On Statistical Distribution of the Largest Eigenvalue of Channel Correlation Matrice in MIMO Multi-Keyhole Environment

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Abstract

MIMO leads to dramatic improvement in channel capacity and link reliability of wireless systems. However, a MIMO channel has only one degree of freedom in the keyhole environment. As this result, it reduces achievable channel capacity and link quality. This paper proposes the MIMO repeater system that is mitigated to reduce MIMO channel capacity, which can provide a multi-stream transmission in the multi-keyhole environment. Limit of the transmission performance of MIMO is given singular value or eigenvalue essentially, hence we consider a representation by an approximate equation of the probability density function of the largest eigenvalue connecting to the theory of space diversity. It is shown that calculated values based on the proposed method agree well with simulated values.

1. INTRODUCTION

Multiple-input multiple-output (MIMO) wireless communication systems have multi-element antenna arrays at the both the transmitter and receiver sides. MIMO has been performed with the objective of higher transmission rate and/or reliable data transmission under finite spectral resources by using array antennas for both transmission and reception [1]-[4]. A large number of papers have been published on stimulating MIMO research opportunities with the motivations of implementing the arrays in terminals such as wireless LANs and WiMAX.

The MIMO channel capacity can be decreased when the MIMO function is relayed using only one antenna, or waveguiding structure, even though the signals at the antenna elements are uncorrelated [5],[6]. This effect has been termed "keyhole" or "pinhole" (hereafter, we call it keyhole). In the keyhole environment, the multi-stream transmission becomes impossible, and high-speed, high-reliability transmission can not be expected.

In this case, to maintain the ability of high-speed and high-reliability data transmission, the establishment of a general idea of a MIMO repeater system is importance that will still confirm the efficient transmission. Research on system channel model validation is essential about singular value or eigenvalue [7]. In the future, it is believed that the service area of the MIMO will spread. Thus, expansion of the service area to the isolated space is anticipated. In general, in the MIMO repeater system, the whole channel is equivalent to a MIMO multi-keyhole environment [8]. The information transmission performance of MIMO is evaluated by average channel capacity and average BER are calculated using the probability density function of singular value of channel response matrix or eigenvalue of correlation matrix [9]. But, the analytic solution of the probability density functions of eigenvalues is not derived in the MIMO multi-keyhole environment yet.

In this paper, the MIMO repeater system (where the multi-keyhole model is adapted) is proposed and the MIMO multi-keyhole model analyses the eigenvalue distribution and average channel capacity. In addition, the approximate expression of the probability density function of the largest eigenvalue is presented in the MIMO multi-keyhole environment.

2. SYSTEM MODEL

2.1. *MIMO REPEATER SYSTEM*

This paper discusses that the MIMO repeater system can expand MIMO function in another isolated space, and it can enlarge the service area of the MIMO function. Figure. 1 shows the concept of the MIMO repeater system.

We consider that the MIMO repeater system with multiple antennas is capable to extend multi-stream transmission from the MIMO service space to the isolated space. We consider the MIMO multi-keyhole channel with M transmits antennas, N receives antennas and K repeater system antennas. Fig. 2 shows structure of the MIMO repeater system.

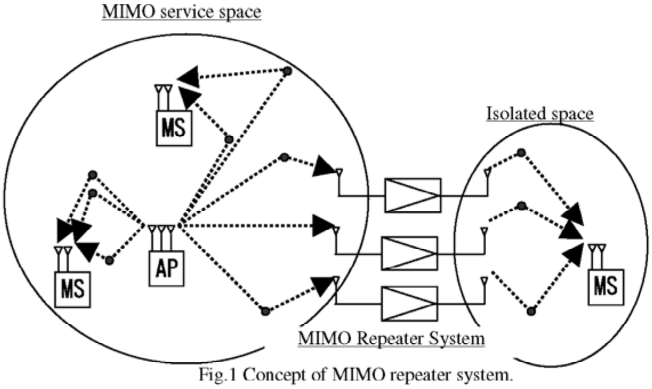

Fig.1 Concept of MIMO repeater system.

The MIMO repeater system consists of Access Point (AP), a repeater system and Mobile Station (MS). The function of the repeater system can be repeating the signal to isolated space. The signal is transmitted from AP side, it is received by the repeater system, and then it retransmitted to MS side. The signal processing of the repeater system is only amplification and relay of received wave. This paper assumes Rayleigh fading environment with i.i.d. (independent and identically distributed) statistics .

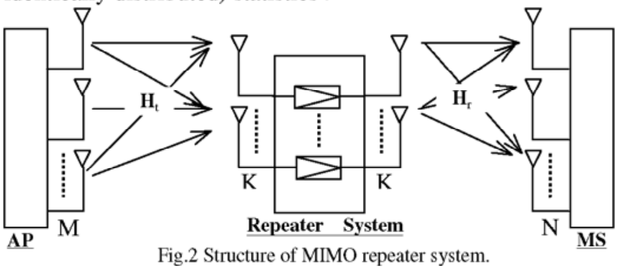

Fig.2 Structure of MIMO repeater system.

2.2. *CHANNEL MODEL*

We consider the MIMO repeater system in the MIMO multi-keyhole environment. The received signal vector **r** is given by

$$\mathbf{r} = \mathbf{H}_r \mathbf{G}(\mathbf{H}_t \mathbf{s} + \mathbf{n}_{rp}) + \mathbf{n}_{rv} \tag{1a}$$

where **s** denotes the transmit signal vector, $\mathbf{n}_{rp}$ denotes the noise vector at the repeater system, $\mathbf{n}_{rv}$ denotes the noise vector at MS, $\mathbf{H}_t$ denotes the channel matrix from AP to the repeater system, $\mathbf{H}_r$ denotes the channel matrix from the repeater system to MS, **G** denotes the gain matrix in amplitude at the repeater system.

Let us define the following matrix notations

$$\mathbf{r} \equiv [r_1, r_2, \cdots, r_N]^T \tag{1b}$$

$$\mathbf{s} \equiv [s_1, s_2, \cdots, s_M]^T \tag{1c}$$

$$\mathbf{H}_r \equiv \begin{bmatrix} h_{11}^r & h_{12}^r & \cdots & h_{1K}^r \\ h_{21}^r & h_{22}^r & \cdots & h_{2K}^r \\ \vdots & \vdots & \ddots & \vdots \\ h_{N1}^r & h_{N2}^r & \cdots & h_{NK}^r \end{bmatrix} \tag{1d}$$

$$\mathbf{H}_t \equiv \begin{bmatrix} h_{11}^t & h_{12}^t & \cdots & h_{1M}^t \\ h_{21}^t & h_{22}^t & \cdots & h_{2M}^t \\ \vdots & \vdots & \ddots & \vdots \\ h_{K1}^t & h_{K2}^t & \cdots & h_{KM}^t \end{bmatrix} \tag{1e}$$

$$\mathbf{n}_{rp} \equiv \left[r_1^{rp}, r_2^{rp}, \cdots, r_K^{rp}\right]^T \tag{1f}$$

$$\mathbf{n}_{rv} \equiv \left[r_1^{rv}, r_2^{rv}, \cdots, r_N^{rv}\right]^T \tag{1g}$$

$$\mathbf{G} \equiv diag[g_1, g_2, \cdots, g_K] \tag{1h}$$

where the superscript ^{T} denotes the matrix transpose, the diag[·] means a matrix composed of diagonal components.

As the first step of eigenvalue analysis of the MIMO repeater system, we will assume the noise $\mathbf{n}_{rp}$ and $\mathbf{n}_{rv}$ are negligible. Under this assumption, the noiseless channel matrix $\mathbf{H}_e$' is given by

$$\mathbf{H}_e^{'} = \mathbf{H}_r \mathbf{G} \mathbf{H}_t \tag{2}$$

The gains between receive and transmit antennas of the repeater system are assumed to be constant, and the gain **G** at the repeater system is normalized by the number of antennas, K.

$$\mathbf{G} = \frac{1}{\sqrt{K}} \mathbf{I}_{K \times K} \tag{3}$$

Consequently, the channel matrix in the MIMO multi-keyhole environment through the MIMO repeater system is given by

$$\mathbf{H}_e = \frac{1}{\sqrt{K}} \mathbf{H}_r \mathbf{H}_t \tag{4}$$

This papers discuses the channel matrix of Equation (4).

3. EIGENVALUE OF MULTI-KEYHOLE ENVIRONMENT

3.1. *EIGENANALYSIS*

As shown below, the channel matrix $\mathbf{H}_e$ of the MIMO multi-keyhole environment can be represented by singular value decomposition (SVD)[2].

$$\mathbf{H}_e = \mathbf{E}_r \mathbf{D} \mathbf{E}_t^H = \sum_{i=1}^{M_0} \sqrt{\lambda_i} \mathbf{e}_{r,i} \mathbf{e}_{t,i}^H \tag{5}$$

where

$$\mathbf{D} \equiv diag\left[\sqrt{\lambda_1} \quad \sqrt{\lambda_2} \quad \cdots \quad \sqrt{\lambda_{M_0}}\right] \tag{6a}$$

$$\mathbf{E}_t \equiv \left[\mathbf{e}_{t,1} \quad \mathbf{e}_{t,2} \quad \cdots \quad \mathbf{e}_{t,M_0}\right] \tag{6b}$$

$$\mathbf{E}_r \equiv \left[\mathbf{e}_{r,1} \quad \mathbf{e}_{r,2} \quad \cdots \quad \mathbf{e}_{r,M_0}\right] \tag{6c}$$

$$M_0 \equiv \min(M, N, K) \tag{6d}$$

where λ_i is the i-th eigenvalue of the correlation matrix $\mathbf{H}_e\mathbf{H}_e^H$ (or $\mathbf{H}_e^H\mathbf{H}_e$) and is ordered according to i=1,2,..,M_0. $\mathbf{e}_{t,i}$ is the eigenvector belonging to eigenvalue λ_i of $\mathbf{H}_e^H\mathbf{H}_e$, and $\mathbf{e}_{r,i}$ is the eigenvector belonging to eigenvalue λ_i of $\mathbf{H}_e\mathbf{H}_e^H$.

3.2. *PROBABILITY DENSITY FUNCTIONS OF EIGENVALUES*

The analytic solution of the probability density functions of the eigenvalues is not yet derived in the general case of the MIMO multi-keyhole environment. However, if the number of repeater system antennas is limited to one, the probability density function of the eigenvalue could be derived as follows: With K=1, the channel matrix of Eq (4) is given by

$$\begin{aligned} \mathbf{H}_e &= \mathbf{H}_r \mathbf{H}_t \\ &= \left[h_{11}^r \quad h_{21}^r \quad \cdots \quad h_{N1}^r\right]^T \left[h_{11}^t \quad h_{12}^t \quad \cdots \quad h_{1M}^t\right] \end{aligned} \tag{7}$$

In matrix notations we can now define the correlation matrix **R** by

$$\mathbf{R} = \mathbf{H}_e \mathbf{H}_e^H = \mathbf{H}_r \mathbf{H}_t \mathbf{H}_t^H \mathbf{H}_r^H \tag{8}$$

The rank of the Eq. (8) is one. Therefore, the largest eigenvalue λ_1 is given by

$$\lambda_1 = Trace[\mathbf{R}] = Trace(\mathbf{H}_r \mathbf{H}_r^H)(\mathbf{H}_t \mathbf{H}_t^H) \tag{9}$$

The Trace($\mathbf{H}_r\mathbf{H}_r^H$) has central chi-square distributed with 2N degrees of freedom (since it is sum of probability variables with an exponential distribution). We can obtain the probability density function $P_{2N}(\lambda_1)$ as,

$$p_{2N}(\lambda_1) = \frac{\lambda_1^{N-1} e^{-\lambda_1}}{\Gamma(N)} \tag{10}$$

In the same way, $\mathbf{H}_t\mathbf{H}_t^H$ has central chi-square distribution with 2M degrees of freedom. Therefore, the probability density function p(λ_1) of the largest eigenvalue is given by

$$p(\lambda_1) = \int_0^\infty \frac{1}{u} p_{2N}(u) p_{2M}\left(\frac{\lambda_1}{u}\right) du \tag{11}$$

$$= \frac{2\lambda_1^{\frac{N+M}{2}-1}}{\Gamma(N)\Gamma(M)} K_{M-N}\left(2\sqrt{\lambda_1}\right)$$

where $\Gamma(\cdot)$ is Gamma function, $K_\alpha(\cdot)$ is the α-th modified Bessel function of the second kind.

Fig. 3 shows the cumulative distribution function of the largest eigenvalue calculated from the theoretical value of the Eq. (11) and simulated value for M=N with M=2,3,and 4. We show the cumulative distribution function for i.i.d. environment for reference depicted by solid lines.

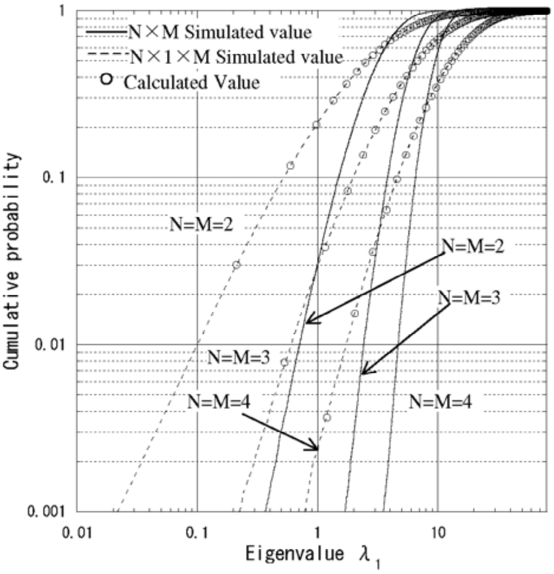

Fig. 3 Comparison of cumulative distribution of the largest eigenvalue (N×M v.s. N×1×M).

Then, Figs. 4 and 5 show the cumulative distribution function of the eigenvalues using Eq. (4) for M=N with 2 and 4, while K=1,2,3,and 4.

Figs.4 and 5 show that the increment of antenna number K of the repeater system increases the rank and diversity order in the MIMO multi-keyhole channel.

3.3. *CHANNEL CAPACITY*

The MIMO repeater system is shown in Fig. 2 decomposed by SVD has a different method for information transmission depending on whether the channel state information for both transmission and reception is known or not. When channel state information is available for both, we have two options: The MRC (Maximal Ratio Combing) transmission where all the information is assigned on the largest eigen-path, and the ET(Eigenmode Transmission) where the power is assigned to each eigen-path, and the information can be transmitted in parallel. The distribution of the optimum power for each channel is determined based on Water Filling (WF) scheme.

The upper bound of the average channel capacity with the power allocation via WF method is given by the following equations based on Shannon's information theory.

$$\langle C_{WF}\rangle \approx \sum_{i=1}^{M_0} \log_2\left(1+\langle\lambda_i\rangle\gamma_i\right) \quad \text{(bit/s/Hz)} \tag{12a}$$

$$\sum_{i=1}^{M_0} \gamma_i = \gamma_0 \tag{12b}$$

C_{WF} denotes the channel capacity in the case of optimal power allocation based on WF. <C> denotes the ensemble average of the channel capacity C. The γ_0 in Eq. (12) is the CNR when the signal is radiated from a single antenna and then propagated through a path with unit gain, and is received by a single antenna.

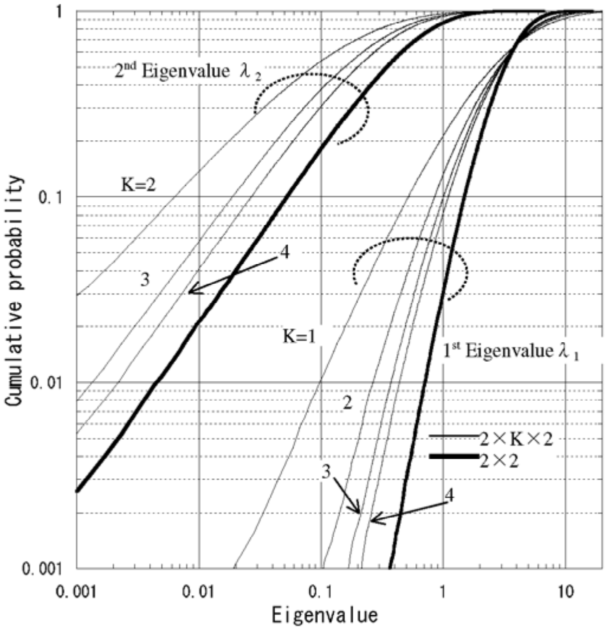

Fig. 4 Cumulative distribution of eigenvalues(N=M=2,K=1～4).

Fig. 6 shows the average channel capacity in ET of the MIMO repeater system for M= N=2, while k=1,2,3,4, and 5. It also plots the result in the case of not using the repeater system (i.e., 2×2 with figure) for comparison. In Fig.6, difference between the largest and the smallest value of the average channel capacity of ET is approximately 1bits/s/Hz, when K repeater system antennas is two or more.

Then, we compare the case of K=2 or more versus K=1 in the repeater system. In this case, the difference of average channel capacity increases, as the increase of CNR. On the other hand, the MRC transmission does not depend on the number of antennas of the repeater system, the nearly same average channel capacity is gained.

4. APPROXIMATION EQUATION OF PROBABILITY DENSITY FUNCTION OF LARGEST EIGENVALUE

In the previous chapter we have discussed the distribution of the eigenvalues which is required in the transmission performance evaluation, and they are obtained by the computer simulation. Closed form of the probability density function of the eigenvalues in the MIMO multi-keyhole environment is not solved yet. Therefore, we consider a representation by an approximate equation linked to space diversity [10][11].

Here, as the first step in eigenvale analysis, we will consider the probability density function of the largest eigenvale.

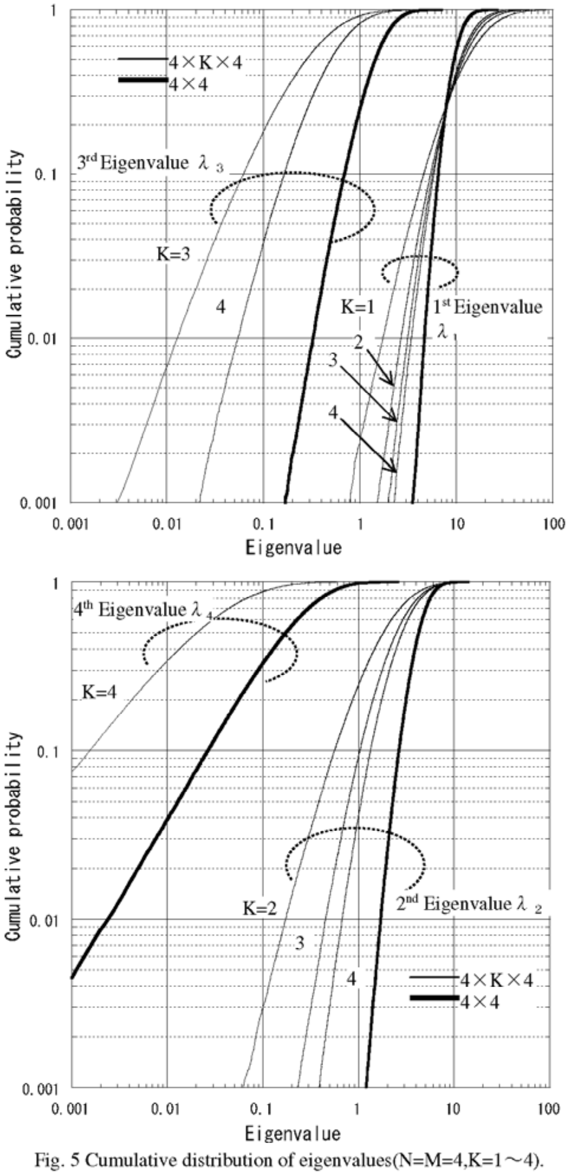

Fig. 5 Cumulative distribution of eigenvalues(N=M=4,K=1～4).

4.1. *APPROXIMATE MODEL*

In this section, we show the approximate equation of the probability density function of the largest eigenvale based on the theory of space diversity. This paper assumes Rayleigh fading environment with i.i.d. (independent and identically distributed) statistics. A simplified estimation for the distribution of the largest eigenvalue of the correlation matrix, that is, key information on estimating MRC transmission characteristic in Rayleigh MIMO channel is given in [10][11]. In the same way, we transform it to the single-keyhole structure that shows the configuration of the MIMO repeater system with K=1 in Fig. 7. To put it more concretely, space diversity theory is applied to AP side and MS side, respective. As a result, the approximate equation of the probability density function of the largest eigenvalue under the MIMO multi-keyhole environment can be solved from the probability density function in the single-keyhole model by Eq. (11). The approximate equation is given by

$$p(\lambda_1)=\frac{2\left(\frac{\lambda_1}{\Lambda}\right)^{\frac{N+M+2L}{2}-1}}{\Lambda\Gamma(N+L)\Gamma(M+L)}K_{M-N}\left(2\sqrt{\frac{\lambda_1}{\Lambda}}\right) \tag{13}$$

where L and Λ denotes the parameter which is related to diversity order, and diversity gain, respectively.

The relationship of parameter L when the structure is converted from K≧2 to K=1 is shown in Fig. 8. From Fig. 8, power function can be approximated by the function L using parameters M, N and K. It can be calculate by

$$L(M,K,N)=\alpha(M\times(K-1)\times N)^{\beta} \tag{14}$$

where α is 0.4343, β is 0.6681. In a similar fashion, the parameter Λ may be given an approximate equation.

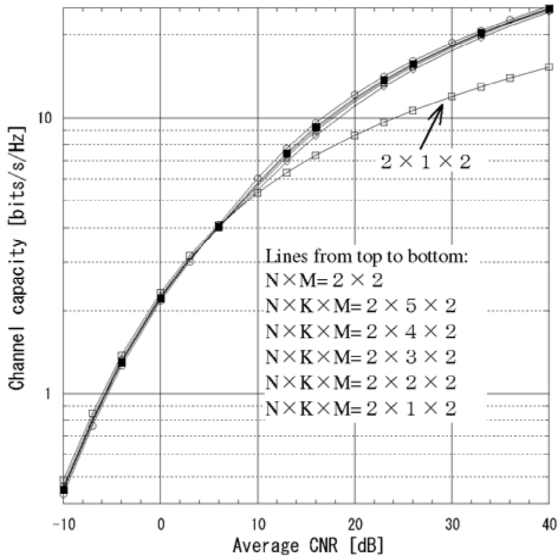

Fig. 6 Throughput performance of MIMO eigenmode transmission (Case of N,M=2) in the MIMO repeater system.

When the calculated values are applied to Eq. (14), Fig. 9 shows the result of comparison between the calculated values and simulated values of parameter (M+N+2L)/2. It is shown that the calculated values coincide well with the simulated values. Then, Fig. 10 shows the probability density function of the largest eigenvalue comparing the calculated values with simulated values. It is shown that calculated values based on the proposed method agree well with simulated values in the MIMO multi-keyhole environment. Here, as the first step of eigenvale analysis, we have shown the approximate equation of the probability density function of the largest eigenvalue in the MIMO multi-keyhole environment.

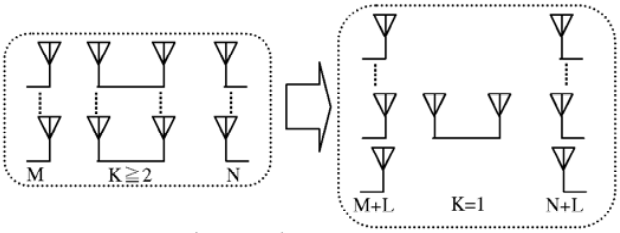

Fig. 7 Equivalent configuration for diversity order analysis concerning the largest eigenvalue.

This calculation model can be applied to simplified calculation and to get good accuracy of the transmission evaluation, although it does not refer mathematical and physical viewpoint of the Eq. (13). However, the Eq. (13) includes theoretical Eq. (11) in the case of K=1, we will consider that it satisfies the portion of the necessary conditions.

5. CONCLUSIONS

We proposed a MIMO multi-keyhole model for the analysis of the MIMO repeater system, which can provide the relay function to the isolated space to realise the high-speed and high-reliability data transmission. Then, we have shown the approximate equation of the probability density function of the largest eigenvalue connecting to the theory of space diversity. It showed that calculated values based on the proposed method agree very well with simulated values in the MIMO multi-keyhole environment.

Then, We have shown the number of relay antennas, which are required to the MIMO repeater system with a viewpoint of average channel capacity.

This paper has shown only the approximate equation of the probability density function of the largest eigenvalue, more investigation is necessary concerning the distribution of exact solutions including those of other eignvalues.

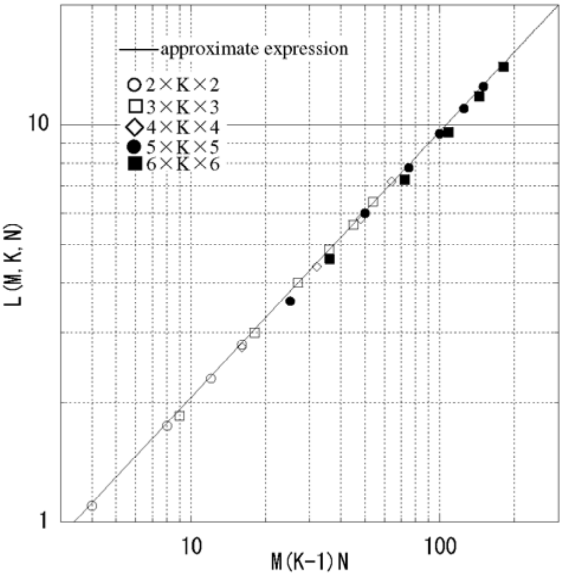

Fig. 8 The relationship between L(M,K,N) and M(K-1)N.

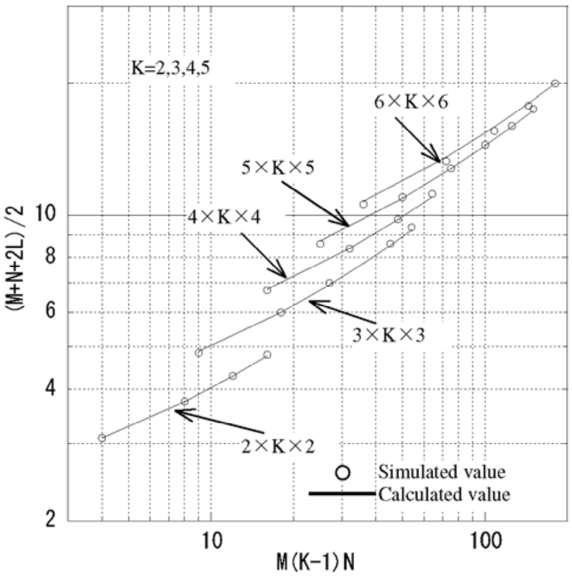

Fig. 9 Parameter (M+N+2L)/2 comparison between simulated and calculated

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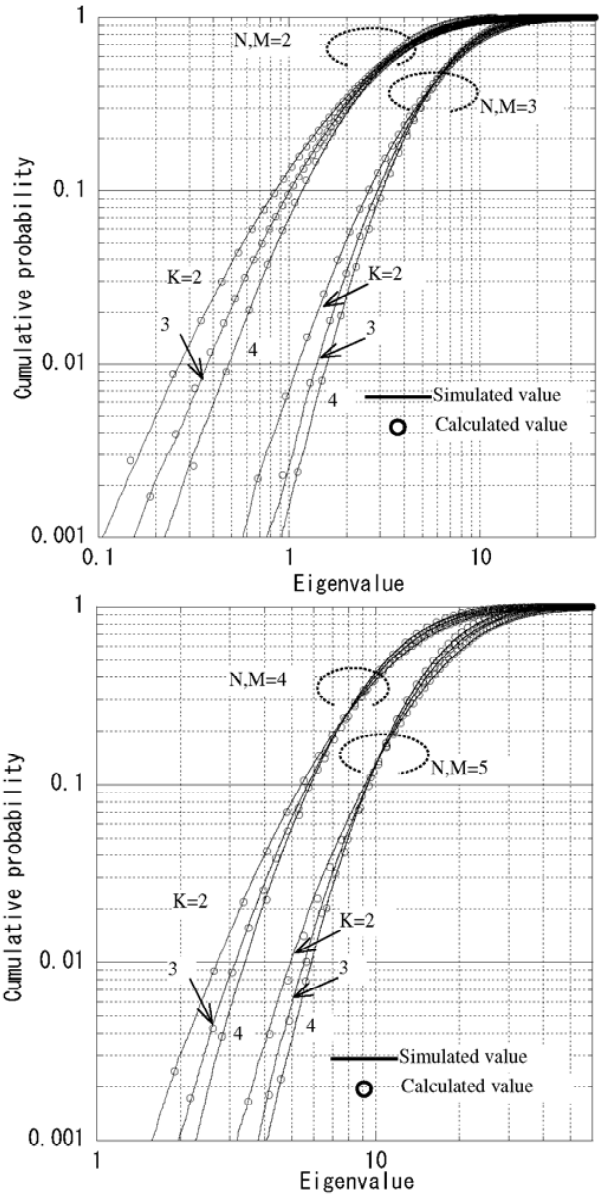

Fig. 10 Cumulative distribution of the largest eigenvale: Comparison between simulated and calculated.