

## A DESIGN METHOD OF LINEAR ADAPTIVE ARRAYS

Yimin Zhang, Department of Radio Engineering,  
Southeast University, China  
Kazuhiro Hirasawa, Kyohei Fujimoto,  
Institute of Applied Physics,  
University of Tsukuba, Japan

## I. INTRODUCTION

Adaptive antenna arrays are considered to be useful for communication systems to suppress undesired signals. The performance of an adaptive array depends on many parameters. Generally speaking, the parameters can be classified into three categories [1]. The first is the signal processing, such as the number of processing loops and the performance of the component devices. The second is the antenna array arrangement, such as the number of antenna elements and their locations. The last one is the signal environments, which, usually, are uncontrollable parameters.

Presently, the cost is still one of the most important problems in the applications of adaptive arrays. One way to reduce the cost of an adaptive array is to properly place the array elements. Usually, the requirements for the locations of array elements are to obtain the longest aperture and to avoid grating nulls [1]. However, as introduced in literatures [2,3], to seek the optimum locations needs complicated computations and often requires a lot of time. In this paper, we present a simple method which approximately provides the optimum locations of linear array elements.

## II. DESIGN CONCEPT

The least mean square (LMS) algorithm [4] and the steady state performance is only considered here. By using the concept of a spatial signal correlation coefficient, the steady state output signal-to-interference-plus-noise ratio (SINR) can be expressed as [5,6]

$$\text{SINR} = N\xi_D \left( 1 - \frac{N\xi_I}{N\xi_I + 1} \beta^2 \right) \quad (1)$$

where  $N$  is the number of antenna elements,  $\xi_D$  and  $\xi_I$  are the relative input power of the desired and the interference signal with respect to the input noise power. The spatial correlation coefficient  $\beta$  of two signals  $s_1$  and  $s_2$  is defined as

$$\beta = \frac{|U_1^* U_2|}{|U_1| |U_2|} \quad (2)$$

where  $U_1$  and  $U_2$  are the phase vectors of the arrived signals  $s_1$  and  $s_2$ , and the superscript  $*$  denotes complex conjugate. Eqn. (1) is plotted with respect to  $\beta$  in Fig.1 when only one

---

This work was supported in part by the National Science Foundation of China under Grant 6862044.

desired and one interference signal arrive. From this figure, it is seen that the output SINR degrades when  $\beta$  increases and it degrades rapidly as  $\beta$  approaches 1. Therefore, it is reasonable to set a threshold value of the spatial correlation coefficient so that the output SINR is acceptably large when the spatial correlation coefficient is under the threshold level. In this paper, this threshold is assumed as 0.8, and the degradations of the output SINR corresponding to it is at most -4.4 dB when  $\xi_1$  tends to an infinity.

### III. ANALYSIS AND RESULTS

Let us first consider a 3-element array, as shown in Fig. 2. For simplicity, all array elements are considered to be isotropic and no mutual coupling exists. The direction-of-arrival (DOA)  $\phi$  of a signal is assumed to be measured from the X-axis.

In the presence of one desired and one interference signal, the spatial correlation coefficient of the desired and the interference signal is provided as

$$\beta = \frac{1}{3} | 1 + e^{j\theta_1} + e^{j\theta_2} | \quad (3)$$

where

$$\begin{aligned} \theta_1 &= kd_1 [\cos \phi_0 - \cos \phi_1] \\ \theta_2 &= kd_2 [\cos \phi_0 - \cos \phi_1] \end{aligned} \quad (4)$$

Assume here that the DOA's ( $\phi_0$  and  $\phi_1$ ) of both the desired and interference signal are uniformly distributed between  $\phi = 0$  and 180 degrees. Then, the term in the parenthesis of eqn.(4) varies between -2 and 2. Thus, if we plot the relation between  $\theta_1$  and  $\theta_2$  obtained from eqn.(4), the locus should be a straight line connecting the two points  $(-2kd_1, -2kd_2)$  and  $(2kd_1, 2kd_2)$ .

The spatial correlation coefficient can also be plotted in the  $(\theta_1, \theta_2)$  plane. We use discrete darkness for  $\beta = 0$  to 1 with a step 0.1 in Fig.3(a). To make the figure more clear, only the area of  $\beta > 0.8$  is shown in Fig.3(b). The desired locus of  $\theta_1$  and  $\theta_2$  in Fig.3(b) is the longest straight line going through the origin and staying within the region of no grating lobes (Note that a grating null of the output SINR appears as a grating lobe of the spatial correlation coefficient). Because of the symmetry property of  $\beta$ , only the positive part of the line is shown. From Fig.3(b), the optimum interelement spacings are obtained as

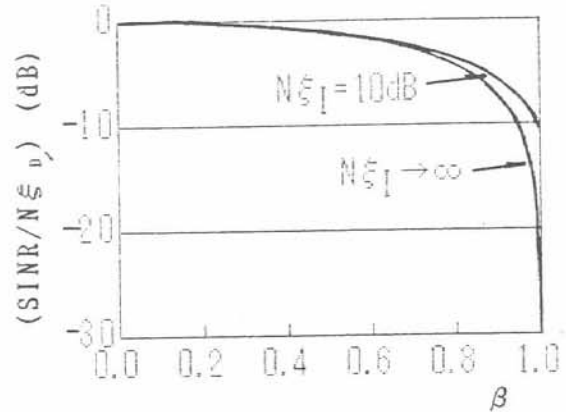


Fig.1 SINR degradation vs.  $\beta$ .

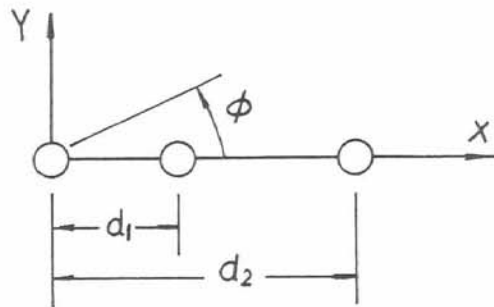


Fig.2 Coordinate system.

$$d_1 = 0.46\lambda, \quad d_2 = 1.85\lambda \quad (5)$$

where  $\lambda$  denotes the wavelength.

The spatial correlation coefficient for eqn.(5) is shown in the  $(\phi_0, \phi_1)$  plane as Fig.4(a). For comparison, the spatial correlation coefficient of a half-wavelength spaced array is also shown in Fig. 4(b). The area of  $\beta > 0.8$  is 13.6% in Fig. 4(a) and 31.2% in Fig.4(b).

Next, we consider a 4-element case. It is assumed that the 4th element is newly added to the three elements spaced with the optimum distances given by eqn.(5). There are two choices for the 4th element location: either on the left or the right side of the existing 3 elements. The spatial correlation coefficient for a 4-element case can be expressed as

$$\beta = \frac{1}{4} | 1 + e^{j\theta_1} + e^{j\theta_2} + e^{j\theta_3} | \quad (6)$$

where

$$\begin{aligned} \theta_1 &= d_1 \theta_3 / d_3, \quad \theta_2 = d_2 \theta_3 / d_3, \\ \theta_3 &= kd_3 [\cos \phi_0 - \cos \phi_1], \end{aligned} \quad (7)$$

and is plotted in Fig.5. The two possible choices of the optimum  $d_3$  are obtained as

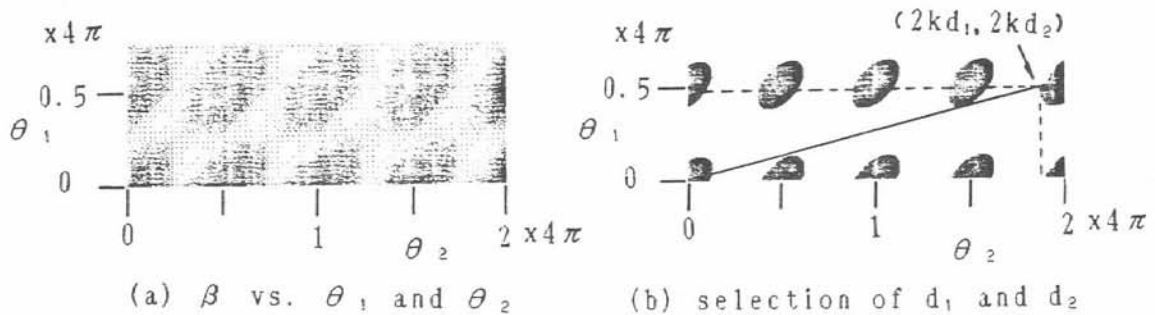


Fig.3 3-element case.

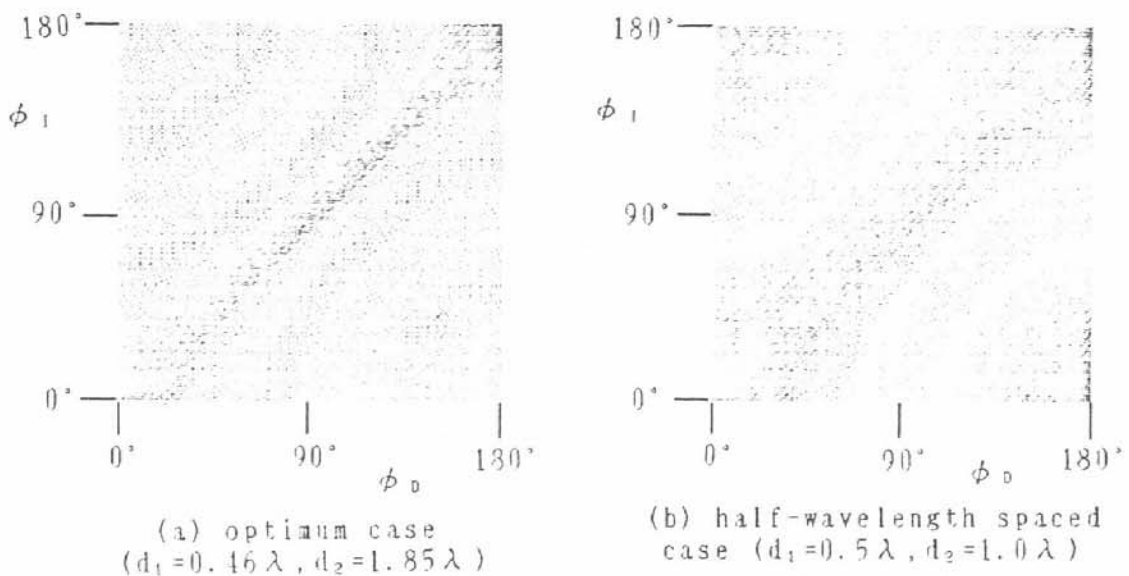


Fig.4  $\beta$  vs.  $\phi_0$  and  $\phi_1$  ( $N=3$ ).

$$d_3 = 4.68\lambda \quad \text{or} \quad d_3 = -2.40\lambda \quad (8)$$

where the first one is selected for the longer aperture. The area of  $\beta > 0.8$  is 6.9% for the obtained array, and is 23.9% for a half-wavelength spaced array.

By continuing the same procedure, an approximate solution of the optimum interelement spacing can be obtained for a multiple element array.

#### IV. DISCUSSIONS AND CONCLUSION

A simple design method using graphical expressions of a spatial signal correlation coefficient is introduced to determine the locations of the array elements in a linear adaptive array. By this method much better output SINR can be obtained compared with the equally spaced linear arrays. In addition, the computation is much simpler and quicker compared with the ordinary optimum seeking methods. Furthermore, because all of the loci are shown graphically, this method provides a clear image to help one to understand the overview of the performance of an adaptive array.

#### V. REFERENCES

- [1] Y. Zhang, et al., Tech. Rep. IEICE, AP86-119, pp.1-6, Jan. 1987.
- [2] N. Goto, Proc. IECE Japan, 63, pp.760-764, July 1980.
- [3] IECE Japan(ed.), Antenna Engineering Handbook, Chpt.11, Ohm-sha, 1980.
- [4] B. Widrow, et al., Proc. IEEE, 55, pp.2143-2159, Dec. 1967.
- [5] H. Lin, IEEE Trans., AP-30, pp.212-223, March 1982.
- [6] I. J. Gupta, et al., IEEE Trans., AES-19, pp.380-388, May 1983.

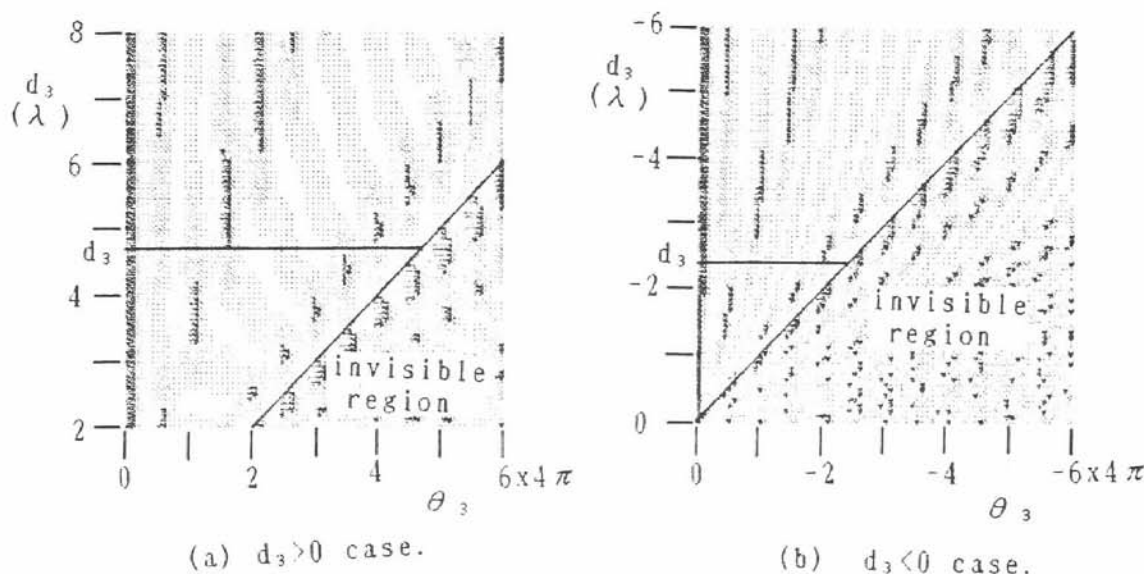


Fig.5 Selection of  $d_3$  ( $N=4$ ).