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ANALYSIS OF COPLANAR WAVEGUIDE WITH THICK, TRAPEZOIDAL ELECTRODES AND MULTILAYERED DIELECTRIC SUBSTRATE

Keren Li and Kazuhiko Atsuki

The University of Electro-communications
1-5-1 Chofugaoka, Chofu-shi, Tokyo 182, Japan
E-mail Address: keren@light.ee.uec.ac.jp

I. INTRODUCTION

Coplanar waveguide (CPW) is one of the basic transmission lines used in microwave or millimeter-wave integrated circuits (MICs) [1]. Recent development on MICs and monolithic MICs (MMICs) has brought many research interests to us in and stimulated extensive studies on the characteristics of the CPWs[2]. A conventional CPW usually consists of one central electrode and two grounded electrodes which are coplanar on a dielectric substrate. CPW has several advantages compared with the conventional microstrip line, another basic transmission line widely used in MICs. CPW makes surface mounting of the electronic components easy while the microstrip line requires a via hole for grounding. In addition, CPW has more design freedom for given characteristic impedance and effective dielectric constant.

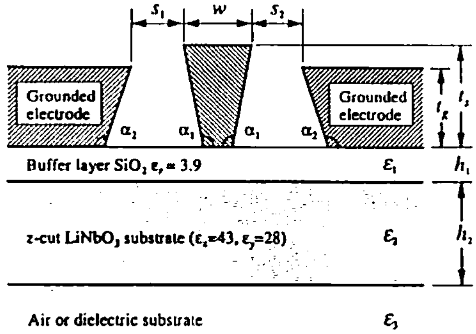

Fig. 1 Configuration of traveling-wave optical modulator using CPW electrodes

Both quasi-TEM wave approximation and full-wave analysis of the CPW have been reported by many researchers [1-3]. For the simplicity of analysis, the CPWs under consideration are usually assumed that their electrodes have infinite thickness or even have finite thickness but the cross-section is rectangular, and their substrate is a one-layered dielectric media [1-3]. In the recently developed MMICs, however, the CPWs may be fabricated on a multilayered substrate and may have thick electrodes from the design aspects such as reduction of conductor loss and achievement of higher integration of the MMICs [4]. The CPW is also used in the broad-band traveling-wave optical modulator of which it is designed as driving electrodes for microwave to modulate the optical wave through the interaction between these two waves [5]. A simplified configuration of the modulator is illustrated in Fig. 1, where the modulator is fabricated to have very thick electrodes for reducing conductor loss and taking the velocity matching between the microwave traveling along the CPW electrodes and the optical wave propagating along the optical waveguide. Below the electrodes, the first dielectric layer, called buffer layer, is introduced to protect the optical waveguide from the contact to the conductor, as well as the velocity matching; the second layer is a electro-optic crystal material, such as $LiNbO_3$, where the microwave and optical wave interact through the Pockels effect; the third layer may be air or some dielectric material. To the analysis of this CPW structure, however, several widely used methods of analysis, such as conformal mapping method spectral domain approach and mode matching method [6-9] may have difficulty which arises because of the thick trapezoidal electrodes and the multilayered substrate.

In this paper, we propose a new method of analysis by using a combination of the conformal mapping and the variational principle under quasi-TEM wave approximation. After a brief description of the proposed method, we present some numerical results to show the effects of the thick trapezoidal electrodes and the multilayered substrate.

II. THEORY

Our analysis includes two steps. First, we transform the upper region including the thick trapezoidal electrodes into an upper semi-infinite plane and then into a welled structure by Schwarz-Christoffel transformation [6], as shown in Fig. 2. The mapping points on the t-plane are obtained by a numerical method called Marquardt's method. Since the electrostatic electric energy is conserved after the conformal mapping, we can easily obtain the energy in the welled structure by Fourier series expansion and express it by an unknown boundary potential function $\psi(x)$ on the interface of upper and lower regions. In contrast, the electric energy in the lower multilayered dielectric region is derived directly by Fourier series expansion or Fourier transform in the horizontal layer direction, i.e., the x-direction in Fig. 3. This derivation for the multilayered region is fairly complicated but the final expression is surprisedly simple and very similar to that for welled structure.

Approximating the function $\psi(x)$ with first-order spline functions and the node potential values ϕ_i as

$$\psi(x) = \sum_{i=1}^{M-1} f_i(x)\phi_i\,,\quad f_i(x) = \begin{cases} (x - x_{i-1})/(x_i - x_{i-1}), & x_{i-1} \le x \le x_i \\ (x_{i+1} - x)/(x_{i+1} - x_i), & x_i \le x \le x_{i+1} \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

we can get the energy expressions in the upper and lower regions as follows:

$$W_{\text{up}} = \frac{\varepsilon_0}{2}\sum_{n=1}^{\infty} n\pi\, \psi_n^2\,,\quad W_{\text{low}} = \frac{\varepsilon_1}{2}\sum_{n=1}^{\infty} n\pi\, \psi_n^2 K_n \tag{2}$$

where

$$K_n = \frac{1}{\varepsilon_1}\,\frac{1 + \varepsilon_1^2\, T_{1,n}(T_{2,n} + T_{3,n}) + \varepsilon_2^2\, T_{2,n}T_{3,n}}{T_{1,n} + T_{2,n} + T_{3,n} + \varepsilon_2^2\, T_{1,n}T_{2,n}T_{3,n}} \tag{3}$$

$$T_{k,n} = \frac{1}{\varepsilon_k}\tanh\left(\frac{n\pi h_k}{a}\right),\quad k = 1, 2, 3$$

for a three-layered dielectric structure and

$$\psi_n = \sum_{i=1}^{M-1} \frac{2p_{i,n}}{n^2\pi}\,\phi_i\,,\quad p_{i,n} = \frac{\sin n\tilde{x}_i - \sin n\tilde{x}_{i-1}}{\tilde{x}_i - \tilde{x}_{i-1}} - \frac{\sin n\tilde{x}_{i+1} - \sin n\tilde{x}_i}{\tilde{x}_{i+1} - \tilde{x}_i} \tag{4}$$

$$\tilde{x}_i = \begin{cases} u_i\,, & \text{for upper region} \\ \pi/a\; x_i\,, & \text{for lower region} \end{cases} \tag{5}$$

From above relations, we can then express the total electric energy in the whole region in per unit length by the node potential values as follows:

$$W = W_{\text{up}} + W_{\text{low}} = \sum_{i=1}^{M-1}\sum_{j=1}^{M-1} H_{ij}\,\phi_i\,\phi_j \tag{6}$$

$$H_{ij} = \frac{\varepsilon_0}{\pi}\sum_{n=1}^{\infty}\frac{1}{n^3}\,p_{i,n}^{\text{up}}p_{j,n}^{\text{up}} + \frac{\varepsilon_1}{\pi}\sum_{n=1}^{\infty}\frac{K_n}{n^3}\,p_{i,n}^{\text{low}}p_{j,n}^{\text{low}} \tag{7}$$

Applying the variational principle to the total energy expression (6) gives a series of simultaneous equations for unknown node potential values

$$\sum_{j=1}^{M-1} H_{ij}\,\phi_j = 0,\quad i = 1, 2, \ldots, M1-1, M1+2, \ldots, M-1 \tag{8}$$

with additional conditions, $\phi_{M1} = \phi_{M1+1} = V_0$. Finding their solution on a computer and back them to equation (6) gives the total energy and the line capacitance C in per unit length and then the characteristic impedance Z, the effective dielectric constant ε_{eff} and the effective index n_{eff} as follows:

$$C = \frac{2W_{\min}}{V_0^2}\,,\quad Z = \frac{1}{v_0\sqrt{C\,C_0}}\,,\quad \varepsilon_{eff} = \frac{C}{C_0}\,,\quad n_{eff} = \sqrt{\varepsilon_{eff}} \tag{9}$$

where C_0 is the line capacitance in per unit length after removed all dielectric material, and v_0 is the speed of light in vacuum [6].

III. NUMERICAL RESULTS

Several numerical results are presented to demonstrate the application of the proposed method for a CPW used in the traveling-wave optical modulator on a z-cut $LiNbO_3$ substrate with a SiO_2 buffer layer, as shown in Fig. 1. The electrodes are assumed perfectly conducting. Width of the lower multilayered region a is taken as more than 100 times of the width of the central electrode in order to simulate the open structure, which is an enough value to eliminate the effects of the outside shield walls. Anisotropy of the $LiNbO_3$ substrate is treated in such way where the anisotropic substrate is replaced by an alternative isotropic substrate with dielectric constant = $(\varepsilon_x \varepsilon_y)^{1/2}$ and thickness = $(\varepsilon_x/\varepsilon_y)^{1/2} h_2$.

Figure 4 is the characteristic impedance and the effective index n_{eff} versus the thickness of the SiO_2 buffer layer. This result shows very good agreement with that presented by Kitazawa [7]. Figure 5 shows the characteristic impedance and the effective index n_{eff} versus the thickness of the electrodes. Figure 6 shows the effects of the angles of the trapezoidal electrodes. The CPU time spent for calculating one set of characteristic data was about 15 seconds on a workstation SUN SPARCstation 20.

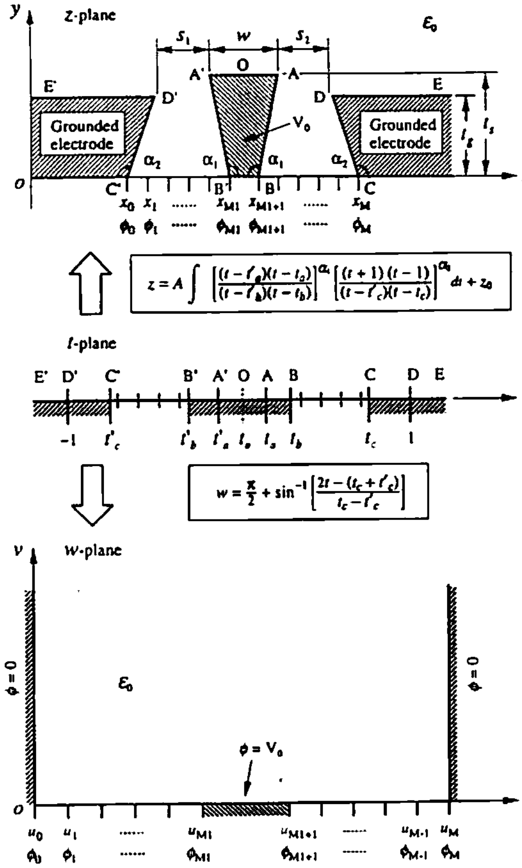

Fig. 2 Conformal mapping of upper electrode region

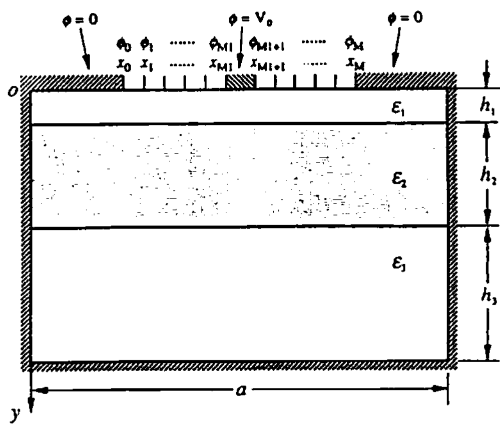

Fig. 3 Lower multilayered dielectric region

IV. CONCLUSION

In this paper, we presented an analysis of the CPW with thick trapezoidal electrodes and multilayered dielectric substrate. To the analysis of such CPW, we proposed a method which uses a combination of the conformal mapping and the variational principle. The conformal mapping makes us easy to deal with the very complicated cross-section of the electrodes. At the same time, by using the energy expression derived in this paper, we have no any difficulty to deal with the multilayered dielectric substrate, even if the waveguide consists of large number of layers. It is also easy and automatic to deal with the case where some layers may be very thin or have zero or infinite thickness. The energy expression for multilayered region can be extended to a general multilayered structure with arbitrary number of layers by using a technique developed in our previous papers [10-11]. The proposed method will provide a very powerful and efficient means to the analysis of the CPWs and microstrip lines, especially the structures with complicated polygonal electrodes and multilayered dielectric substrate.

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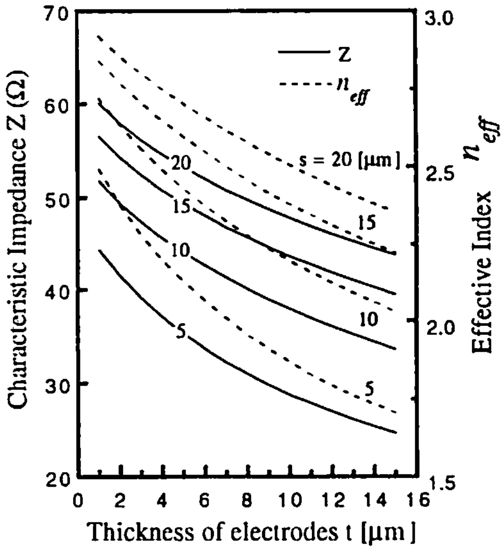
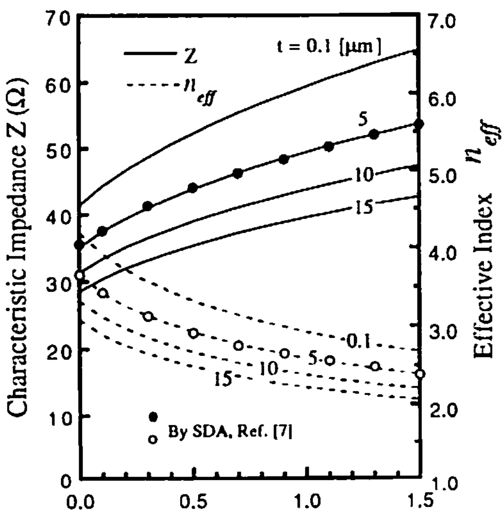


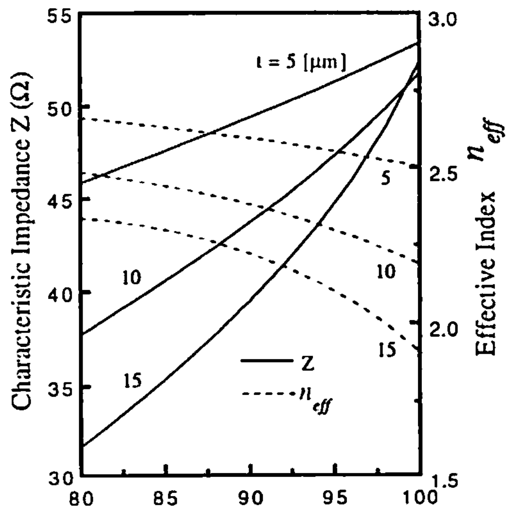
$w = 8$ μm, $t_g = t_s = t$, $s_1 = s_2 = s$, $\alpha_1 = \alpha_2 = 90°$
$h_1 = 1.0$ μm, $h_2 = 500$ μm, $h_3 = \infty$
$\varepsilon_{1r} = 3.9$, $\varepsilon_{2r} = 34.7$, $\varepsilon_{3r} = 1.0$

Fig. 5 Characteristic impedance and effective index n_{eff} versus thickness of electrodes



$w = 8$ μm, $t_g = t_s = t$, $s_1 = s_2 = 15.0$ μm, $\alpha_1 = \alpha_2 = 90°$
$h_2 = 500$ μm, $h_3 = \infty$
$\varepsilon_{1r} = 3.9$, $\varepsilon_{2r} = 34.7$, $\varepsilon_{3r} = 1.0$

Fig. 4 Characteristic impedance and effective index n_{eff} versus thickness of SiO_2 buffer layer



$w = 8$ μm, $t_g = t_s = t$, $s_1 = s_2 = 15.0$ μm, $\alpha_1 = \alpha_2 = \alpha$
$h_1 = 1.0$ μm, $h_2 = 500$ μm, $h_3 = \infty$
$\varepsilon_{1r} = 3.9$, $\varepsilon_{2r} = 34.7$, $\varepsilon_{3r} = 1.0$

Fig. 6 Variation of the characteristics versus angle of trapezoidal electrodes