THE EFFECT OF ELEMENT POSITION ERRORS OF AN ADAPTIVE ARRAY

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1 Introduction

The effects of the imperfection of an adaptive array, such as mutual coupling or random input errors have been reported in several papers [1] [2]. In practice, the positions of the array elements are also subject to random variations because of practical limitations on mechanical accuracy and changes in environment conditions. It is therefore important to consider the effect of the element position errors on the performance of the adaptive array. In the case of the directionally constrained minimization of power (DCMP) [3] adaptive array, the constraint condition consists not only of the incident angle and frequency of the desired signal, but also of the positions of the array elements. In the presence of element position errors, there arises some discrepency between the phases of the input desired signal and those assumed in setting the constraint condition. Consequently, the desired signal is regarded as the unwanted one and becomes the target of suppression. Under such a condition, however, the Tamed Adaptive Array [1] is shown to be robust, in the sense that it can satisfactorily protect the desired signal from suppression.

2 Element misplacement and SINR deterioration

The DCMP adaptive array tries to minimize the output power while maintaining a constant response to the incoming signal from a specified direction. The optimum weight is given by [3]

$$\boldsymbol{W}_{\mathrm{opt}} = R_{xx}^{-1} C \left[C^{\mathrm{T}} R_{xx}^{-1} C \right]^{-1} \boldsymbol{H}, \tag{1}$$

where R_{xx} is the correlation matirx of the input, C is the constraint matrix and $\boldsymbol{H}$ is the constrained response vector. The superscript T denotes the transpose.

The phase of an entity of C (the input at the ℓ-th tap of the k-th antenna of the array is numbered as i) is given by the following:

$$\Phi_i(\theta_{\mathrm{S}}, f) = -2\pi f \left(\frac{d_{0k}}{c} \sin\theta_{\mathrm{S}} + \tau_\ell \right), \tag{2}$$

where θ_{S} is the arrival angle of the desired signal, f is the frequency, d_{0k} is the nominal setting position of the k-th antenna element, c is the light velocity and τ_ℓ is the delay time of the ℓ-th tap.

When the actual position of the k-th element is shifted by an error x_k from the setting position d_{0k}, the above phase in (2) becomes

$$\Phi_i'(\theta_{\mathrm{S}}, f) = \Phi_i(\theta_{\mathrm{S}}, f) - 2\pi f \frac{x_k}{c} \sin\theta_{\mathrm{S}}. \tag{3}$$

Thus, phasal differences arise between the contraint and the input desired signal due to the error x_k. This results in the deterioration of the output signal to interference plus noise ratio (SINR), since the desired signal is taken for as unwanted one and becomes the target of suppression, although a complete null is not generally formed in the constraint direction. (In this case, the system is forced to supress the signal that is very close to the constraint. Consequently, the weights become extremely large, thermal noise becomes dominating, and the result is a "super directive" situation.)

The simulation experiments were carried out on a 4-element 2-tap linear array of isotropic elements when the incident angle of the desired signal is scanned from −90° to 90°. The parameters for input radio environment are shown in Table 1. It is assumed that each weight produces thermal noise of equal power, independently. Nominally, the array is equally spaced with the spacing of half wavelength. It is assumed that the random fluctuation of each element position follows Gaussian distribution of the same standard deviation σ. With the randomly generated x_k's thus defined, 100 different sets of array were composed, and their output SINR's are averaged and shown in the ordinate of Fig.1. In the figure, it is evident that the output SINR deteriorates with the increase of position errors and the incident angle of the desired signal.

3 Tamed Adaptive Array as a robust system

To gaurantee the protection of the desired signal, we recall the Tamed Adaptive Array which desensitize the nulling capability of the system by introducing pseudo noise in its feedback loop. The pseudo noise moderates the cancellation of the desired signal and prevents the variable weights from becoming too large in magnitude. In practice, a high output SINR will be maintained by reinforcing the diagonal elements of the input correlation matrix R_{xx} in (1) with an appropriate amount of pseudo noise as follows:

$$R'_{xx} = R_{xx} + P_{\mathrm{pn}} U, \tag{4}$$

where U is the identity matrix and P_{pn} is the pseudo noise power. $\hat{P}_{\mathrm{pn}}$, the optimum value of P_{pn} is given by the following equation.

$$\hat{P}_{\mathrm{pn}} = \begin{cases} \hat{P}_{\mathrm{n0}} - P_{\mathrm{n}} & \text{for} \quad \hat{P}_{\mathrm{n0}} > P_{\mathrm{n}} \\ 0 & \text{for} \quad \hat{P}_{\mathrm{n0}} \leq P_{\mathrm{n}}, \end{cases} \tag{5}$$

where P_{n} is the real thermal noise power and $\hat{P}_{\mathrm{n0}}$ is the optimum thermal noise power that is analytically derived by maximizing the output SNR under the assumption that the position errors follow a Gaussian distribution. It is given by

$$\hat{P}_{\mathrm{n0}} = P_{\mathrm{S}} \sqrt{K(K-1)} \left| \frac{\pi\sigma}{\lambda} \sin\theta_{\mathrm{S}} \right| \quad (\text{for } \sigma \ll 1), \tag{6}$$

where P_{S} is the power of the desired signal, K is the number of elements.

Fig.2 shows the relation between the pseudo noise power and the output SINR while changing the magnitude of position error (σ/λ). The thermal noise and incident angle of the desired signal are fixed at −20dB and 30°, respectively. The other parameters are similar to those in Table 1. Let us decide the $\hat{P}_{\mathrm{pn}}$ for the worst case by setting the largest permissible error to be $\sigma = 0.01\lambda$. From (6) and (5) we obtain $\hat{P}_{\mathrm{pn}} = 0.1$ when $\theta_{\mathrm{S}} = 90°$, $P_{\mathrm{S}}=1$ and $K=4$. Referring to Fig.2, this value of pseudo noise almost always produces the highest attainable SINR for any values of σ. Therefore it may be concluded that $\hat{P}_{\rm pn}$ for the worst case guarantees good performance for all cases of smaller error. The simulations on a tamed array by adopting this value of pseudo noise were carried out under the same condition as in Fig.1. The results, shown in Fig.3, prove a remarkable improvement, that the ill effect of the element position errors is hardly recognized. Tamed Adaptive Array is found to be robust also against element position errors.

Acknowledgement

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	incident angle	power[dB]
desired signal (S)	$-90° \sim 90°$	0
interference (I)	$-60°$	20
thermal noise	-	-20

Table 1: Input data used in computation

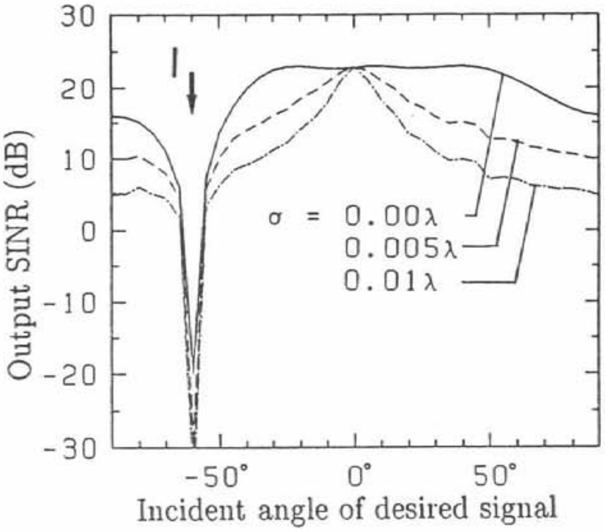

Fig.1: The relation between incident angle of desired signal and averaged output SINR in the presence of position errors for conventional DCMP system.

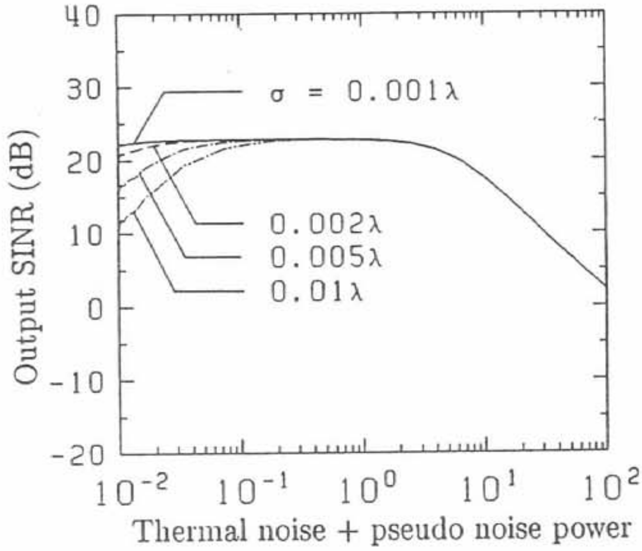

Fig.2: The relation between pseudo noise and output SINR with θ_S=30°, thermal noise=−20 dB.

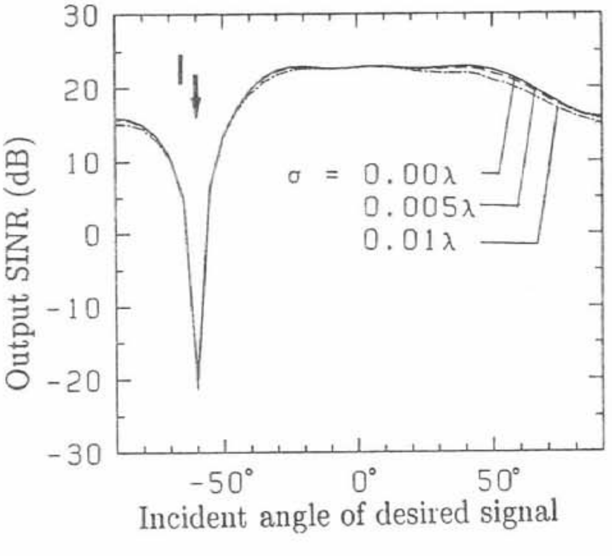

Fig.3: The relation between incident angle of desired signal and averaged output SINR in the presence of position errors for the tamed system.