NEW EXPLANATION OF THE LEAKY MODE PHENOMENA IN A COPLANAR STRIP LINE

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Abstract

Catastrophe and bifurcation theories are applied for the rigorous analysis of the leaky mode phenomena in a coplanar strip line. It is shown that a Morse critical point (MCP) and degenerate critical points CP's (fold or turning points) exist in the vicinity of a leaky mode region. A Taylor series expansion in the neighborhood of the MCP and CP's reproduce improper real and complex (leaky) solutions with good accuracy. The type of MCP defines the structural stability of a system. Introduction of additional small perturbations change the coordinates of MCP's, but the type of nondegenerate critical points and their qualitative local structures remain the same. Bifurcation (leakage) appears when a complex conjugate pair of fold or turning points is degenerated into pure real values. Evolution of MCP's and CP's makes it possible to predict regions of structural stability and regions of mode leakage.

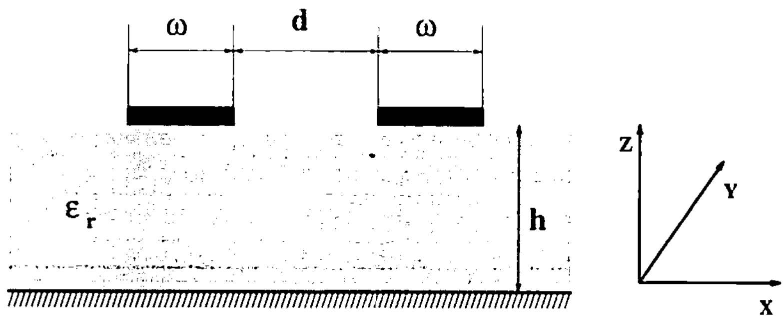
Introduction

Simultaneous propagation of both the bound and leaky dominant modes in slot and coplanar strip lines has been recently studied [1], [2], and a new improper real (nonphysical) solution has been discovered. It is shown that small changes of geometrical parameters can dramatically change improper real solutions and generate a new improper complex (leaky) solution. It is contended that the discovered effects can exist in most printed-circuit transmission lines.

A new explanation of existence and appearance of improper real and complex solutions, and bifurcation (leaky) regions is proposed in the present paper. The principles of catastrophe theory [3] and bifurcation theory [4] are applied for the rigorous analysis of the above mentioned phenomena.

Theory

The geometry of a coplanar strip line on an isotropic substrate is shown in Fig. 1. The spectrum of all possible physical and nonphysical, real and complex solutions is determined using a rigorous electric field integral equation formulation, similar to [5]. A set of homogeneous integral equations is obtained by enforcing an electric field boundary condition along all conducting strips. The layered background environment is accounted for by defining an appropriate dyadic Green's function, which serves as the kernel of the resulting integral expressions. The method is theoretically exact to within the limits of perfectly conducting, infinitely thin strips, as all wave phenomena are included in the formulation. Numerical solutions for various types of modes (proper real, improper real, etc.) can be obtained by correctly choosing the spatial Fourier transform inversion path, as discussed in [5], [6]. Dispersion characteristics are obtained by converting the set of integral equations into a system



h=1 cm; d=0.25 cm; ε_r=2.25

Fig. 1. Geometry of a coplanar strip line

of linear equations:

$$A(k,f)X = 0\ , \tag{1}$$

where X is the vector of unknown coefficients of the current density expansion, $A(k,f)$ is a matrix operator, k is the normalized propagation constant k_y/k_0, and f is the frequency. The operator equation (1) is reduced to finding a discrete set of roots of an operator-function $H(k,f) = det[A(k,f)]$. According to [3], the set of Morse critical points Ω_{MCP} can be determined for a smooth function $H(k,f)$ if

$$\Omega_{MCP}\ :\quad \nabla_{k,f}H(k,f) = 0, \tag{2}$$

and if the Hessian matrix is non-singular, and the Hessian determinant is not equal to zero:

$$[H''_{kk}H''_{ff} - (H''_{kf})^2]|_{(k_m,f_m)} \neq 0 \tag{3}$$

The Hessian determinant is calculated at the MCP (k_m, f_m). The Morse Lemma is satisfied for this situation, and the determinant $H(k,f)$ can be written locally in a quadratic canonical form. This guarantees a smooth change of spectral parameters and leads to structural stability of a system in a local region. If the MCP is defined in the mode coupling region, the local structure can be reproduced using a Taylor series expansion:

$$H(k,f) = H(k_m,f_m) + H'_{k|(k_m,f_m)}(k-k_m) + H'_{f|(k_m,f_m)}(f-f_m) +$$

$$\frac{1}{2}[H''_{kk|(k_m,f_m)}(k-k_m)^2 + 2H''_{kf|(k_m,f_m)}(k-k_m)(f-f_m) + H''_{ff|(k_m,f_m)}(f-f_m)^2] + O^3 = 0, \tag{4}$$

where O^3 are cubically small terms. According to the condition (2), the partial derivatives $H'_{k|(k_m,f_m)}$ and $H'_{f|(k_m,f_m)}$ are equal to zero , and the local structure is completely defined by coefficients of the Hessian matrix and the coupling factor $H(k_m, f_m)$. The intensity of the mode coupling is determined by the value of the characteristic determinant $H(k,f)$ at (k_m, f_m).

The set of critical points (CP's) Ω_{CP} : $\{(k_c, f_c)\}$, called fold or turning points, obeys the following set of equations [4] :

$$\Omega_{CP}\ :\quad H(k,f)|_{(k_c,f_c)} = H'_k(k,f)|_{(k_c,f_c)} = 0; \quad H''_{kk}(k,f)H'_f(k,f)|_{(k_c,f_c)} \neq 0 \tag{5}$$

The fold catastrophe is defined under conditions (5) with the normal form $k^2 \pm f$. A Taylor series expansion (4) reproduces a local structure of the function $H(k,f)$ in the vicinity of a fold point

(k_c, f_c). The fold catastrophe is globally determined by $H_f'(k, f)$ and coefficients of the Hessian matrix at (k_c, f_c).

Numerical results

The spectrum of all possible solutions presented in [1], including real improper and proper modes, and improper complex (leaky) modes, has been generated using the above mentioned rigorous electric field integral equation formulation. Fig. 2 demonstrates dispersion curves for the improper real and the new improper real solutions in a coplanar strip line. It was discovered that a real MCP exists in the interaction region with coordinates $(k_m, f_m) = (1.51102664, 7.9101394)$. The local structure is generated using a Taylor series expansion (4). A pair of complex conjugate CP's (fold or turning points) also exists in the local region shown in Fig.'s 2 and 3 with coordinates $(k_{c1,2}, f_{c1,2}) = (1.51071229 \pm \jmath 0.011917855, 7.89373303 \mp \jmath 0.246129795)$. Local structures in the vicinity of the MCP and CP's reproduce improper real solutions with good accuracy. Small changes of strip width dramatically change dispersion curves and generate a new nonphysical improper complex (leaky) solution, as depicted in Fig.'s 4 and 5, and discussed in [1]. Appearance of the bifurcation situation can be explained using the above mentioned definition (5). It was found that a pair of complex conjugate CP's is degenerated into two real values with coordinates $(k_{c1}, f_{c1}) = (1.53154225, 7.4473463)$ and $(k_{c2}, f_{c2}) = (1.486483738, 8.36879119)$. The location of the MCP is slightly changed to $(k_m, f_m) = (1.50792505, 7.85158451)$ in compare with significant changes of CP's. Fig.'s 6 and 7 show leakage constant dispersion curves, local structures generated by the MCP and CP's, and location of discussed critical points.

Conclusion

Determination of critical point (MCP's and CP's) enables the investigation of regions of structural stability and unstability. Evolution of critical points can predict appearance and disappearance of improper complex (leaky) modes. It should be emphasized that MCP's define the structural stability of a system. It is shown that introduction of the additional small perturbation as variation in the strip width does not change the type of MCP's and their qualitative local structures. A reason of existence of leaky regions is in the qualitative change of fold points. Degeneration of complex fold points leads to appearance of unstable bifurcation situation.

References

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Fig. 2. Dispersion curves of the improper real solutions, local structure, and the MCP in a coplanar strip line

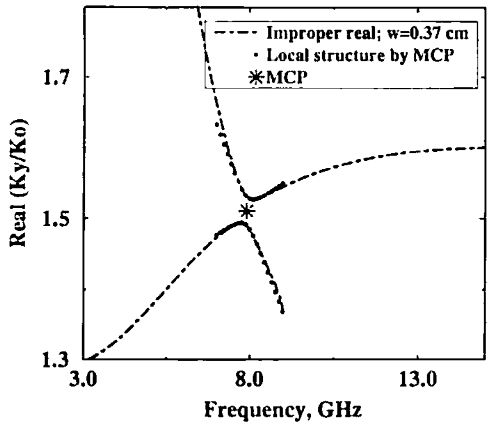

Fig. 3. Local structures in the vicinity of the MCP and CP's in a coplanar strip line with w=0.37 cm

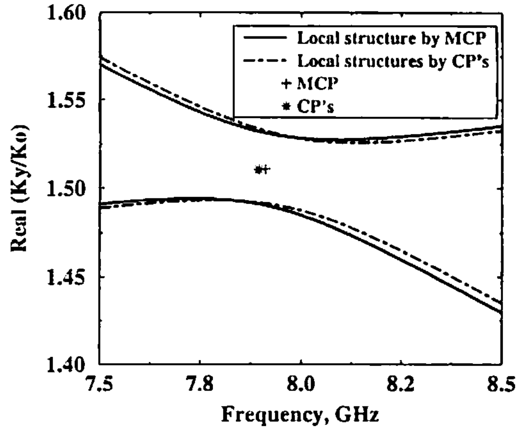

Fig. 4. Improper real and complex solutions, local structure, and the MCP in a coplanar strip line

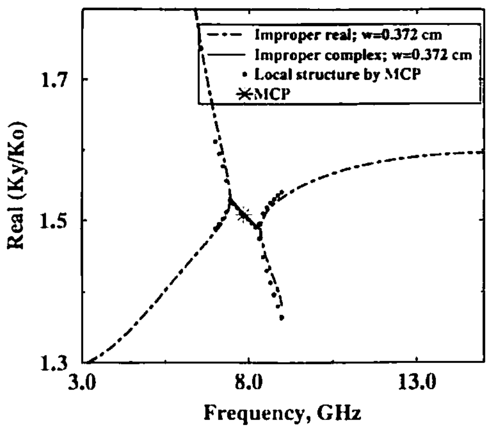

Fig. 5. Improper real and complex solutions, local structures, and CP's in a coplanar strip line

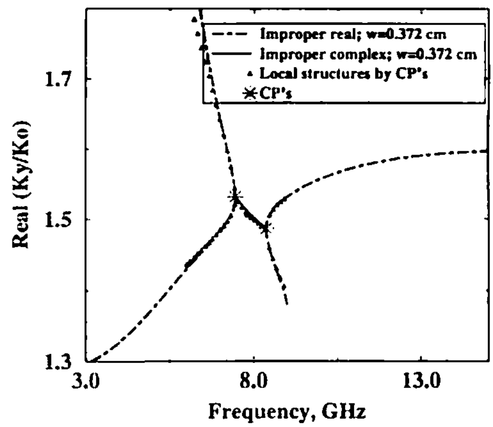

Fig. 6. Leakage constant Imag(Ky/Ko), local structure, and the MCP in a coplanar strip line with w=0.372 cm

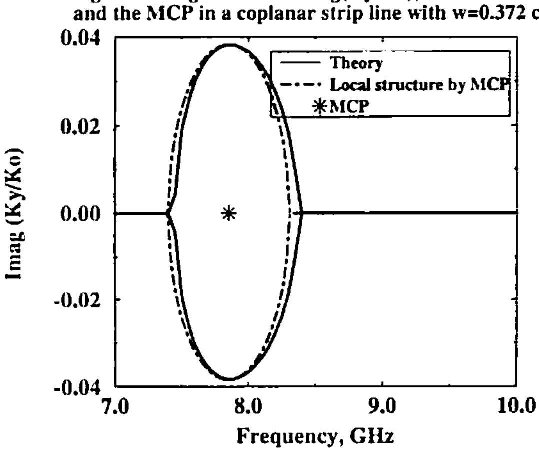

Fig. 7. Leakage constant Imag(Ky/Ko), local structures, and CP's in a coplanar strip line with w=0.372 cm