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SPACA-MLFACA Algorithm for Fast Solution of Electromagnetic Scattering Problems

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Abstract - **This paper presents a fast solution of electromagnetic scattering from perfect electric conductors by using a sparfied adaptive cross approximation-multilevel fast adaptive cross approximation (SPACA-MLFACA) algorithm. In the new method, the conventional SPACA and MLFACA is used to deal with the interactions between well-separated blocks with relative small and large size, respectively. As a result, the new method can save more memory and iterative time than the conventional SPACA and MLFACA for electrically large targets.**

Index Terms — **Computational electromagnetics (CEM), method of moments (MoM), adaptive cross approximation (ACA), sparsified adaptive cross approximation (SPACA), multilevel fast adaptive cross approximation (MLFACA).**

1. Introduction

In the past decades, many methods have been proposed to speed up the method of moments (MoM) [1], which is very popular for solving electrometric scattering and radiation problems. In these methods, the adaptive cross approximation (ACA)-based approaches have attracted considerable attention, such as the ACA [2][3], ACA-singular value decomposition (ACA-SVD) [4][5], multilevel ACA (MLACA) [6], sparsified ACA (SPACA) [7][8], sparsified MLACA [9], fast ACA (FACA) [10] and multilevel fast ACA (MLFACA) [10]. In these ACA-based methods, the MLACA has the smallest computation and storage complexities. The CPU time and memory used in the MLFACA scale $O(N\log^2N)$, where N is the number of unknowns.

In this paper, a new method is proposed to improve the conventional MLFACA. In the new method, we combine the SPACA and MLFACA together. The SPACA is employed to compress the interactions between relative small blocks in the tree structure. Thus, the compression ratio can be improved, because the SPACA can achieve a complexity close to $O(N\log N)$ [7][8] for relative small blocks.

2. Formulations

(1) Conventional MLFACA

To use the MLFACA, the target needs to be divided into blocks using the octal tree. This step is same as the classic multilevel fast multipole algorithm (MLFMA) [10]. The MLFACA is used to compress the interactions related to far-block pairs.

The MLFACA is a multilevel version of the FACA [8], and the multilevel scheme used in the MFLACA is borrowed from the multilevel matrix decomposition algorithm (MLMDA) [12]. However, compared with MLMDA, the MLFACA does not need to construct equivalent sources with the same complexity.

Taking the L-level MLFACA as an example, the impedance submatrix related to the ith and jth block is compressed as

$$\mathbf{Z}_{i,j} \approx \left(\prod_{p=1}^{L} \mathbf{C}^{(p)} \mathbf{D}^{(p)} \right) \mathbf{R}^{(L)}. \qquad (1)$$

where $\mathbf{C}^{(p)}$, $\mathbf{D}^{(p)}$, and $\mathbf{R}^{(L)}$ are sparse block matrices. And more details of these sparse matrices can be find in [10].

For both CPU time and storage, the MLFACA can achieve the asymptotic complexity $O(N\log^2N)$ for large target and the asymptotic complexity $O(M\log M)$ for large block, where N and M is the number of unknowns in a target and a block. However, the MLFACA is not the best choice for relative small targets and relative small blocks.

(2) SPACA-MLFACA

To improve the conventional MLFACA, the SPACA is used to compress the interactions between relative small blocks in the MLFACA.

For relative small targets and blocks, the SPACA can achieve a complexity close to $O(N\log N)$ and $O(M)$, respectively. It compress a submatrix as a product of five matrices as

$$\mathbf{Z}_{i,j} \approx \mathbf{A}_i \hat{\mathbf{Q}}_i \mathbf{S}_R \hat{\mathbf{Q}}_j^T \mathbf{A}_j^T \qquad (2)$$

where $\mathbf{A}_i$ and $\mathbf{A}_j^T$ are block diagonal matrices. $\mathbf{S}_R$ is a diagonal matrix. Details of obtaining the five matrices can be find in [8].

With the help of the SPACA, the SPACA-MLFACA can save more memory and iterative time than the conventional MLFACA.

3. Numerical Example

In this section, the proposed SPACA-MLFACA is implemented and applied to solve the electromagnetic scattering from a PEC sphere with radius of 10 m. The work frequency is 300 MHz. The combined field integral equation (CFIE) with GMRES is used. 450522 RWG [13] with the average side length is obtained. The octal tree has 6 levels.

The comparison of the performance of the MLFACA and the SPACA-MLFACA is shown in Table 1. Fig. 1 gives the bistatic RCS of the target calculated by the SPACA-MLFACA. It can be seen that a good agreement between the SPACA-MLFACA solution and the Mie solution.

TABLE I

Comparison of storage and CPU time for different methods

method	memory of far-block pairs	iterative time
MLFACA	12.1 GB	140 s
SPACA-MLFACA	8.0 GB	113 s

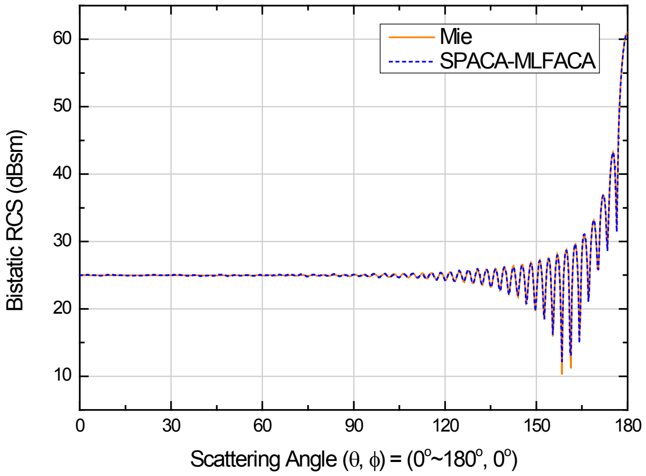

Fig. 1. Bistatic RCS of PEC sphere with radio of 10 m at 300 MHz.

4. Conclusion

This paper proposes a fast iterative solver termed as SPACA-MLFACA for the MoM. In the new method, the MLFACA is replaced with the SPACA to compress the interactions between relative small blocks. Numerical results show that the proposed method can save memory and CPU time than the conventional MLFACA.

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