PROCEEDINGS OF ISAP '96, CHIBA, JAPAN

Modeling Neptune's Atmosphere and an Application of Ray Tracing

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1 Introduction

The Voyager 2 spacecraft, which was launched in 1977, encountered the Neptune system on August 25, 1989. When a spacecraft passes by the opposite side of a planet, the signal from the spacecraft will be perturbated and/or absorbed. This phenomenon is called occultation. Voyager radio occultation data of the Neptunian system were collected using three antennas on Earth on August 25, 1989. Unmodulated carriers of frequency 8.4 GHz and 2.3 GHz were transmitted from the spacecraft toward Earth with the transmission power of 18 watts and 6.5 watts, respectively. A parabolic antenna at Canberra, Australia, of 70 meter diameter received both 2.3 GHz and 8.4 GHz carriers. The 64 meter parabolic antennas at Parkes, Australia and Usuda, Japan, received 8.4 GHz and 2.3 GHz carriers, respectively. Due to the large distance from Earth to the spacecraft, of ~4.5 billion kilometers (~ 30 A.U.), the received signal powers were as weak as -138.9 dBm for the Canberra 8.4 GHz signal, -141.4 dBm for the Parkes 8.4 GHz, -154.4 dBm for the Canberra 2.3 GHz, and -156.0 dBm for the Usuda 2.3 GHz.

From these data, the T–p profile of the Neptune's atmosphere was acquired, and the profile was analyzed up to $\sim 1.2 \times 10^5$ Pa where the signal decreased into the noise-floor[1]. We have attempted to increase the signal-to-noise ratio (SNR) of the data and to get a deeper profile.

To increase the SNR of the received data, it was planned to array (average) the signals collected at the different stations coherently. To do this, it is necessary to remove the differential signal phase $\Delta\phi$ between the stations with the accuracy much better than one cycle. But, in the period where the signals are weak, this did not work well. To avoid this, we have attempted to get the proper $\Delta\phi$ using the ray tracing method. This paper describes the technique of modeling Neptune's atmosphere and the ray tracing for the coherent signal arraying.

2 Radio Occultation Observation

When the signal from a spacecraft to Earth is refracted by the atmosphere of planet, the received signal has its phase influenced, which seems like a doppler perturbation. Analyzing this radio occultation data, we can obtain the T–p profile[1]. In this analysis, the vertical profiles, $n(r)$, $p(r)$, and $T(r)$, are also obtained on the specified position. In this paper, $n(r)$ is necessary for the atmospheric model.

3 Coherent Signal Arraying

From our estimation of all the possible factors which affect $\Delta\phi$, the largest unpredictable error source, the error in UT1, can change $\Delta\phi$ by as much as 1800 cycles and 2900 cycles for 8.4 GHz and 2.3 GHz signals, respectively, but the change is systematic. Therefore, it is necessary to remove $\Delta\phi$ using the $\Delta\phi$ estimated from the data themselves. The largest random error is the effect of the fluctuation in the Earth's troposphere (~0.8 cycles) for the 8.4 GHz signal, and the fluctuation in Earth's ionosphere (~1.3 cycles) for the 2.3 GHz signal with the characteristic time scales of 100~1000 sec for the troposphere, and ~30 sec for the ionosphere.

We can get $\Delta\phi$ with an ambiguity of n cycles (or in $(-\pi,\pi)$) by taking the arctangent of the complex samples. The true phase $\Delta\phi$ is reconstructed from this phase by the phase unwrapping procedure. To do this, however, is not simple with the real data which includes noise. The reconstructed phase can cause cycle slips, which is not permitted for the coherent signal arraying.

At first, to avoid cycle slips, we tried to unwrap the phase assuming that $\Delta\phi$ can be approximated by a second order polynomial for a short time interval. By coherently arraying between the Canberra 8.4 GHz and the Parkes 8.4 GHz signals and between the Canberra 2.3 GHz and the Usuda 2.3 GHz signals we succeeded in increasing the SNR by as much as 1.63 ~ 1.65 dB and 1.75 ~ 1.80 dB for most of the experimental period, respectively. These increases are almost the expected values. But the data in the period where the SNR of the original signal is low were not properly arrayed and not reliable due presumably to noise caused cycle slips. This is because the low SNR prevented a correct estimate of $\Delta\phi$ using the empirical prediction from the polynomial approximation [2].

Thus, to get the expected SNR increase even in the low SNR period, we have tried to use not an empirical but an analytical method to get a model of $\Delta\phi$ for proper unwrapping. To do this, we have applied the ray tracing technique to estimate the path length of the ray. Since the largest factor of the $\Delta\phi$ change is the ray bending due to the atmosphere of Neptune, we have modeled this effect first. We can consider the ray separation which could affect the systematic $\Delta\phi$ change to get better prediction using the ray tracing technique.

4 Modeling of Jovian Atmosphere

It is necessary to model the atmosphere to simulate the ray refraction, because the ray is refracted with the effect of refractive index which is simply calculated from the gas number density at the microwave band.

Voyager 2 is the only mission that has explored Neptune, and the radio occultation data were acquired at only two planetocentric latitudes: 61°N(ingress) and 44°S(egress). From both of these, the profile of the atmosphere is calculated. To simulate the ray path within the atmosphere, we must extrapolate these profiles and model the global structure of the atmosphere.

On Jovian planets (Jupiter~Neptune), the geoid may be determined from data on the gravitational field and the zonal wind velocities. Here, we assume that acceleration and potential are symmetric with respect to the spin axis. The gravity potential ($U(r,\phi)$) and the gravity acceleration (g) are well-known formula as follows:

$$U(r,\phi) = -\frac{GM}{r} + \frac{GM}{r}\sum_{i=1}^{\infty} J_{2i}\left(\frac{R_{\rm ref}}{r}\right)^{2i} P_{2i}(\sin\phi) + \frac{1}{2}\omega_{\rm III}^2 r^2 \cos^2\phi \qquad (1)$$

$$g = -\mathrm{grad}\ U \qquad (2)$$

where r is the radius, ϕ is the latitude, $P_{2i}(x)$ is the Legendre polynomial of degree $2i$, $R_{\rm ref}$ is the reference radius to which the zonal harmonic coefficients have been normalized, and $\omega_{\rm III}$ is the angular velocity corresponding to the system III rotation period. $U(r,\phi)$ is defined to be equal to zero on the spin axis at infinity. The reference geoid is calculated with the equation (1).

On Jovian planets, eastward zonal winds perturb the geoid. The altitude of a geoid ($h(\phi)$) is calculated relative to the reference geoid ($R_{\rm ref}$). From images of Neptune obtained by the narrow-angle camera of Voyager 2, the zonal wind velocity($V_{\rm W}$) is given as a function of latitude. Here, we have the well-known results [5] :

$$h(\phi) \approx \frac{2\omega_{\rm III}}{\langle g \rangle} \int_{\phi}^{\frac{\pi}{2}} V_{\rm W} \sin(\phi) d\phi \tag{3}$$

$$V_{\rm W} = (\omega - \omega_{\rm III}) r \cos\phi \tag{4}$$

where $\langle g \rangle$ denotes the average acceleration due to gravity, and is a slowly varying function of $h(\phi)$. The geoid is determined from the reference geoid and the adjustment of $h(\phi)$ [3].

If we assume the fluid is in hydrostatic equilibrium, we can assume that the pressure is constant along the geoid. And pressure is strongly related to the gas number density which is already measured through the radio occultation observation. Finally, we can convert the gas number density on limited points to global structure. For example, the measured gas number density at 60° N can be converted to the pressure at first, then, as this is same with the pressure at any latitude ϕ, it can be converted to the gas number density at the latitude ϕ. Figure 1 illustrates this procedure.

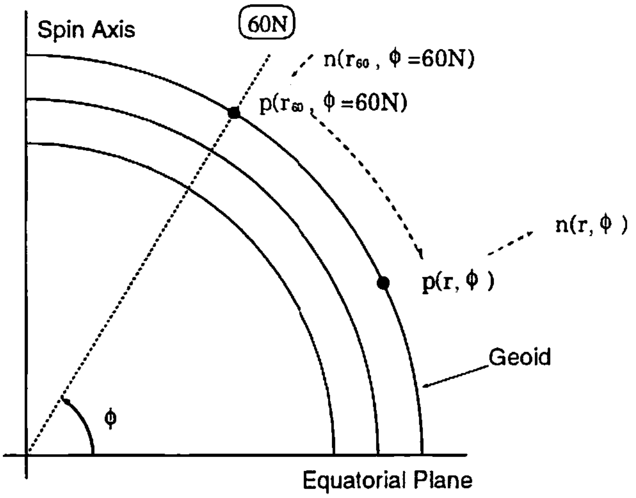

Figure 1: Procedure to Extrapolate Atmosphere's Structure

5 Method of Ray Tracing

The atmosphere modeled as above is useful for simulating the ray path in the atmosphere. For our application to estimate the whole ray path from the spacecraft to the Earth, ray tracing with many shells which consist of unified refractive index is not correct. We cannot determine the transmitting angle to calculate the path length towards the Earth, because the refraction angle does not change smoothly, due to the rapid and discontinuous change of the refraction angle near the boundary between the shells.

To avoid this, we use the integration method. Here, we have Bouguer's ray equation:

$$\frac{\mathrm{d}}{\mathrm{d}s}\left(\mu \frac{\mathrm{d}r}{\mathrm{d}s}\right) = \mathrm{grad}\ \mu \tag{5}$$

where r is a position vector on the ray, μ is the refractive index($\mu(r)$), and s is the length of the ray. $\mu(r)$ and grad $\mu(r)$ must be continuous functions to avoid roughness of ray path.

Using this equation, we can calculate the ray path as a summation of partial refraction steps. On every calculation step, the maximum bending angle and maximum/minimum propagating distance is adjusted in order to suppress the deviation of the total bending angle and to ensure a reasonable computational time. We have already confirmed this to be appropriate for the determination of the transmitting vector, and it is useful for the conversion from time to (differential) ray path.

This method has the following merits also. First, as we can calculate the refractive index and its gradient using the model, we can easily calculate the refraction on every step using Bouguer's ray equation. Second, we use much smaller memory to calculate refractive index. We can create the global model from about 1000 points × 91 (on every degree of latitude) samples of refractive index. Refractive index is interpolated using these samples. If we use thousands of layered shells, we get rough bending angle ($\sim 0.1^\circ$) and more than 10^6 bytes memory is needed. This is not realistic for calculation on workstations. And finally, it is easy to control a balance of speed and accuracy of refraction angle varying the distance between the steps. To simulate the propagation in Neptune's atmosphere, $10^4 \sim 10^7$ calculation steps are needed for one simulation, and we must iterate the calculation to find the true ray path. So, it is important to adjust the program to calculate within a realistic period with enough accuracy.

6 Conclusion

The Neptunian atmosphere, which gives refractive index and its gradient, has been modeled. To do this, an extrapolation technique using a geoid is proposed. And a method of ray tracing using Bouguer's ray equation is presented. Its merits are accuracy and ease of programming to calculate the ray path. Using the model and the method, we can find the ray path very accurately which is effective for the coherent signal arraying even under the condition that the signal is very weak.

References

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