z/a , where n(z) is the refractive index in the atmosphere and a is the radius of curvature of the earth , and the coordinate systems (x , z) are shown in Fig.1(a) and Fig.1(b) , respectively. The modified refractive index m(z) decreases as the function of the height z from the earth's surface , which assumes the occurrence of the ground-based tropospheric duct. The earth's surface is described by a constant impedance surface Z_a [3],[6], which is defined as the ratio of the tangential electric field to the tangential magnetic field at the air-ground interface.

When the vertical electric dipole source A of current moment $I_a dl$ is located on the earth's surface , the vertical electric field E at the distance x may be represented by

$$E = E_0 W(x) \quad , \quad E_0 = \frac{i\omega\mu_0 I_a dl}{2\pi x} e^{ikx} \qquad (1)$$

where E_0 is the vertical electric field excited by the same electric source located on the flat , perfectly conducting surface. The W(x) is the attenuation function containing the information about the surface impedance Z_a , the distance x , the duct parameter , and the frequency.

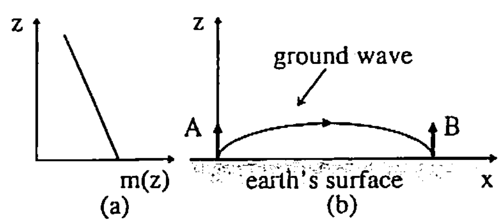


Fig.1 (a) Modified refractive index.
(b) Constant impedance surface and coordinate systems(x , z)

When the m(z) is approximated by m(z) $\fallingdotseq 1 - z/a_0$, where a_0 is the constant parameter , the attenuation function can be given by the following from:

$$W(\hat{x})=\sqrt{\pi x}\,e^{i\pi/4}\frac{i}{2\pi}\int_{-\infty}^{\infty}\frac{\exp(i\hat{x}t)}{Ai'(t)/Ai(t)+q}dt\ ,\ \hat{x}=mx/a_0\ ,\ m=(ka_0/2)^{1/3}\ ,\ q=im\sqrt{\varepsilon_r}\,\hat{Z}_a \tag{2}$$

where ε_r and $\hat{Z}_a(=Z_a/120\pi)$ are the dielectric constant of the air at the interface and the normalized surface impedance of the earth , respectively. Since the radio waves are confined to the earth's surface , we have used the Airy functions Ai(t) and Ai'(t) in Eq.(2).

Employing large argument expansions for the Airy functions Ai(t) and Ai'(t) , and applying the following transformations:

$$t=sq^2\exp(-i\pi)\ ,\ \hat{x}=p\left(iq^2\right)^{-1}\exp(-i\pi)\ ,\ q=2^{-1/3}\delta^{-1}\exp(i\pi) \tag{3}$$

in Eq.(2) , one may obtain the following integral.

$$W(p)=\sqrt{p}\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{e^{ps}}{s}\left[\frac{i\sqrt{\pi}\,s}{1+i\sqrt{s}}+\frac{i\sqrt{\pi}}{2\left(1+i\sqrt{s}\right)}\delta^3+i\sqrt{\pi}\left\{\frac{1}{4s\left(1+i\sqrt{s}\right)^3}-i^{5/8}\frac{1}{\left(1+i\sqrt{s}\right)^2 s^{3/2}}\right\}\delta^6\right.$$
$$\left.+i\frac{\sqrt{\pi}}{8}\left\{\frac{1}{\left(1+i\sqrt{s}\right)^4 s^2}-\frac{15}{\left(1+i\sqrt{s}\right)^2 s^3}-\frac{5i}{\left(1+i\sqrt{s}\right)^3 s^{5/2}}\right\}\delta^9\right]ds \tag{4}$$

The inverse Laplace transform in Eq.(4) is evaluated by applying the formula[3] given by Bremmer. One may obtain the following representation.

$$W(p)=F(p)-\frac{1}{2}\left\{(1+2p)F(p)-1-i\sqrt{\pi p}\right\}\delta^3+\left\{\left(\frac{1}{2}p^2-1\right)F(p)+i\sqrt{\pi p}(1-p)+1-2p+\frac{5}{6}p^2\right\}\delta^6$$
$$-\left\{\left(\frac{35}{8}-\frac{1}{4}p^2+\frac{1}{6}p^3\right)F(p)-i\sqrt{\pi p}\left(\frac{35}{8}-\frac{35}{8}p+\frac{31}{16}p^2-\frac{5}{16}p^2\right)-\frac{35}{8}+\frac{35}{4}p-\frac{67}{12}p^2+\frac{5}{3}p^3\right\}\delta^9\cdots \tag{5}$$

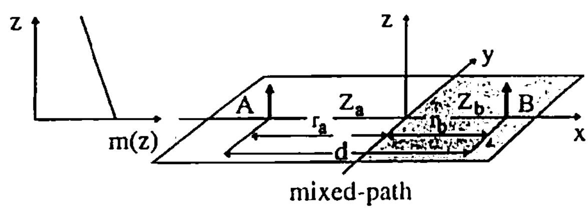
The Sommerfeld function F(p) , the numerical distance p , and the duct parameter δ are defined as follows.

$$F(p)=1+i\sqrt{\pi p}\,e^{-p}\mathrm{erfc}\left(-ip^{1/2}\right)\ ,\ p=\frac{ikx}{2}\hat{Z}_a\ ,\ \delta=i2^{-1/3}\Big/\left\{(ka_0/2)^{1/3}\hat{Z}_a\right\} \tag{6}$$

The attenuation function in Eq.(5) for the surface duct problem may be reduced to the attenuation function for the spherical earth[3] of radius a_0 by the analytic continuation from a_0 to $-a_0\ (=a_0\exp(i\pi))$.

3. FORMULATION FOR INHOMOGENEOUS EARTH'S SURFACE

In this section , we derive the electromagnetic field representation which takes into account the existence of the tropospheric surface duct. The modified refractive index $m(z)(=1-z/a_0)$ and the mixed-path associated with the coordinate systems (x , y , z) are shown in Fig.2(a)and Fig.2(b) ,respectively. The surface impedance assumes the value Z_a for x<0 and Z_b for x>0. $d(=r_a+r_b)$ is the total distance between the transmitting antenna A and the receiving antenna B.



(a) (b)

Fig.2 (a) Modified refractive index.
(b) Mixed-path and coordinate systems(x , y , z).

Following the general treatment of Wait[4] , one may obtain the vertical electric field at the observation point B as follows:

$$E = E_0 W'(p_0) \tag{7}$$

$$W'(p_0) = W(p_0) + i\left(\frac{p_0}{\pi}\right)^{1/2}\left(1-\sqrt{K}\right)\int_0^{p_0V/K}\frac{W(p)W(p_0-Kp)}{\sqrt{p(p_0-Kp)}}dp \tag{8}$$

$$p_0 = \frac{ikd}{2}\hat{Z}_a^2 \quad , \quad K = \frac{\sigma_b - i\omega\varepsilon_b}{\sigma_a - i\omega\varepsilon_a} \quad , \quad V = \frac{r_b}{d} = \frac{r_b}{r_a + r_b} \tag{8a}$$

where E_0 is the reference field defined in Eq.(1) . The $W'(p_0)$ is the attenuation function over the mixed-path and the $W(p_0)$ is the attenuation function over the constant impedance surface Z_a derived in Eq.(5). The second term in the right hand side of Eq.(8) is the contribution from the portion of the impedance surface Z_b.

The integral in Eq.(8) may be divided into two parts:

$$\int_0^{p_0V/K} G(p)dp = \int_0^{p'} G(p)dp + \int_{p'}^{p_0V/K} G(p)dp. \tag{9}$$

where G(p) is the integrand in Eq.(8) and p′ is an arbitrary value which satisfies:

$$0 < |p'| \ll |p_0V/K| \quad , \quad |Kp'| \ll |p_0| \quad , \quad |p_0| \gg 1. \tag{10}$$

Then the first term in Eq.(9) is evaluated analytically by applying the following approximation and the power series expansion for the Sommerfeld function:

$$\frac{W(p_0-Kp)}{\sqrt{p_0-Kp}} \approx \frac{W(p_0)}{\sqrt{p_0}} \quad , \quad F(p) = 1 + i(\pi p)^{1/2} - 2p + i\sqrt{\pi}\,p^{3/2} + \cdots \tag{11}$$

After performing the straightforward calculation , one may obtain:

$$\begin{aligned} W'(p_0) = W(p_0) + i\left(\frac{p_0}{\pi}\right)^{1/2}\left(1-\sqrt{K}\right)\Bigg\{ & \frac{W(p_0)}{\sqrt{p_0}}\Bigg[2(p')^{1/2} + i\sqrt{\pi}\,p' - \frac{4}{3}(p')^{3/2} - \frac{i\sqrt{\pi}}{2}(p')^2 - \frac{1}{2}\left\{\frac{i\sqrt{\pi}}{2}(p')^2\right. \\ & \left. - \frac{8}{5}(p')^{5/2} - \frac{4i\sqrt{\pi}}{7}(p')^{3/2}\right\}\delta^3 + \left\{\frac{i\sqrt{\pi}}{4}(p')^2 + \frac{8}{15}(p')^{5/2} - \frac{2}{7}(p')^{7/2} - \frac{i\sqrt{\pi}}{8}(p')^4\right\}\delta^6 + \cdots\Bigg] \\ & + \int_{p'}^{p_0V/K}\frac{W(p)W(p_0-Kp)}{\sqrt{p(p_0-Kp)}}dp\Bigg\} \end{aligned} \tag{12}$$

The integral in Eq.(12) can be evaluated numerically.

4. NUMERICAL RESULTS AND COMPARISONS WITH MEASUREMENTS

We performed the extensive numerical calculations to investigate the propagation phenomena of the electromagnetic wave propagation over the mixed-path. We have shown in Fig.3 the calculation results for the field strength as the function of the distance at three frequencies. The propagation path changes abruptly from the land to the sea at 40km(see Fig.2). We assume that the conductivity and the dielectric constant are σ_a=0.01 S/m and ε_a=10 for the land , and σ_b=5 S/m and ε_b=80 for the seawater. The duct parameter δ defined in Eq.(6) is assumed to be zero (i.e., δ=0) in Fig.3. The radio waves over the sea surface are strengthened compared with those for the all-land path (dotted curves). In Fig.4 , we have calculated the field

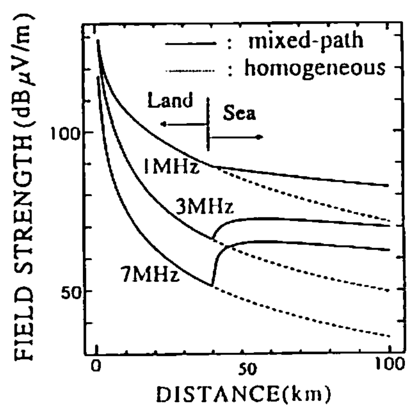


Fig.3 Field strength for land-to-sea path. Radiated power: 100 kW

strength for four values of the duct parameter. The signal levels are enhanced more strongly as the duct parameters δ increases .

We performed the experiments in Kanto area including Tokyo in Japan. The measurements were made on two frequencies 954 kHz and 1134 kHz. Since we observed the similar tendency for both frequencies , we show here the experimental data for 1134 kHz taken in the route I and the route II in Fig.5. The coverage by man-made objects is quite different between two routes. The route I passes through the low density area while the route II is on the high density area including the city centers. The results are shown in Fig.6. The experimental result on the route I agrees very well with the theoretical one obtained from Eq.(1) and Eq.(5) for the constant impedance. We used σ_a=0.01 S/m and ε_a=10.

The experimental result on the route II agrees very well with the mixed-path theory in Eq.(7) and Eq.(12). The portions extending from the transmitting antenna to the distance 30km are the high density area and the remaining portions are the low density area. We used the normalized surface impedance $\hat{Z}_a$=0.161exp(-i76°) for the high density area and $\hat{Z}_b$ with σ_b=0.03 S/m and ε_b=10 for the semi-rural area.

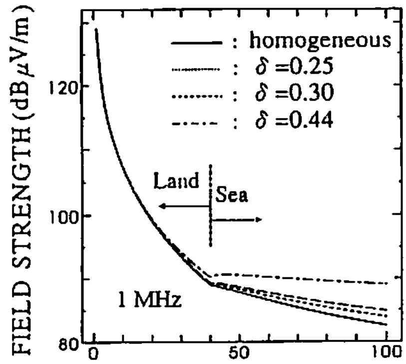


Fig.4 Effects of the surface duct.

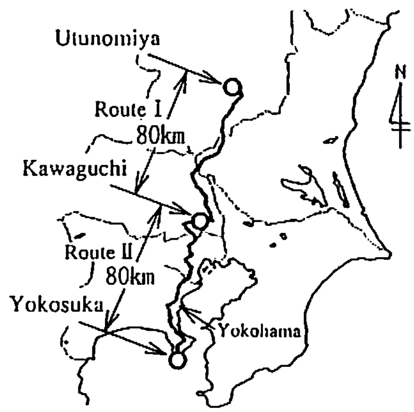


Fig.5 Route I and Route II used for experiment.

5. CONCLUSION

We have derived the electromagnetic field representation over the mixed-path , taking into account the existence of the tropospheric duct. We have clarified numerically the propagation phenomena over the mixed-path where the propagation path changes from the land to the sea. We have also shown that the experimental results obtained in Tokyo areas agree very well with the numerical results caluculated by using the theory for either the constant impedance surface or the inhomogeneous impedance surface , depending on the coverage by man-made objects.

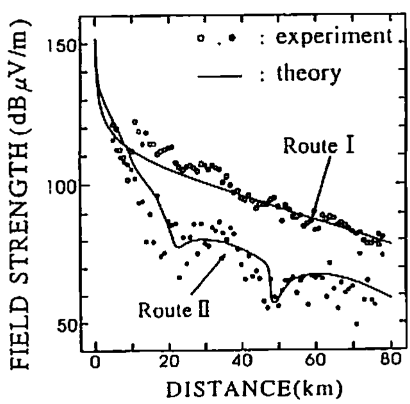


Fig.6 Measurements along Route I and Route II .

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