Finite Element Formulation of Radio Wave Propagation in Tunnels

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Abstract

Characterization of VHF-UHF electromagnetic wave propagation in tunnels has important applications in the field of mobile communications. Radio wave propagation in tunnels are influenced by several factors like shape, transverse dimensions of tunnels, frequency and electrical parameters of the surrounding material of tunnels. In this paper, a vectorial parabolic equation has been derived and applied to model VHF/UHF wave propagation in tunnels. Vectorial parabolic equation model is solved using finite element method and results are analysed for straight and curved tunnels.

1. Introduction

There has been a tremendous growth in the number of mobile device users in the past decade. Mobile operators have deployed a greater number of wireless infrastructure in order to satisfy the demand and achieve a great coverage. Radio wave propagation in tunnels is also of high interest to security and other emergency services who have a need to communicate in tunnels. Tunnels form a major part of the transport infrastructure of most modern metropolitan cities and mountainous regions. It is estimated that Austria for example, has 10% of its transport in high priority roads made up of tunnels [8]. Because of their confined nature, normal cell planning techniques are not applicable to tunnels. There is therefore a need to develop accurate techniques to characterize the propagation of radio waves in tunnels.Many researchers have therefore been focusing their attention on radio propagation in tunnels [2].

To solve for the field distribution in tunnels, a number of factors must be taken into account, such as shape and transverse dimension, frequency, direction of polarization, permittivity and conductivity of the surrounding material. One way of communication in tunnels is by use of a leaky coaxial cable placed near the tunnel wall [3], [4].The basic principle has been described, in many papers, for both leaky feeders working above or below the tunnel cut-off [5]. However, at high frequencies, typically above few hundred MHz for a road tunnel, the natural propagation of the waves does not lead to significant attenuation and thus at least for short tunnels, radiating devices are not needed [6].

Modal analysis [7] and Geometrical Optics (GO) [8] are two major approaches to model radio wave propagation in tunnels. The analogy between a tunnel and a common electromagnetic waveguide suggests the application of the modal approach. Mode theory is effective if one mode dominates, a very rare occurrence in real tunnels. An extension of modal approach to a more general class of wide smoothly curved waveguide has been presented in [9], so called adiabatic mode theory. Although adiabatic mode theory enables one to study radio wave propagation in realistic tunnels, it is not always convenient for practical applications because of computational efficiency [10]. Geometrical optics is complicated at long ranges due to growing number of contributing rays and breaks down in caustic regions. While reasonable numerical results can be obtained by estimating wave amplitude via the calculated ray density [11], this approach requires excessive computational resources. So apart from extremely high frequency problems, a full wave solution seems to be preferable [10].

Multi-path propagation model may also be used for predicting radio channel in tunnels [12]. But the traditional model is a simplified one and cannot represent the real propagation characteristics of radio wave in tunnel and therefore cannot be used directly for predicting the propagation properties in a tunnel. To use multi-path model, it is extremely important how multi-path distances are calculated and then time delay of each path. Recently, an algorithm has been proposed for calculating multi-path distances in very simple rectangular tunnel [13].

Several factors are key to characterizing accurately radio wave propagation in tunnels, the electrical properties of the surrounding material of the tunnel is an important one. In [14] a detailed analysis is provided of the effect of electrical parameters and it was shown that the effect of conductivity is low and may be neglected.

Parabolic Equation Method (PEM) has previously been successfully applied to other propagation phenomena [15], [16] and short wave diffraction problems [17]. The computational efficiency of vectorial parabolic equation method has been demonstrated in [18], [20], where realistic problems of wave propagation and scattering were solved on a desktop with normal configuration. Parabolic equation method seems to be an adequate mathematical model of wave propagation in tunnels due to selective wall absorption filtering out higher Brillouin angles and forming a paraxial wave packet even if tunnel axis is a curved one [10]. In this work, finite element method (FEM) has been applied to solve vectorial PEM propagation model in curved tunnels. FEM has previously been developed for tropospheric wave propagation [15] and propagation in the presence of vegetation canopies [19]. In contrast to other methods, like sparse matrix solver [20] or FD splitting techniques [21], FEM gives the field in whole domain and complex types of tunnel structures can easily be modelled.

The remainder of the paper is organized as follows. In section 2, vectorial parabolic equation for the curved tunnel along with boundary conditions is given. Finite element formulation of the model is given in section 3. Finally, results and discussion is given in section 4. Section 5 concludes this paper.

2. Parabolic Equation Modelling for Curved Tunnel

Consider a curved tunnel shown in figure 1, where y and z are the transverse dimensions and $\rho(s)$ denotes the curvature radius so for straight tunnels, $\rho(s) = \infty$. Suppose D is the cross sectional dimension of the tunnel so in UHF/VHF band it can be assumed that $\lambda/D \simeq \nu$, $\beta \simeq \nu$, $D/\rho \simeq \nu^2$ with $\nu \ll 1$ where λ is the wavelength and β is the Brillouin angle. From Maxwell's equations, an asymptotic solution can

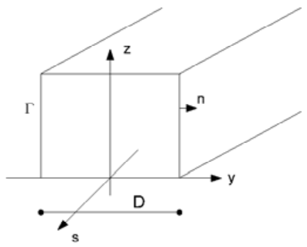


Fig. 1: Cross section of a curved Tunnel with radius of curvature ρ

be found as a power series in ν [9]. By combining the first two terms of this series, the transverse electric field can be approximated as:

$$E_\perp = We^{-jks} \tag{1}$$

The attenuation function $W(s, y, z) = (W_y, W_z)$ must satisfy following equation [10]:

$$\frac{\partial^2 W}{\partial y^2} + \frac{\partial^2 W}{\partial z^2} - 2k^2 \frac{y\cos\theta(s) + z\sin\theta(s)}{\rho(s)} W = 2kj\frac{\partial W}{\partial s} \tag{2}$$

where θ is the rotational angle and k is the wave number. At the boundary contour Γ of the tunnel cross section, Leontovich impedance boundary conditions can be implemented as an approximation of the wall electrical properties [22].

$$W = \frac{j}{k}\tilde{A}\tilde{B}\tilde{A}\frac{\partial W}{\partial n} \tag{3}$$

with,

$$\tilde{A} = \begin{bmatrix} n_y & n_z \\ n_z & -n_y \end{bmatrix} \tag{4}$$

$$\tilde{B} = \begin{bmatrix} 1/Z & 0 \\ 0 & Z \end{bmatrix}$$

Here, $n = (n_y, n_z)$ is the unit normal to Γ and Z is the impedance of the tunnel walls. For a wall material with relative permittivity ϵ_r and conductivity σ, wall impedance can be approximated as $Z = \frac{1}{\sqrt{\epsilon_r - j60\lambda\sigma}}$ [22]. Equation (2) is the vectorial version of scalar parabolic equation describing creeping and whispering gallery waves [17]. Equation (2) accounts for transversal diffusion of wave amplitude $W(s, y, z)$ whereas the matrix boundary condition (3) governs the effects of grazing angle reflection, selective mode absorption and depolarization in the tunnel walls.

3. Finite Element Formulation of Vectorial Parabolic Equation Model

Applying the finite element method (FEM) over the domain $y_{min} \leq y \leq y_{max}$ and $z_{min} \leq z \leq z_{max}$ to equation (2) will yield the following equations,

$$-j2k[M]\frac{\partial W}{\partial s} + ([K] - k^2[1 - 2(\frac{y\cos\theta + z\sin\theta}{\rho(s)})][M])W + [K]_\Gamma W = 0 \tag{5}$$

where,

$$[K] = \Sigma_e \int\int_e \{N\}\{N\}^T dydz$$

$$[K]_\Gamma = \acute{\Sigma}_e \int_\Gamma jk\{N\}_\Gamma\{N\}_\Gamma^T dS$$

$$[M] = \Sigma_e \int\int_e [k^2\{N\}\{N\}^T - \frac{\partial\{N\}}{\partial y}.\frac{\partial\{N\}^T}{\partial y} - \frac{\partial\{N\}}{\partial z}.\frac{\partial\{N\}^T}{\partial z}]dydz \tag{6}$$

where W is the attenuation function $W = (W_y, W_z)$, N is the shape function vector and N_Γ is the shape function vector on the computational window edge Γ as shown in figure 2.

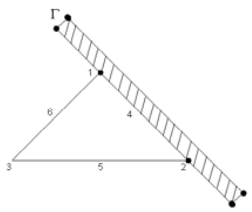


Fig. 2: Triangular element touching the computational window edge

so equation 5 becomes,

$$-j2k[M\frac{\partial\{W\}}{\partial s} + ([\tilde{K}] - k^2\{1 - 2(\frac{y\cos\theta + z\sin\theta}{\rho(s)})\}[M])W = 0 \tag{7}$$

If the Crank-Nicholson algorithm is now applied to equation (7) in the propagation direction s the following is obtained,

$$[A]_i\{W\}_{i+1} = [B]\{W\}_i \tag{8}$$

where,

$$[A]_i = j2k\sqrt{1 - 2(\frac{y\cos\theta + z\sin\theta}{\rho(s)})}[M]_i + 0.5\Delta s([\tilde{K}]_i - k^2\{1 - 2(\frac{y\cos\theta + z\sin\theta}{\rho(s)})\}[M]_i) \quad (9)$$

$$[B]_i = j2k\sqrt{1 - 2(\frac{y\cos\theta + z\sin\theta}{\rho(s)})}[M]_i - 0.5\Delta s([\tilde{K}]_i - k^2\{1 - 2(\frac{y\cos\theta + z\sin\theta}{\rho(s)})\}[M]_i) \quad (10)$$

and Δs is propagation step size and subscript i and $(i+1)$ denotes the quantities related to the i and $(i+1)$ step sizes respectively. The crank-Nicholson scheme is unconditionally stable and the global truncation error is $O(\delta s)^2$ [23].

4. Results and Discussions

To analyze radio wave propagation in rectangular tunnels, consider a rectangular tunnel of cross section 10m x 5m, and the transmitting device is centered in the middle of the tunnel. Length of the tunnel is assumed to be 300m. From equation (8) it is evident that an initial field is required to apply the crank-nicholson scheme. A gaussian beam profile is choosen because of its excellent numerical properties as well as it can be adapted to any propagating mode in a particular structure [24]. In these simulations, a gaussian shaped initial field is used and defined as follows:

$$\phi(y, z) = e^{-\frac{(y-H_y)^2}{k_f}} . e^{-\frac{(z-H_z)^2}{k_f}} \quad (11)$$

where H_y and H_z is the y and z positions of the altitude of transmitting device and k_f is the coefficient which determines beam width. It is assumed that transmitting antenna is in the middle of tunnel so $H_y = 5$m, $H_z = 2.5$m and $k_f = 11$ unless stated otherwise. Rotational angle θ is assumed to be 0 in all simulations. Various radio frequencies in the VHF and UHF range were used in the analysis. The walls of the tunnel are characterised by relative permittivity of $\epsilon_r = 4.0$ and conductivity of $\sigma = 0.01$.

Configuration of the tunnel plays a crucial role in determining the radio coverage. Straight tunnels are modelled by using $\rho = \infty$ while for curved tunnels $\rho = 1000$m. If the tunnel is straight and antenna is located in the tunnel, the signal's primary component will be a result of line of sight transmission. As the tunnel changes direction, the signal experiences more loss due to reflections and scattering. The more abruptly the tunnel changes direction, the greater the loss is and lower the signal level will be. Figure 3 to 7 represents electric field distribution at 300m from the entrance of the tunnel for straight and curved tunnel. Signals propagating through curved tunnel experience a dramatic decrease in signal performance compared with that in straight tunnel. Mode distortion because of the curvature effects of tunnel can be shown clearly in figures **??** to 7. These results are in good agreement with the analytical results in [25]. For simple tunnel profiles it is possible to calculate field analytically based on separation of variables, but straightforward numerical solution of vectorial PEM is more efficient and effective [10].

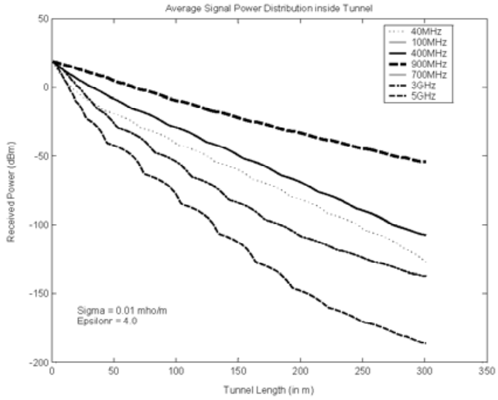


Fig. 8: Average Signal Power across tunnel cross section versus tunnel length

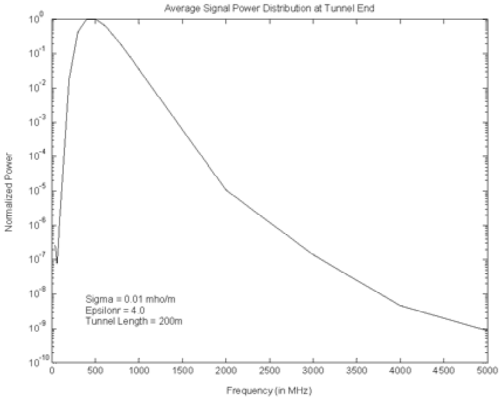
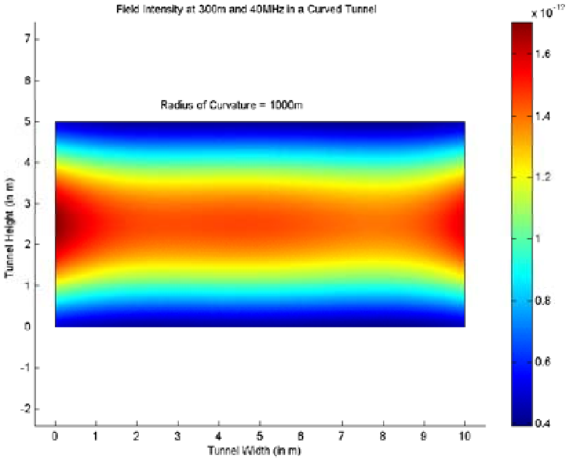


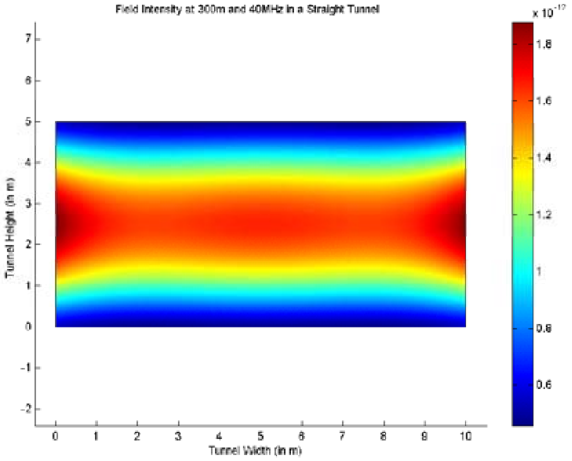
Fig. 9: Normalized Signal Power at 200m from tunnel entrance versus frequency

Figure 8 shows average signal power calculated across tunnel cross section at various range steps in a straight tunnel. In figure 9, average power is calculated at 200m from the tunnel entrance and plotted versus frequency. Simulations were carried out at $\sigma = 0.01$mho/m and $\epsilon_r = 4.0$. Results indicated that in straight tunnels, 800 - 900MHz signals travel significantly farther as they have more power as compare to other frequencies. These results are in good agreement with experimental results presented in Public Safety Wireless Network (PSWN) report [2].

Figure 10 and 11 shows average power distribution across tunnel cross section at 300m from tunnel entrance at 900MHz for different values of permittivities and conductivities of tunnel wall. Signal leaving transmitter is partially absorbed and partially reflected by tunnel walls. Due to the electrical properties of the tunnel walls, a signal may propagate more efficiently so analysis is carried out by choosing different values of conductivity and relative permittivity in a curved tunnel. Figure 10 shows clearly that average power is independent of

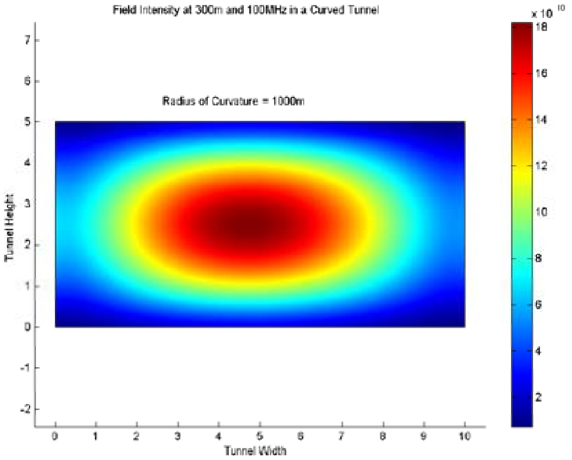


(a) Curved Tunnel

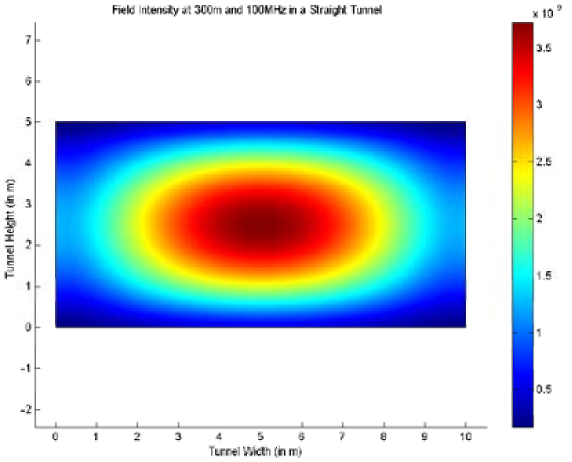


(b) Straight Tunnel

Fig. 3: Signal Intensity at the end of tunnel at 40MHz

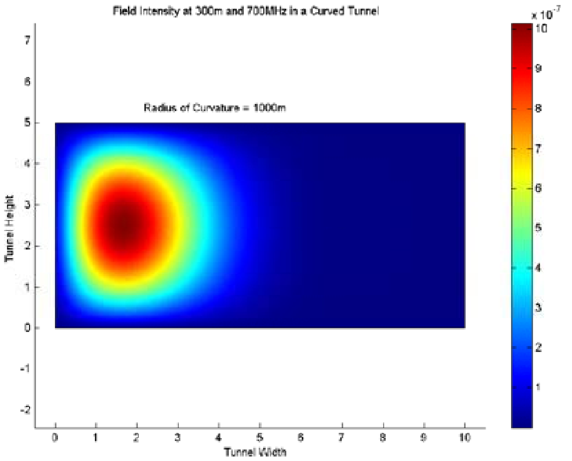


(a) Curved Tunnel

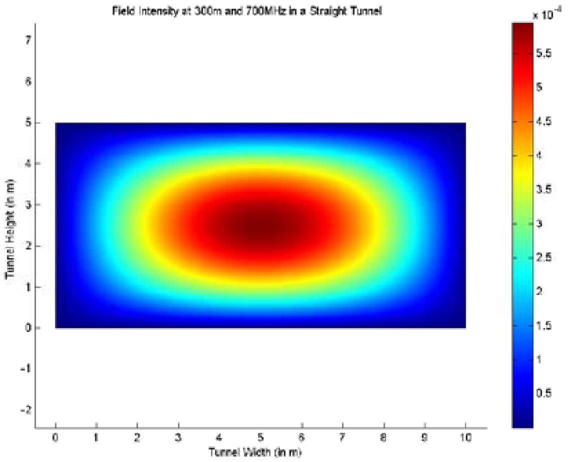


(b) Straight Tunnel

Fig. 4: Signal Intensity at the end of tunnel at 100MHz

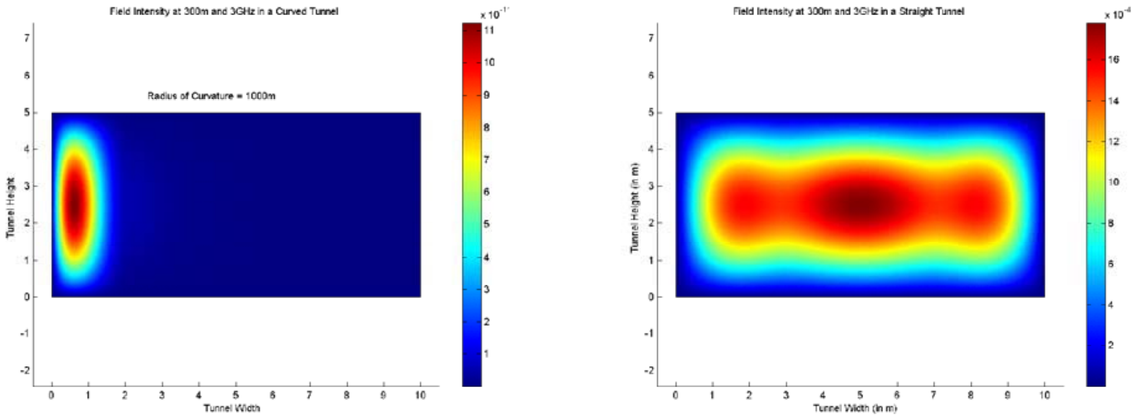


(a) Curved Tunnel



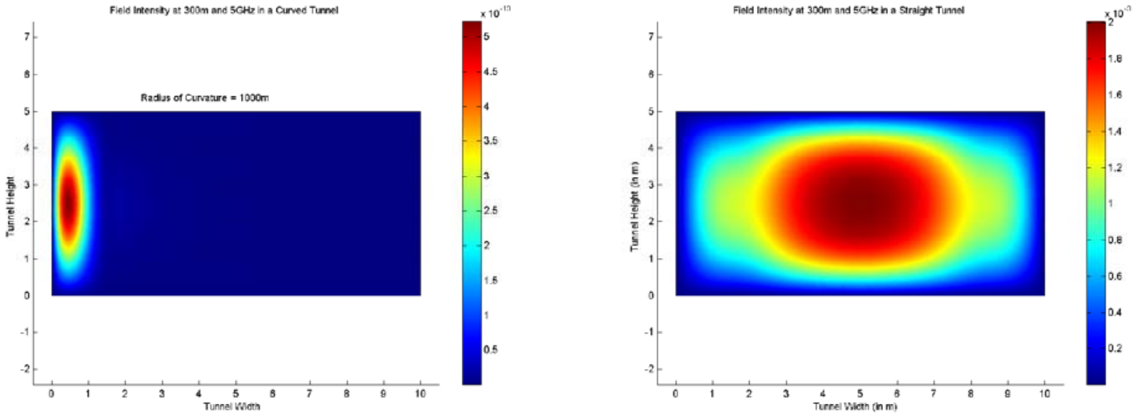
(b) Straight Tunnel

Fig. 5: Signal Intensity at the end of tunnel at 700MHz



(a) Curved Tunnel (b) Straight Tunnel

Fig. 6: Signal Intensity at the end of tunnel at 3GHz



(a) Curved Tunnel (b) Straight Tunnel

Fig. 7: Signal Intensity at the end of tunnel at 5GHz

conductivity σ for low values of σ. Figure 12 represents the power distance variation for three different positions of the transmit antenna, left corner, middle and right most corner of the tunnel. Simulations were carried out at 900MHz and $\epsilon_r = 4.0$, $\sigma = 0.01$mho/m. Power loss for the case of left and middle position is almost same while for right position there is more power loss.

5. Conclusion

An efficient finite element based parabolic equation method is capable of modelling VHF/UHF radio communications in straight and curved tunnels beyond the limits of existing methods. 3D field distributions and power loss diagrams obtained by FE solution of vectorial PEM help to gain deeper insight into the physical effects of radio wave propagation inside tunnels. Effects of curvature in tunnels has been analyzed and an optimum frequency band is suggested for natural propagation of waves. Effect of electrical parameters such as conductivity

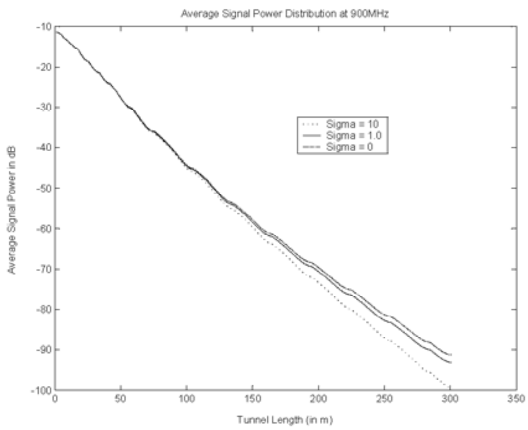


Fig. 10: Average Signal Power at 900MHz for different σ

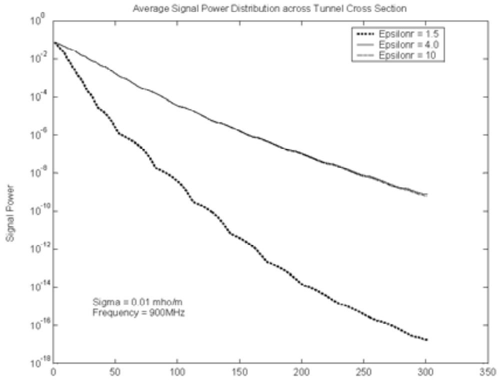


Fig. 11: Average Signal Power at 900MHz for different ϵ_r

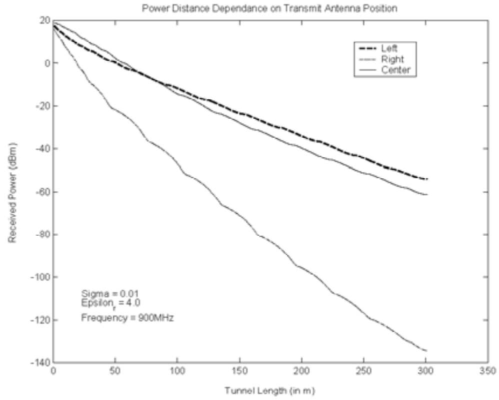


Fig. 12: Plot of power distance dependence on transmit antenna positions

and relative permittivity is analyzed and concluded that for underground street tunnels where conductivity is low, effect of conductivity on wave propagation can be neglected.

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