B-5-3 THE MAXIMUM APERTURE EFFICIENCY OF CASSEGRAIN ANTENNAS WITH A SPECIFIED SIDELOBE LEVEL

Fumio Watanabe and Naohisa Goto
Tokyo Institute of Technology
Tokyo , Japan

1. Introduction

In Cassegrain antennas, the shape of reflector cross section is often determined such that electromagnetic waves in the aperture plane become desired amplitude and phase distribution [1]. Even if a Taylor distribution [2] is applied to Cassegrain antenna in order to obtain a narrow beamwidth and low sidelobe radiation pattern, the pattern has relatively high sidelobes due to blocking of electromagnetic waves by a subreflector. A synthesis of amplitude distribution for a circular aperture which has the subreflector located at the center of a main reflector has been investigated [3]. In the reference [3], however, amplitude distribution was designed to terminate abruptly in a "pedestal" at an edge of the main reflector. This paper deals with a synthesis of amplitude distribution which vanishes linearly at its edge, considering a blocking by the subreflector. It is shown that we can obtain aperture distributions for the radiation pattern which has the maximum aperture efficiency under strict constraints on the specified sidelobe level.

2. Design approach

Let the geometry of a circular aperture (radius a) blocked by a concentric subreflector (radius αa) be as illustrated in Fig. 1. We can write a radiation pattern as

$$E(x) = \int_{\alpha}^{1} g(p)\, J_0(p x)\, p\, dp \qquad (1)$$

$$x = k a \sin\theta \ , \quad p = \rho/a$$

where k is the free space wave number, θ is an angle from the reflector axis and g(p) is a scalar amplitude function assumed to be rotationally symmetric. In this paper we treat amplitude distributions designed to vanish linearly with distance from the edge of a main reflector, then we express g(p) by superposition of first $\bar{n}$ terms of Fourier-Bessel expansion [4] as

$$g(p) = \sum_{n=1}^{\bar{n}} D_n\, J_0(b_n p) \qquad (2)$$

where $b_1, b_2, \ldots$ denote the positive zeros of $J_0(x)$ arranged in ascending order of magnitude, and $J_n(x)$ is a Bessel function of order n. The aperture efficiency η is defined as [5]

$$\eta = \frac{2\left\{\int_{\alpha}^{1} g(p)\, p\, dp\right\}^2}{(1-\alpha^2)\int_{\alpha}^{1} g^2(p)\, p\, dp} = \frac{2\,E^2(0)}{(1-\alpha^2)\sum_{n=1}^{\bar{n}}\sum_{m=1}^{\bar{n}} r_{n,m}\, D_n\, D_m} \qquad (3)$$

$$r_{n,m} = \frac{-\alpha}{(b_n^2 - b_m^2)}\left\{ b_n J_1(\alpha b_n) J_0(\alpha b_m) - b_m J_0(\alpha b_n) J_1(\alpha b_m) \right\} \qquad (n \neq m)$$

$$r_{n,n} = \frac{1}{2} J_1^2(b_n) - \frac{\alpha^2}{2} \{ J_0^2(\alpha b_n) + J_1^2(\alpha b_n) \}$$

If the amplitude distribution is uniform, the aperture efficiency becomes 100 %, but the sidelobe level of the radiation pattern is -17.6 dB for $\alpha = 0$ % and -15.2 dB for $\alpha = 20$ %. We synthesize a radiation pattern with high aperture efficiency and low sidelobes. The optimization problem of interest here is that of choosing D_n so as to

$$\text{maximize} \quad \eta \tag{4-1}$$

$$\text{subject to} \quad E(0) = 1 \tag{4-2}$$

$$|E(x)| \leq R \quad \text{for sidelobe region,} \tag{4-3}$$

where R is the specified sidelobe level. The denominator of the objective function (4-1) is a quadratic form of the amplitude coefficient D_n which is represented by $\bar{n} \times \bar{n}$ symmetric matrix. The matrix is positive definite since it corresponds to a square integral of amplitude function. If we consider finite points of x in the sidelobe region in (4-3), equality and inequality constraints can be written in the form of linear combination of D_n. Therefore the opimum solution for (4) can be obtained by using a quadratic programming technique. We use the algorism proposed by Wolf [6].

3. Numerical results

Fig. 2 shows amplitude distributions for radiation patterns (Fig. 3) which have the maximum aperture efficiency and -25 dB sidelobe level. Solid lines in these figures denote the case of $\alpha = 10$ % and broken lines denote the case of $\alpha = 0$ %, i.e. without blocking. The aperture distributions are cutted sharply at the edge of the main reflector. They have higher ripples than that of the optimum solution of a "pedestal" type [3], since a uniform distribution has the 100 % aperture efficiency. It is seen from Fig. 3 that a few sidelobes of radiation patterns near the main beam are suppressed to the specified level and other sidelobes decrease gradually in magnitude.

Fig. 4 shows the relationship between the maximum aperture efficiency and number of terms $\bar{n}$ for various size of subreflector. It is seen from this figure that the aperture efficiency decreases as the subreflector is made larger or lower sidelobe level is specified. Though higher aperture efficiency can be obtained with large number of terms, increasing a number of terms forms a very complicated amplitude distribution in return. The case of $R = -25$ dB, $\alpha = 15$ % and $\bar{n} = 3$ is not plotted in this figure since the limit of sidelobe suppression [3] is greater than -25 dB, so no solution satisfies the constraints of (4-2) and (4-3). Fig. 5 shows the relationship between the maximum aperture efficiency and specified sidelobe level for the case of $\bar{n} = 5$. The maximum aperture efficiency is equal to about 93 % for the specified sidelobe level higher than a certain level which is, for example, -17.4 dB for $\alpha = 0$ % and -14.9 dB for $\alpha = 20$ %. It has become clear that we can determine the radiation pattern with low sidelobe level and high aperture efficiency for Cassegrain antennas with relatively large subreflector.

Now we consider another example, which is constrained on an amplitude at the edge of the subreflector. Let a new constraint $g(\alpha) = g_\alpha$ add to the problem (4). Since the added constraint is represented in the form of linear combination of D_n, we can solve this new problem by quadratic programming again. The solid line in Fig. 6 is the amplitude distribution of problem (4) and broken line is that of new problem. Though the latter distribution has higher ripples and lower aperture efficiency than that of the former, such a distribution constrained on an amplitude at the edge of the subreflector will be practically available.

4. Conclusion

The quadratic programming technique is used to control the sidelobe level of the radiation pattern and to maximize aperture efficiency of a circular aperture antenna which has a subreflector located at the center of a main reflector. Amplitude distributions are designed to vanish linearly at an edge of the main reflector. In resulting radiation pattern a few sidelobes near the main beam are suppressed to the specified level.

References

[1] K. A. Green : "Modified Cassegrain antenna for arbitrary aperture illumination", IEEE Trans. AP-11, p. 589 (Sept. 1963)

[2] T. T. Taylor : "Design of circular aperture for narrow beamwidth and low sidelobes", IRE Trans. AP-8, p. 17 (Jan. 1960)

[3] N. Goto and F. Watanabe : "The optimum aperture efficiency of Cassegrain antennas with a specified sidelobe level (in Japanease)", Trans. IECE Japan (to be published)

[4] G. N. Watson : "Theory of Bessel functions", ch. 18, Cambridge University Press (1966)

[5] R. C. Hansen : "Microwave scanning antennas", ch. 1, Academic press (1964)

[6] P. Wolf : "The simplex method for quadratic programming", Econometrica, vol. 27, p. 382 (July 1959)

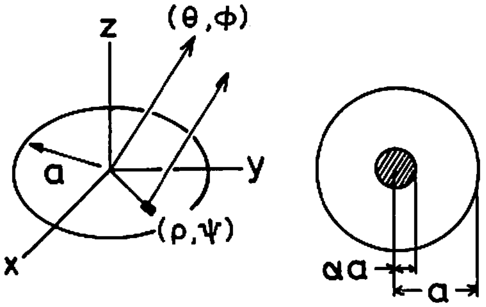

Fig. 1 A coordinate system for a circular aperture.

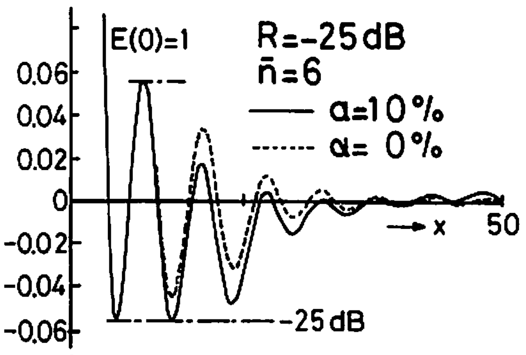

Fig. 3 The optimum radiation pattern specified by the -25 dB sidelobe level.

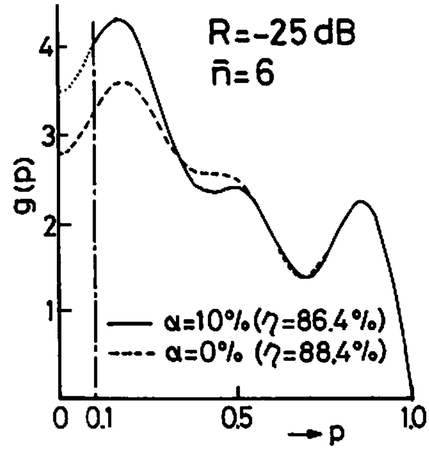

Fig. 2 The optimum aperture distribution for a Cassegrain antenna.

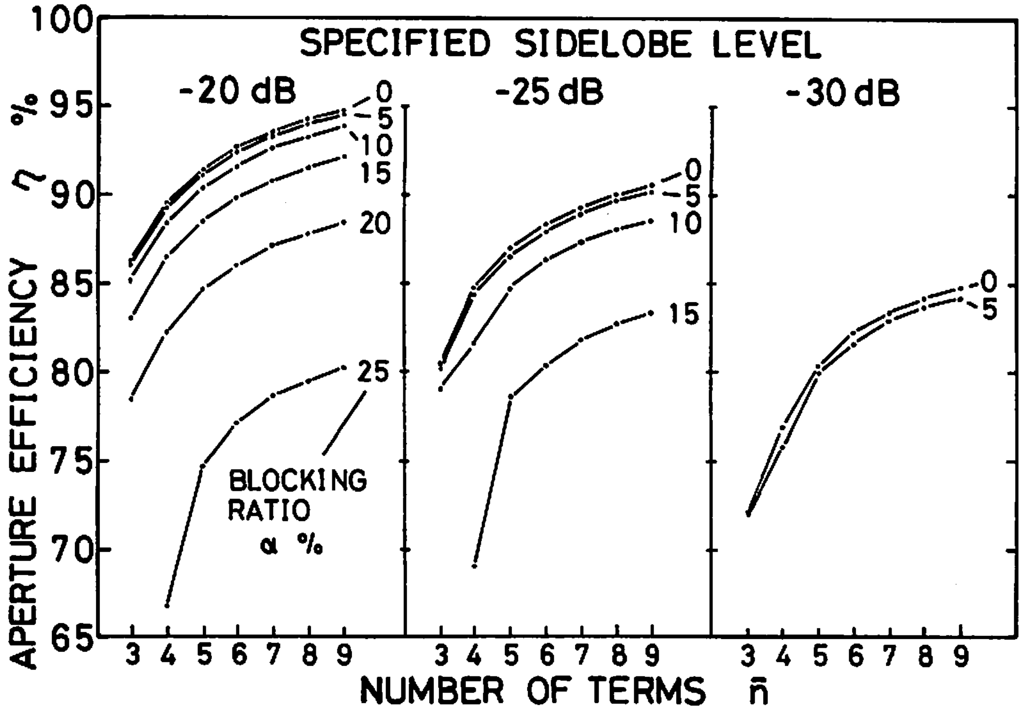

Fig. 4 The maximum aperture efficiency with number of terms.

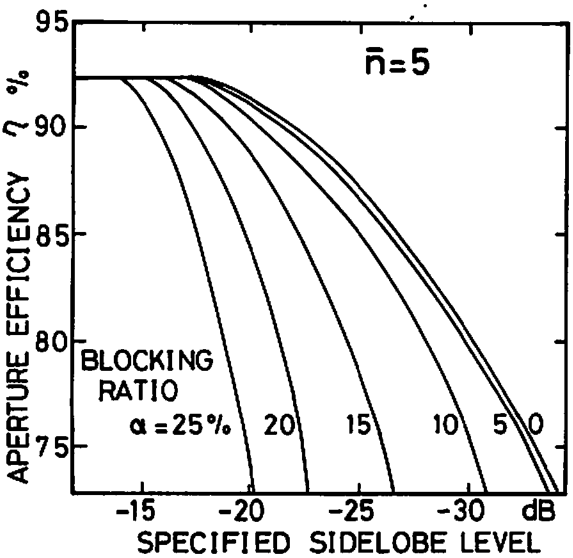

Fig. 5 The maximum aperture efficiency as a function of the specified side-lobe level and of the blocking ratio.

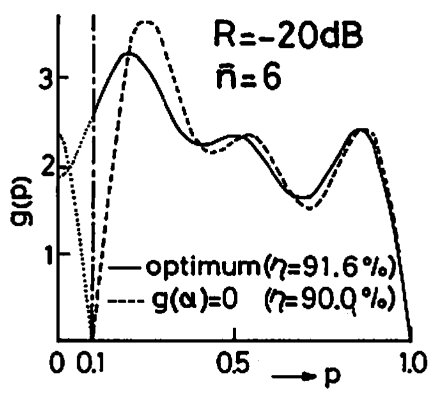

Fig. 6 The aperture distribution under constraint on the subreflector edge.