New boundary treatments to avoid undesired reflection from matched DNG slab in FDTD method

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1. Introduction

In recent years, Metamaterial $[1] \sim [3]$ are expecting for developing innovetive antenna devices from that's unique property. A Double NeGative (DNG) material is one of the Metamaterial which indicate that negative refraction index. The DNG material is very interesting, because, the possibility of realization of extremely high gain small antenna by using the DNG material have been reported[3]. On the other hand, it is needed that a numerical analysis to apply or to invent new technology using the DNG materials such as FEM (The Finite Element Method) or FDTD (The Finite Difference Time Domain method). However, in these numerical analysis, it is effective to investigate novel DNG application or novel DNG phenomena, using only the DNG nature such as negative permittivity and negative permeability rather than including the DNG structure as wire and SRR. In these calculations, DNG materials are often realized by loss less Drude dispersion models for the electric permittivity and magnetic permeability in FDTD analysis. The dispersive mediums are modeled by RC (Recursive Convolution) [4] or PLRC(Piecewise Linear Recursive Convolution)[5] method in FDTD analysis. Especially PLRC method is well known as quite accurate method. However, undesired reflection is found from matched DNG slab in RC- or PLRC-FDTD analysis. The one reason of undesired reflection is considered that an uncertainty modeling of the boundary between DNG slab and free space. In this paper, we will indicate the undesired reflection from matched DNG slab in FDTD analysis, in this calculation, the DNG slab is modeled by loss-less Drude dispersion, and we will propose its reducing method.

2. Undesired reflection from matched DNG slab in FDTD analysis

The DNG material is consist of negative permittivity and negative permeability, in the FDTD analysis, these negative parameters are realized by using dispersive material such as loss less Drude dispersion as follows

$$\begin{cases} \varepsilon(\omega) = \varepsilon_0 \left(1 - \frac{\omega_{pe}^2}{\omega} \right) \\ \mu(\omega) = \mu_0 \left(1 - \frac{\omega_{pm}^2}{\omega} \right) \end{cases}$$
(1)

Where ε_0 and μ_0 are the permittivity and the permeability of free space respectively, ω_{pe} and ω_{pm} are resonant frequency. From eq.(1), the permittivity and permeability become negative value while $\omega_{pe}^2 > \omega$ and $\omega_{pm}^2 > \omega$, if the value of $\frac{\omega_{pe}}{\omega}$ and $\frac{\omega_{pm}}{\omega}$ are 2 than the refraction index *n* become -1. This condition is one of the mathod DNG. In order to model dispersive media, several methods have been proposed, especially, The Recursive Convolution (RC) Method[4] and the Piecewise Liner Recursive Convolution (PLRC) Method[5] are known as very low memory method to analyze dispersive media, because, the method calculate convolution integral of electric flux density and magnetic flux density recursively, and the methods do not use auxiliary field such as Auxiliary Difference Equation (ADE) method[6].

Numerical example by calculating RC-FDTD method is shown in Fig2(a). The analysis model is 2-dimensinal model as shown in Fig.1, in this calculation, $\omega_{pe} = \omega_{pm} = 2\pi \times 1 \times 10^9/2[rad/sec]$ and analysis frequency is 10GHz, therefore, the index of refraction and the reflection coefficient are n = -1



Figure 1: The 2-Dimensional DNG Model



(b) The exact solution

Figure 2: The electric fields of 2-Dimensional DNG analysis

and $\Gamma = 0$, the FDTD parameters are set as $\Delta x = \Delta y = 0.75 \times 10^{-3} [m]$, the DNG slab is set from 100 cell to 180 cell in computation region. In this calculation, PML absorbing boundary condition is used whole computation space including the DNG region. The PML for the DNG material is applied that the PML for dispersive media.[7] From Fig.2(a), the analyzed electric fields by the RC-FDTD method is very disturbed, in contrast, the Fig.2(b) which is calculated by the exact solution[8] shows continuous fields. The disturbance occur due to uncertain definition of the boundary between air and matched DNG slab. Furthermore, in order to investigate the magnitude of undesired reflection from matched DNG slab, we calculate 1-dimensinal model. The FDTD parameters are set as $\Delta z = 1 \times 10^{-4} [m]$ and $\Delta t = 3.333 \times 10^{-14}$ [sec], analysis region is 2000 cells, the DNG slab is set from 1000 cells to 1500 cells in computation region. The analyzed result of whole computation space is shown in Fig.3. In this analysis, the sinusoidal wave is fed at 400 cells from edge of computation space. The input wave is one-way wave which is realized by feeding electric field and magnetic field simultaneously. Fig.3 shows transmitted electric field at 5000 time steps. In Fig.3 left-handed property is found in DNG region. Fig.4 shows electric fields of 0 cell to 400 cells, the input one-way wave is fed at 400 cells, therefore, this figure shows reflection waves. Form Fig.4, undesired reflection from matched DNG slab is found in RCand PLRC-FDTD analysis. In this example, 1 % reflection is found for input wave.

3. New modeling method

In order to reduce the undesired reflection from matched DNG slab in the RC-FDTD method, we propose new treatment technique for boundary between DNG slab and free space. The DNG material in the Yee's cell is shown in Fig.5, in this case the boundary between DNG and free space is defined at electric filed E_x as Fig.5. However, DNG has negative permittivity and negative permeability simultaneously, therefore the permittivity and permeability can not be modeled simultaneously at the boundary. We have proposed new placement in Ref.[7], in this method, the electric and magnetic fields are placed same place, however, in this paper, the electric and magnetic fields are placed diffrent place as original FDTD method. In the geometry of Fig.5, the boundary is set on the electric field. Then, the matching condition is needed to satisfy matching condition, the matching condition is indicated as

$$\left\{E_{xboundary} = \frac{E_{xfreespace} + E_{xDNG}}{2}\right\}$$
(2)





Figure 4: The reflection waves



Figure 5: The 1-Dimensional Yee's cell

where E_{xDNG} is electric field at boundary which is calculated by using negative permittivity and $E_{xfreespace}$ is electric field at boundary which is calculated by using free space permittivity, therefore field at boundary is calculated two times, after these calculations, eq.(2) is used.

4. Numerical demonstration

We calculate same model as previous example to confirm effectiveness of our proposed method, therefore DNG slab is from 1000 cells to 1500 cells in computation space, and FDTD parameters are same as previous calculation. Fig.6 and Fig.7 shows transmitted and reflected electric field at 50000 time steps. In our proposed method, we used RC-FDTD method. The result of our proposed method is almost same in transmitted region, and from Fig.7, the effectiveness of our proposed method is confirmed. The reflection wave of our method is almost 0, because, the result of our method is almost agree with DPS result in Fig.4.

5. Conclusions

In this paper, we indicated that undesired reflection is occurred from matched DNG slab in RC-FDTD analysis and we proposed its improving method. In our proposed method, the electric and magnetic field are placed same location, furthermore, we used new boundary treatment technique in which we used averaging value of free space and DNG metamaterial to obtain the field at boundary. From results, our propose method is very effective to model matched DNG slab. However, we have confirmed effectiveness of our proposed method only 1-dimensionla problem. Therefore applying 2- and 3-dimensional analysis will be our future works.



Figure 6: The result of whole region



Figure 7: The improved reflection wave

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