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A STATISTICAL MODEL FOR BASE STATION DIVERSITY RECEPTION

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Abstract A model is proposed to predict the base-station signal statistics in urban areas where there is no local scattering around the base. The model eliminates some restrictive assumptions inherent in previous models and provides simple yet accurate expressions for fading correlation. A generalization concerning local scattering in the vicinity of the base is also indicated.

1 Introduction

The only existing signal reception model for a base station with no local scatterers, which has been proposed by Lee (1973), assumes that the scattering in the vicinity of the mobile is caused by a ring of uniformly placed scatterers. This does not portray the actual situation where the scattering is due to randomly positioned scatterers in an area, giving signals arriving at the base with random time delays. The model used in this paper assumes an amorphous scattering area around the mobile. Associating random time delays with individual scatterers, it gives a better representation of the real channel.

In the paper we consider a case in which a two-branch hybrid diversity system, where the antennas are separated in space and frequency, is used at the base station. The mobile unit has a single transmitting/receiving antenna.

2 Proposed Model

2.1 Basic Assumptions

Suppose that two signals of different frequencies ω_1 and ω_2, which are transmitted from the mobile, propagate by way of scattering from N scatterers randomly placed over an area around the mobile and are received at two base antennas, each tuned to one of the frequencies (see Fig.1). Then, at both receiving points there will be N component waves, all with random amplitudes, phases and angles of arrival. The random amplitudes can have any distribution. The phases are assumed to be uniformly distributed over $[0,2\pi)$. The angles of arrival at both receiving points at the base are approximately the same since the propagation distance is large compared to the antenna separation. These angles of arrival will be assumed to be identically distributed for each of N scatterers and its distribution will be taken as uniform over a very narrow range. We assume that there are no local scatterers around the base. The propagation time delay and angle of arrival of each component wave will be considered to be independent, and the distribution of time delay difference τ, which is defined relative to the shortest propagation delay T_0, will be taken to be exponential (Clarke, 1968). It is also assumed that isotropic antennas are used at the base.

2.2 Derivation of correlation coefficient

Signal statistics at the base station can be determined from the proposed model, and in particular the envelope correlation of the received signals separated in space and frequency. At each of the receiving antennas A and B, the received signal will be the superposition of N waves, one of them being the time-shifted version of the other. Thus

$$v_A = \sum_{i=1}^{N} a_i \, e^{-j\omega_1[T_i + \frac{\xi}{2c}\cos\alpha_i]} e^{j\omega_1 t} \tag{1a}$$

and

$$v_B = \sum_{i=1}^{N} a_i e^{-j\omega_2[T_i - \frac{\xi}{2c}\cos\alpha_i]} e^{j\omega_2 t} \tag{1b}$$

where a_i and α_i are, respectively, the amplitude and angle of arrival of the ith waveform. T_i is the propagation time of the ith waveform arriving at the centre point M between A and B. The antenna separation is denoted by ξ and c is the speed of light.

The covariance function of v_A and v_B, denoted by $R_v(\xi,\Omega)$, is the mean of their conjugate product; namely,

$$R_v(\xi,\Omega) = \mathrm{E}\left\{ \sum_{i=1}^{N} a_i^* e^{j(\omega_c - \frac{\Omega}{2})[T_i + \frac{\xi}{2c}\cos\alpha_i]} . \sum_{i=1}^{N} a_i e^{-j(\omega_c + \frac{\Omega}{2})[T_i - \frac{\xi}{2c}\cos\alpha_i]} \right\} \tag{2}$$

where $\Omega = \omega_1 - \omega_2$ and $\omega_c = (\omega_1 + \omega_2)/2$. To a good approximation, the correlation coefficient of the signal envelopes is equal to the squared magnitude of the correlation coefficient of the complex signals (Clarke, 1968).Thus taking the expectation in (2) under the assumptions given in Section 2.1 and simplifying, the correlation coefficient of signal envelopes can be shown to be equal to

$$\rho_{|v|}(\xi,\Omega) = |R_\nu(\xi,\Omega)|^2 = \frac{1}{1+\Omega^2 S^2}\,\frac{1}{\beta^2}\left| \int_{\alpha_o-\beta/2}^{\alpha_o+\beta/2} \exp(jk\xi\cos\alpha)\,d\alpha \right|^2 \tag{3}$$

where S is the time-delay spread (Clarke, 1968) and $k = \omega_c/c$. In obtaining (3) the probability distribution of α is taken to be uniform over the range $[\alpha_o - \beta/2,\ \alpha_o + \beta/2,]$. Note that the term $1/(1+\Omega^2 S^2)$ is the frequency correlation coefficient whereas the rest corresponds to the spatial correlation coefficient. Frequency-space correlation coefficient $\rho_{|v|}(\xi,\Omega)$ can be obtained by numerically evaluating the above integral for whatever the mean and width of the angular sector is.

2.3 Simplified expressions for $\alpha_o = 0$ and $\alpha_o = \pi/2$.

The integral in (3) can be solved analytically for specific values of α_o, and in particular 0 and $\pi/2$ when β is very small. Consider the case when α_o=0, that is when the mean angle of arrival coincides with the axis of the diversity array. To obtained the spatial correlation coefficient of signal envelopes, $\rho_{|v|}(\xi)$, one can use Taylor series expansion for $\cos\alpha$ around 0 up to the second order, that is $\cos\alpha \approx 1 - \alpha^2/2$. This yields

$$\rho_{|v|}(\xi) \approx \frac{|\mathfrak{F}(u)|^2}{u^2} \tag{4}$$

where $\mathfrak{F}(u)$ is the Fresnel integral in complex form (Clarke and Brown, 1980) and $u = \beta(\xi/2\lambda)^{1/2}$. In the case of $\alpha_o = \pi/2$, that is when the mean direction of arrival is perpendicular to the axis of the antennas (broadside case), one can use the approximation $\cos\alpha = \sin(\pi/2 - \alpha) \approx \pi/2 - \alpha$.Thus we obtain

$$\rho_{|v|}(\xi) \approx \mathrm{sinc}^2(\beta\xi/\lambda). \tag{5}$$

2.4 Comparison of theoretical results with experimental data

The spatial correlations, $\rho_{|v|}(\xi)$, can be calculated numerically from (3) for arbitrary α_o. Figure 2 shows $\rho_{|v|}(\xi)$ against normalized antenna separation, for various α_o. Here, $\beta = 1^\circ$. The curves indicate that, to achieve a correlation of about 0.7, a separation of at least 18λ is necessary when the antennas are placed perpendicular to the mean direction of arrival. However, for $\alpha_o = 0^\circ$, the correlation does not decrease significantly up to separations larger than 100λ. That the in-line case correlations are much higher than those obtained in the broadside case is reasonable, since the two received fading signals will tend to be the same in the in-line case. When $\alpha_0 > 10^\circ$ the correlation falls off fast with increasing α_0 and for $\alpha_0 > 50^\circ$ α_0 has little effect on the correlation.

The experimental data due to Lee (1971, 1973) are used to verify these results. The theoretical curves calculated from (3) for various β as well as some experimental data taken when no local scatterers are present are illustrated in Fig.3 and 4 for $\alpha_0 = 30^\circ$ and $\alpha_0 = 60^\circ$, respectively. It can be seen that the agreement between the theoretical curves and various data sets is good. Also, the β values for which the theoretical curves fit experimental data are consistent for the $\alpha_0 = 30^\circ$ and 60° cases. Value of β in the range $1^\circ \sim 1.8^\circ$ give the best fit to Lee's data. Also note that the correlation increases as β decreases; that means, directivity features of the incoming signal affect the correlation.

Theoretical correlations suggested by Lee's model (Lee, 1973) are considerably lower than most of the measured data except when β is very small (0.4°). There is also inconsistency between values of β for different incoming directions. In this sense, the model proposed in this paper seems to give a better agreement with the measurements compared to the model proposed by Lee (1973).

3 A More General Model

In the proposed model we have assumed that the angle of arrival and propagation time delay of a component wave are independent. This is a valid assumption for the case when the scattering area surrounding the mobile is far away from the base and no local scatterers are present around it. In this case, a particular angle of arrival can be associated with several possible time delays due to different scatterers within the area. Thus a correlation between the incoming direction and time delay would not be likely. However, in the case of local scatterers around the base longer time delays can be associated with certain directions depending on the shape of this scattering area. Therefore, in this more general case, (2) would be given in the form of

$$R_\nu(\xi,\Omega) = \exp\,[-j\Omega T_0] \int_\alpha \int_\tau g(\alpha)p(\alpha,\tau)\ \exp[-j\Omega\tau]\ \exp[jk\xi\cos\alpha]\ d\alpha\ d\tau. \qquad (6)$$

That means, there will be coupling between the frequency and space correlations as they are determined by the joint pdf $p(\alpha,\tau)$. Note that in (6) the antenna gain function $g(\alpha)$ is included to represent cases where nonisotropic antennas are used at the base.

On the other hand, the model can also be generalized to take the effect of possible local scatterers by a proper choice of $p(\alpha)$. In such cases $p(\alpha)$ can be considered as a superposition of two pdf's, one of them being non-zero over a wide range (possibly $[0,2\pi)$) to represent the local scattering effects and the other being uniform to represent main incoming signal.

4 Conclusions

A statistical model for base-station diversity reception has been developed. Using the model, analytical relations for the envelope correlation have been found. Comparisons between the theoretical results and some available experimental data provide adequate justification for the model. The model proposed in this paper has more flexibility and applicability than the one due to Lee (1973), which is the only other available model. It is also more consistent with the measured data.

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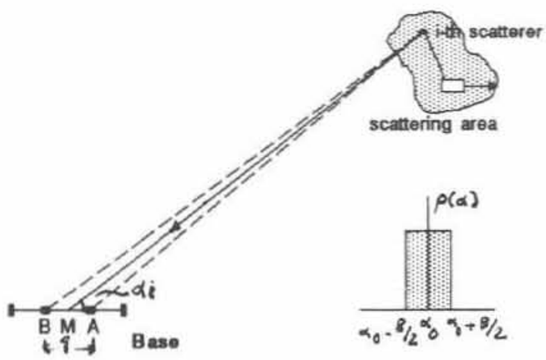

Fig. 1 Model geometry

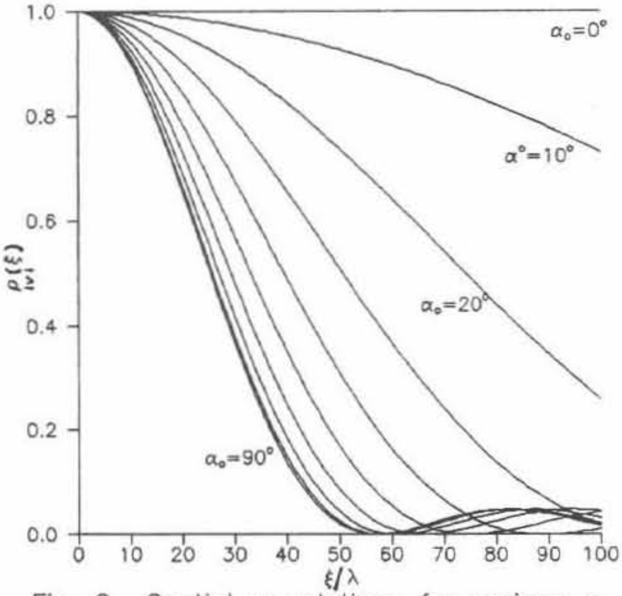

Fig. 2 Spatial correlations for various α_o

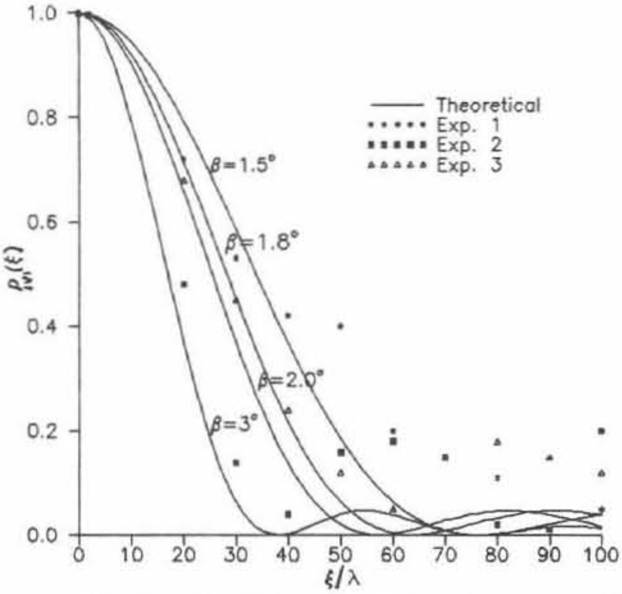

Fig. 3 Spatial correlations for α_o=30°

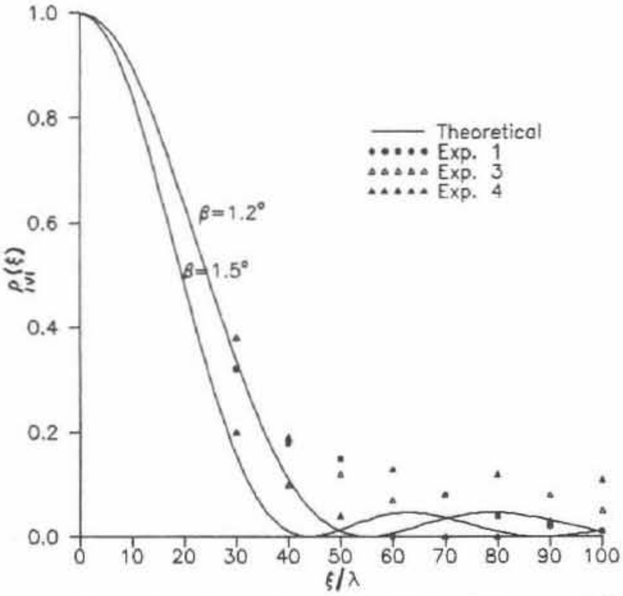

Fig. 4 Spatial correlations for α_o=60°