B-5-2

AN ANALYTICAL APPROACH TO THE PROBLEM OF BORESIGHT CROSS POLARIZATION IN REFLECTOR ANTENNAS WITH SURFACE ERRORS

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Introduction

It has been established that boresight cross polarization may result in axisymmetrical reflector antennas if the surface of the reflector is not perfect(1-4). Ghobrial(1) showed that the worst case takes place if surface errors are periodic with two periods about the axis of the reflector. Thomas and Rayner(4) considered periodic errors in both radial direction and about the axis of symmetry. They concluded that boresight cross polarization is negligible if the number of cycles is large.

In practice surface errors are not periodic but rather random. A statistical study is thus needed. An empirical formula based on measurements was suggested by the present author(3), however, a theoretical study is still lacking. In this paper an attempt to tackle this problem theoretically will be made.

Theory

Fig 1 shows the model that will be adopted. In this model the aperture is divided into N sectors. It is assumed that surface errors are constant over any of the sectors while errors in different sectors are uncorrelated. It is futher assumed that surface errors are uniformly distributed with peak error δ_m, thus

$$p(\delta) = 1/2\delta_m$$

In the presence of surface errors the cross polar voltage may be written as(1)

$$V_{cross} = (\pi D^2/4) \int_0^1 \int_0^{2\pi} \sin\phi\cos\phi \; f(r) \; e^{j\beta} r\,dr\,d\phi \qquad (1)$$

where: β is a random phase error function of position,
f(r) is a funtion that depends on the polarization efficiency of the antenna and the illumination taper.

Using the model of Fig 1 equation (1) may be written as

$$V_{cross} = A \sum_{1}^{N} a_n \exp(j\beta_n) \qquad (2)$$

where: $A = (\pi D^2/4) \int_0^1 f(r) r dr, \quad a_n = \int_{\phi_n}^{\phi_{n+1}} \sin\phi \cos\phi d\phi$

It can be shown that the real and imaginary parts of V_{cross} are normally distributed(5) with zero mean and variances

$$\sigma_x^2 = (A^2 \beta_m^4/45) \sum a_n^2 \quad , \quad \sigma_y^2 = (A^2 \beta_m^2/3) \sum a_n^2$$

respectively; β_m being the peak phase error = $(4\pi \delta_m/\lambda)$.

The probability density of the magnitude of the cross polar voltage is then obtained as

$$p(V) = \frac{V}{c} \exp(aV^2) \, I_o(bV^2) \tag{3}$$

where: $V = V_{cross}$

I_o is the zero order modified Bessel function,

a, b, and c are constants and are given by:

$$a = (\sigma_x^{-2} + \sigma_y^{-2})/4, \quad b = (\sigma_x^{-2} - \sigma_y^{-2})/4, \quad c = \sigma_x \sigma_y$$

The mean and mean square are obtained as

$$\overline{V} = \sqrt{2/\pi} \; \sigma_y \, E\left(\sqrt{1 - \sigma_x^2 \sigma_y^{-2}}\right); \quad \overline{V^2} = \sigma_x^2 + \sigma_y^2$$

where E() is the complete elliptic integral.

Computations

Fig 2 shows the probability density distribution of the normalized cross polar voltage for antennas with an rms surface error of 0.029 λ and polarization efficiencies of 99% and 95%. The number of uncorrelated sectors, N, was taken as 8. The general shape of this distribution is similar to the well known Rayleigh distribution. In fact it can be easily shown that the distribution of equation (3) reduces to the Rayleigh distribution if $\sigma_x = \sigma_y$.

Fig 3 shows a plot of the mean normalized cross polar voltage versus the rms surface error for antennas with different polarization efficiencies and for N = 8. The following may be concluded: (i) For small errors the mean cross polar voltage is almost directly proportional to the rms surface error.

(ii) Antennas with low polarization efficiency are more susceptible to boresight cross polarization.

Conclusions

An expression for the probability density of cross polar voltage in terms of the reflector surface errors has been derived. The mean cross polar voltage was found to increase with decreasing polarization efficiency and is almost directly proportional to the rms surface error.

References

1- S I Ghobrial "Cross polarization effect of paraboloidal reflector antenna surface errors" presented at IEE Conf. Satellite Communication Systems Technology, London, England, April 7-10, 1975; IEE Cof. Publication 126, pp 246-252, 1975.

2- S I Ghobrial "Boresight cross polarization in reflector antennas with surface errors", presented at the 1977 IEEE AP-S Symposium at Stanford University, California, 20-22 June 1977.

3- S I Ghobrial "Loss in gain and boresight cross polarization in reflector antennas with surface errors" Electronics Letters Vol.13;No20 ;pp623-4 ; September 1977.

4- B M Thomas and P T Rayner "Effect of surface distortions on the crosspolarisation performance of paraboloidal reflectors" Electronics Letters, Vol.13, No 16, pp 478-480, August 1977.

5- Petr Beckmann "Probability in communication engineering", Harcourt, Brace & World, Inc. 1967.

Figure Captions

Fig 1: Aperture divided into sectors.

Fig 2: Cross-polar voltage probability density.

Fig 3: Cross-polar voltage as a function of the rms surface error for different polarization efficiencies.

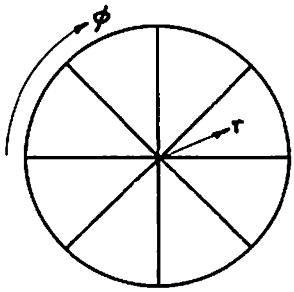

FIG.1

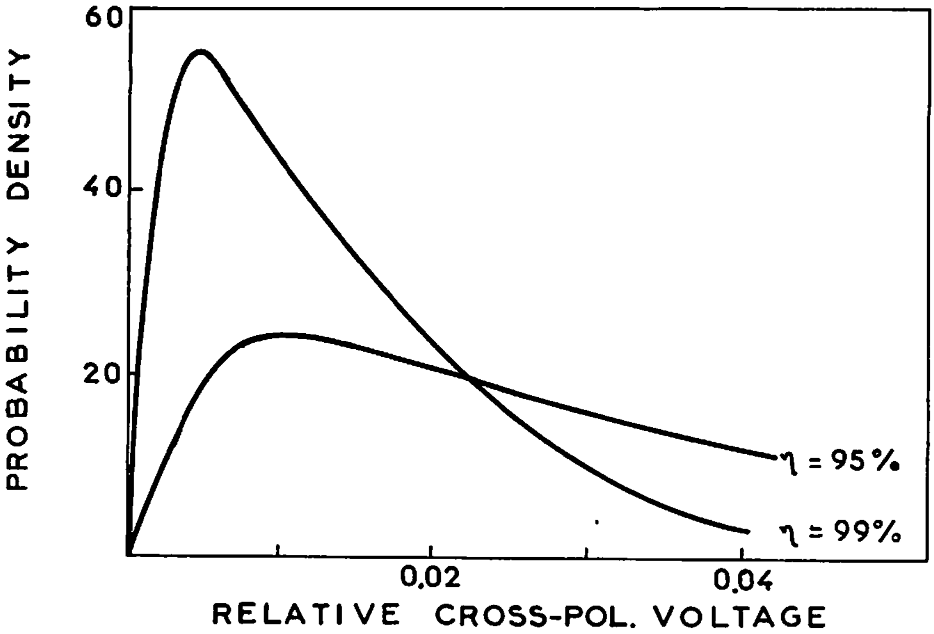
60
40
20
PROBABILITY DENSITY
η = 95%
η = 99%
0.02
0.04
RELATIVE CROSS-POL. VOLTAGE

FIG 2

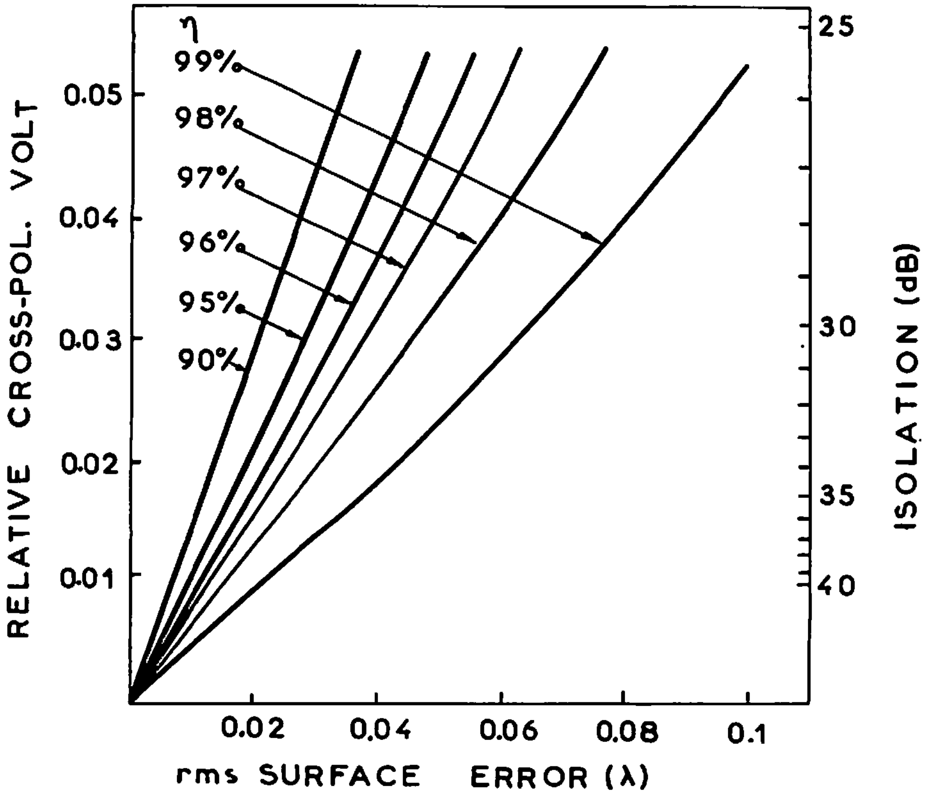
η
99%
98%
97%
96%
95%
90%
0.05
0.04
0.03
0.02
0.01
RELATIVE CROSS-POL. VOLT
25
30
35
40
ISOLATION (dB)
0.02
0.04
0.06
0.08
0.1
rms SURFACE ERROR (λ)

FIG 3