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NUMERICAL ANALYSIS OF GENERALIZED SLOTLINE ON THE BASIS OF DOMAIN PRODUCT TECHNIQUE

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1. Introduction.

In most cases papers devoted to theory of the slotlines and striplines deal with structures having coordinate bounds. Both technological peculiarities of the constructions and the restrictions caused by required numerical methods are taken into account. For further investigation of such transmission lines and the optimization their parameters it is important to develop flexible and efficient procedures providing the possibility of the substantial variation of the structure's geometry within the framework of the chosen mathematical model.

This paper presents an algorithm for the numerical study of generalized slotline having the arbitrary polygonal cross-section form. The solution of the problem is based on the domain product (DP) technique. Some years ago this method was developed and successfully applied to solving problems of wave diffraction on the objects bounded by polygonal surfaces [1,2].

2. Formulation of the problem.

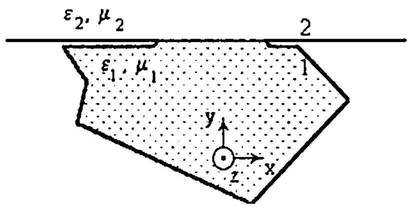

Fig. 1

Fig.1 shows the cross-section contour of the slotline under consideration. Outside the slot some boundary links of the region 1 may be "magnetic wall". The homogeneous, isotropic and lossless mediums are characterized by the relative permittivity/permeability (ε_1,μ_1) and (ε_2,μ_2) in regions 1 and 2, respectively. The time factor is assumed to be $e^{i\omega t}$, where ω is the radiation frequency.

It is known that the eigen waves problem for given structure is reduced to solving of simultaneous Helmholtz equations for the longitudinal components of electric and magnetic fields: $E_z=U(x,y)e^{-ihz}$, $H_z=V(x,y)e^{-ihz}$, where h is transmission constant, U and V are some scalar functions, subject to definition. Functions U and V must satisfy respectively homogeneous Dirichlet and Neumann boundary conditions on the conductive part of the boundary. On the "magnetic walls" those are converse. Necessity of joint solving equations for U and V is stipulated by mixed condition for their normal and tangential derivatives on the slot:

$$\alpha_2\frac{\partial V_2}{\partial y}-\alpha_1\frac{\partial V_1}{\partial y}=\frac{\partial U_2}{\partial x},\quad \nu_2\frac{\partial U_2}{\partial y}-\nu_1\frac{\partial U_1}{\partial y}=\frac{\partial V_2}{\partial x} \tag{1}$$

Here, functions U_1, V_1 and U_2, V_2 correspond to the longitudinal fields components in regions 1 and 2; α_1, α_2, ν_1, ν_2, β are coefficients dependent on medium parameters: $\alpha_i=\frac{\mu_i\cdot\mu_0\cdot\omega}{\chi_i^2}$, $\nu_i=\frac{\varepsilon_i\cdot\varepsilon_0\cdot\omega}{\chi_i^2}$, $\beta=h\cdot\left(\frac{1}{\chi_2^2}-\frac{1}{\chi_1^2}\right)$, $i=1,2$

μ_0, ε_0 are absolute magnetic and electric permittivity, χ_i is the transversal wavenumber in ith medium $\chi_i = \sqrt{\varepsilon_i \cdot \mu_i \cdot k^2 - h^2}$, $k = \frac{2 \cdot \pi}{\lambda}$, $i = 1,2$, λ is the free-space wavelength. Beside conditions (1), continuity condition on the slot $U_1 = U_2$, $V_1 = V_2$, the on-edge condition and the condition at infinity in region 2 must be satisfied.

3. Algorithm.

Let us for every segment of the contour introduce the local Cartesian (x_i,y_i) and elliptical (ξ_i,η_i) coordinate systems, related by:

$$x_i = f_i \cdot cosh\xi_i \cdot cos\eta_i, \quad y_i = f_i \cdot sinh\xi_i \cdot sin\eta_i, \quad i = \overline{1,N}$$

where f_i is the half a length of the ith segment, N is the number of region 1 boundary links, first of them corresponded to the slot.

According to the DP technique, we consider domain 1 as the common part (the product) of simple base regions. Each of them is associated with boundary segment and represented by the whole plane except the set of points belonging to the corresponding segment itself. Functions $U_1(x,y)$ and $V_1(x,y)$ are written down in the form:

$$U_1 = \sum_{i=1}^{N} U_{1i}, \quad V_1 = \sum_{i=1}^{N} V_{1i}. \tag{2}$$

where V_{1i}, U_{1i} are some sought-for functions satisfying the Helmholtz equation in base regions.

Considering each boundary link of the domain 1 as a degenerated ellipse and solving Helmholtz equation by means of separating variables in elliptical coordinates it is possible to obtain representation of the quantities V_{1i}, U_{1i} in the form of series in terms of Mathieu functions se_n and ce_n

$$U_{11} = \sum_{n=1}^{\infty} {}^1A_n^1 \frac{Ne_n^{(2)}(\xi_1,q_1^1)}{Ne_n^{(2)}(0,q_1^1)} \cdot se_n(\eta_1,q_1^1), \; U_{1i} = \sum_{n=1}^{\infty} {}^1A_n^i \frac{Me_{n-1}^{(2)}(\xi_i,q_i^1)}{Me_{n-1}^{(2)}(0,q_i^1)} \cdot ce_{n-1}(\eta_i,q_i^1), \; i = \overline{2,N},$$
$$V_{1i} = \sum_{n=1}^{\infty} {}^1B_n^i \frac{Me_{n-1}^{(2)}(\xi_i,q_i^1)}{Me_{n-1}^{(2)}(0,q_i^1)} \cdot ce_{n-1}(\eta_i,q_i^1), \; i = \overline{1,N} \tag{3}$$

Values U_2 and V_2 are expanded in terms of Mathieu functions too:

$$V_2 = \sum_{n=1}^{\infty} {}^2B_n^1 \frac{Mc_{n-1}^{(3)}(\xi_1,q_1^2)}{Mc_{n-1}^{(3)}(0,q_1^2)} \cdot ce_{n-1}(\eta_1,q_1^2), \quad U_2 = \sum_{n=1}^{\infty} {}^2A_n^1 \frac{Ms_n^{(3)}(\xi_1,q_1^2)}{Ms_n^{(3)}(0,q_1^2)} \cdot se_n(\eta_1,q_1^2) \tag{4}$$

In (3) and (4) $Me_n^{(2)}(\xi,q)$, $Ne_n^{(2)}(\xi,q)$, $Mc_n^{(3)}(\xi,q)$, $Ms_n^{(3)}(\xi,q)$ are modified Mathieu functions [3], $q_i^j = f_i^2 \chi_j^2 / 4$.

Substituting representations (2-4) into boundary conditions and projecting the found equalities onto orthogonal basises $\{se_m(\eta_i,q_i^j)\}$, $\{ce_m(\eta_i,q_i^j)\}$, $\eta_i \in (0,\pi)$, $(j=1,2;$ $i = \overline{1,N})$ we obtain homogeneous set of linear algebraic equations (SLAE) for the unknown coefficients $\{{}^1A_n^i\}$, $\{{}^1B_n^i\}$, $\{{}^2A_n^1\}$, $\{{}^2B_n^1\}$ in the expansions of U and V. In [1] it has been shown, that similar SLAE can be solved by the reduction method. The propagation constant is obtained by solving for values of h that render the SLAE determinant zero.

4. Numerical results.

On the basis of the algorithm described above a computer program was developed. Numerical experiments have been performed to study the convergence of the results with respect to the number of basis functions used. It was established that for practically important geometry of the structure the convergence speed is rather high. This can be seen from Fig. 4a where the computed values of ε_{eff} as a function of 1/M are given. Here $\varepsilon_{eff}=(h/k)^2$, M is a number of terms being taken in the expansions (3,4). Results refer to the object shown in Fig. 2a.

In order to check the results which have been obtained using the technique, they were compared with the corresponding data given in [4,5]. Fig. 4b shows data conforming the line in Fig. 2b. Table 1 gives a comparison between the results obtained for the structure shown on Fig. 2a (r=3b) and data for the bilateral slotline which are available in [5]. Such comparison is possible because the value ε_{eff} stabilizes quickly with increasing r. From Fig.4c one can see that for r>3b lengths of the lateral channels do not practically effect the represented characteristics of the dominant mode. Thus, in both cases (Table 1, Fig. 4b) a good agreement between results obtained and data of the other authors exists.

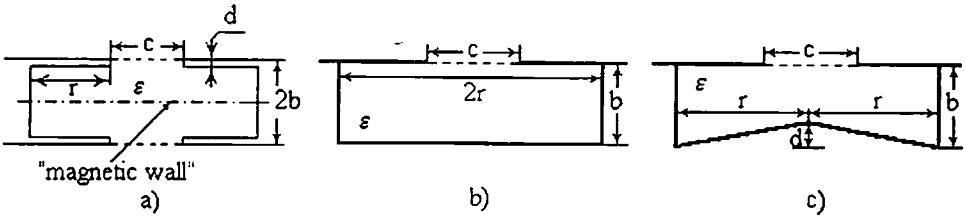

Fig. 2.

Table 1.

b/λ	c/b	ε_{eff}	ε_{eff},[5]
0.026	1	1.377	1.37
	4	1.255	1.24
0.171	1	1.652	1.65
	4	1.650	1.65

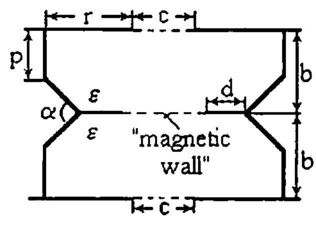

Fig. 3.

In Fig. 4d, 4e and 4f illustrative examples demonstrating some possibilities of the technique are given. Fig. 4d presents the plots of ε_{eff} as a function of the conductor thickness d for the bilateral slotline. Fig.4e illustrates dependence of the effective dielectric constant from the deformation altitude of the lower part of the boundary contour for the line shown in Fig. 2c. Fig. 4f shows a dispersion characteristic of the structure brought in Fig. 3 (c=b, r=2b, p=0.5b, d=b, a=p/2).

5. Conclusion.

A new approach to the numerical study of the open transmission lines having polygonal boundary contours is proposed. Efficiency of the method has been demonstrated using testing and illustrative examples.

Literature.

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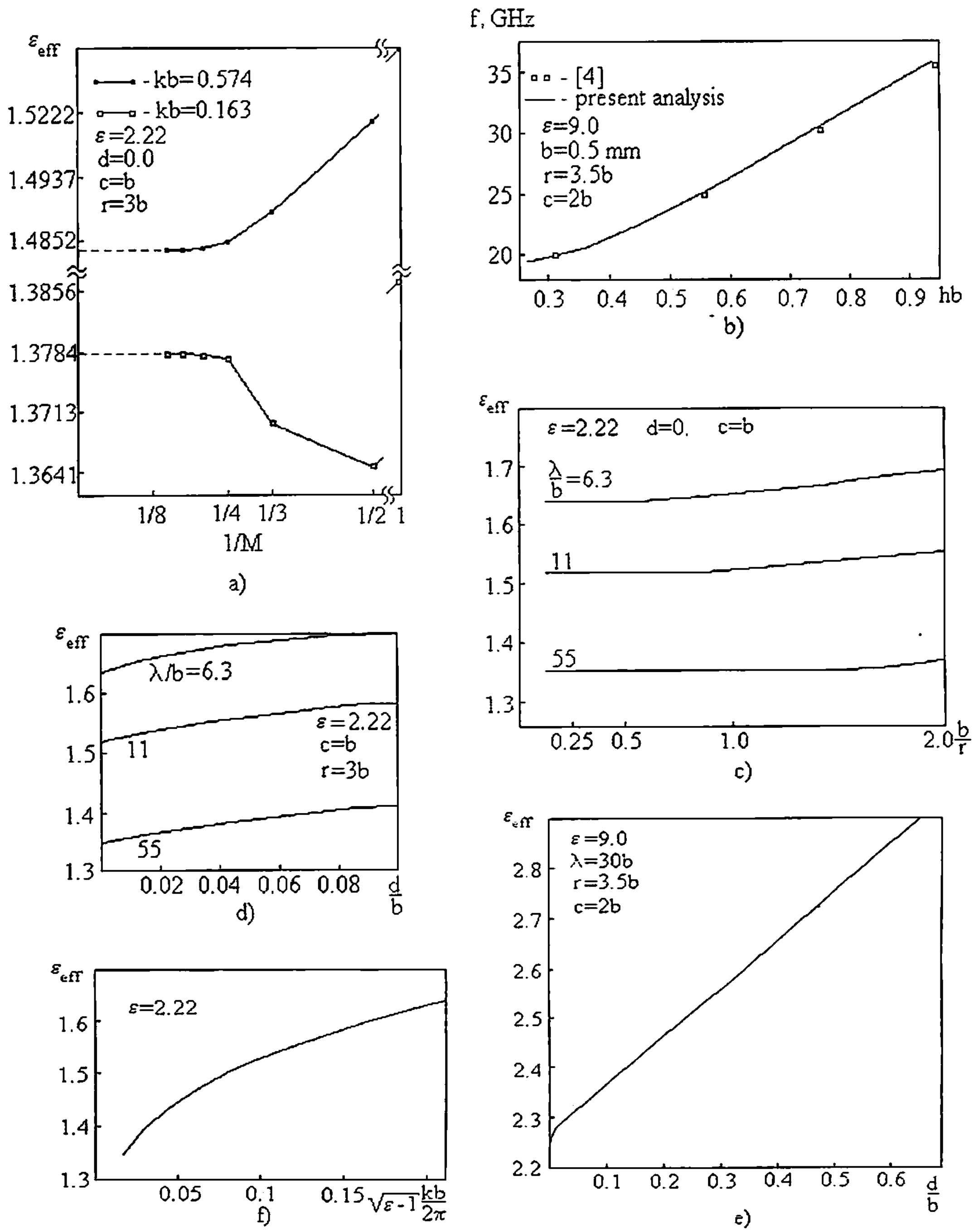

Fig. 4.