2B2-1

A Remark on the Definition of Correlation Coefficient by Propagation Modeling

[#] Daisuke Uchida [1], Takero Asai [1], and Hiroyuki Arai [1]
[1] Graduate School of Engineering, Yokohama National University
79-5, Tokiwadai, Hodogaya-ku, Yokohama-shi, Kanagawa, 240-8501, Japan
uchida-daisuke-vd@ynu.jp

1. Introduction

Spatial correlation is an important index to evaluate performance of multi-antenna systems. Low correlation leads to good diversity performance [1] and large channel capacity [2]. The equations of spatial correlation are derived from Bessel function [3], radiation patterns [4]- [6], S-parameters [6], and the channel matrix [7]. Although the values of the correlation coefficient are different according to the derivation methods, previous researches have not focused on the issue. This paper clarifies applicable conditions for the equation of each correlation coefficient. Then, we reveal that under Rayleigh fading environments, the spatial correlation is properly evaluated by the equation based on three-dimensional radiation patterns, however, under environments with strong direct waves, the equation based on the channel matrix should be used for the evaluation.

2. Equations to Calculate Spatial Correlation Coefficient

Spatial correlation is divided into two types, such as complex correlation coefficient ρ and envelop correlation coefficient ρ_e [8]. They represent correlation for amplitude and power, respectively and satisfy $\rho_e \approx \rho^2$. This paper focuses on complex correlation coefficient in two-element arrays.

In Rayleigh fading channel, spatial correlation between two positions are represented using Bessel function, $\rho_{bessel} = |J_0(kd)|$ [3]. Here, k is the vacuum wave number, J_0 is the 0th order Bessel function, and d is the spacing of the two-element array. Then, when incoming wave with N plane waves is concentrated in the horizontal plane under Rayleigh fading environment, spatial correlation between two receive antennas are given as follows [4]:

$$\rho_{2D} = \frac{\left|\int_0^{2\pi} A_1^*(\phi)A_2(\phi)d\phi\right|}{\sqrt{\int_0^{2\pi} A_1^*(\phi)A_1(\phi)d\phi}\sqrt{\int_0^{2\pi} A_2^*(\phi)A_2(\phi)d\phi}} \tag{1}$$

where $A_1(\phi)$ and $A_2(\phi)$ denote the complex electric field radiation patterns in the horizontal plane for antenna elements, #1 and #2, respectively, and $\{\cdot\}^*$ represents the complex conjugate. As a more general equation, ρ is given by three dimensional radiation patterns, $A_1(\theta,\phi)$ and $A_2(\theta,\phi)$, as follows [6]:

$$\rho_{3D} = \frac{\left|\int_0^{2\pi}\int_0^{\pi} A_1(\theta,\phi)A_2^*(\theta,\phi)\sin\theta d\theta d\phi\right|}{\sqrt{\int_0^{2\pi}\int_0^{\pi} A_1(\theta,\phi)A_1^*(\theta,\phi)\sin\theta d\theta d\phi}\sqrt{\int_0^{2\pi}\int_0^{\pi} A_2(\theta,\phi)A_2^*(\theta,\phi)\sin\theta d\theta d\phi}} \tag{2}$$

In addition, Taga proposed the equation considering the distribution of incoming wave and cross polarization power ratio (XPR) as follows [5]:

$$\rho_{3Dex} = \frac{\left|\int_0^{2\pi}\int_0^{\pi}\left(XPR\cdot E_{\theta1}E_{\theta2}^*P_\theta + E_{\phi1}E_{\phi2}^*P_\phi\right)\sin\theta d\theta d\phi\right|}{\sqrt{\int_0^{2\pi}\int_0^{\pi}\left(XPR\cdot E_{\theta1}E_{\theta1}^*P_\theta + E_{\phi1}E_{\phi1}^*P_\phi\right)\sin\theta d\theta d\phi}\sqrt{\int_0^{2\pi}\int_0^{\pi}\left(XPR\cdot E_{\theta2}E_{\theta2}^*P_\theta + E_{\phi2}E_{\phi2}^*P_\phi\right)\sin\theta d\theta d\phi}} \tag{3}$$

where P_θ and P_ϕ are the angular density functions of incoming plane waves for the θ and ϕ components. $E_{\theta k}$ and $E_{\phi k}$ (k=1, 2) are the θ and ϕ components of the complex electric field radiation pattern of each antenna element.

In [6], Blanch et al. revealed that the equation of spatial correlation derived from radiation patterns could be transformed using S-parameters as follows:

$$\rho_{spara} = \sqrt{\frac{\left|S_{11}^{*}S_{12} + S_{21}^{*}S_{22}\right|}{\left(1-(|S_{11}|^2 + |S_{21}|^2)\right)\left(1-(|S_{22}|^2 + |S_{12}|^2)\right)}} \tag{4}$$

Spatial correlation is also calculated from a channel matrix $\mathbf{H}$ as follows [7]:

$$\rho_{matrix} = \frac{\mathbf{R}_r(1,2)}{\sqrt{\mathbf{R}_r(1,1)}\sqrt{\mathbf{R}_r(2,2)}}, \quad \mathbf{R}_r = E[\mathbf{H}\mathbf{H}^H] \tag{5}$$

where $E[\cdot]$ and $\{\cdot\}^H$ denote the ensemble average and the complex conjugate transpose, respectively.

3. Simulation-Based Evaluation of Each Correlation Coefficient

Spatial correlation for each equation is evaluated using a two-element dipole array and inverted-F array, which were designed by CST Microwave Studio. Figure 1 shows the correlation coefficient for each derivation method. Here, ρ_{2D} and ρ_{3D} are calculated using the radiation patterns of co-polarization. Since only receiving positions are considered without antenna characteristics in ρ_{bessel}, the mutual coupling effect is not included in it. In ρ_{2D}, ρ_{3D}, and ρ_{spara}, mutual coupling effect is considered, and the correlation coefficients are less than ρ_{bessel} due to the radiation pattern distortion. Although these three types of correlations have similar characteristics in narrow inter-element spacing, the characteristics does not appear in wide inter-element spacing. Since three dimensional model is more accurate, ρ_{3D} and ρ_{spara} are appropriate to calculate correlation. Here, ρ_{bessel}, ρ_{2D}, ρ_{3D}, and ρ_{spara} can be applied when distribution of incoming wave is uniform. Then, the components of XPR is not included in those equations, and ρ_{3D} calculated from radiation patterns of co-polarization is equivalent to the characteristics for $XPR = \infty$.

The performances of (3) including the components of distribution of incoming wave and XPR are shown in Fig. 2. Here, Gaussian and uniform distributions are assumed in the vertical and horizontal planes, respectively (Fig. 2(a)). The equations of P_θ and P_ϕ are according to those in [5]. Figure 2(b) shows ρ_{3Dex} for the mean elevation angle m in the incoming wave with Gaussian distribution. Here, the standard deviation σ of Gaussian distribution is 20° [9]. In this figure, although ρ_{3Dex} becomes high with the increase of m, ρ_{spara} cannot respond to the change. Figure 2(c) shows ρ_{3Dex} under each XPR environment for m=20° and σ=20°. When cross polarization level of an antenna is large as inverted-F antennas, ρ_{3Dex} varies according to the XPR. Although ρ_{3Dex} is broadly-applicable, Rayleigh fading channel, which is generated when waves with random phase are received, is a condition for applying it.

Then, we examine free space propagation and Nakagami-Rice fading channel generated when waves with identical phase are received. We confirmed that ρ_{3Dex} was similar to ρ_{2D} when uniform and Gaussian distributions with small angular spread were assumed in the horizontal and vertical planes, respectively. In this scenario, the dimension to be considered is reduced, and therefore two-dimensional channel model is assumed for the investigation. This scenario is obtained by channel matrix $\mathbf{H} = \sqrt{K/(K+1)}\mathbf{H}_D + \sqrt{1/(K+1)}\mathbf{H}_S$. Here, K is the Rice factor, and $\mathbf{H}_D$ is the channel matrix for direct wave. Here, it is assumed that the inter-element spacing of Tx antennas is 1λ, and the array arrangement is parallel to that of Rx antennas, as shown in Fig. 3(a). Then, $\mathbf{H}_S$ represents a channel matrix for scattering wave given by Kronecker model. The effects of radiation patterns and mutual coupling are included in this modeling. Figure 3(b) shows the spatial correlation for each Rice factor K. In small K channel with many scattering waves, ρ_{matrix} is similar to ρ_{2D}. However, large K increases the spatial correlation because direct wave is dominant and incoming waves to two antenna elements are similar. Thus, spatial correlation adapting to a changing environment can be calculated using channel matrix.

Finally, applicable conditions for each correlation formula are summarized in Table 1. Here, we mention angular profile of incoming wave, inter-element spacing, XPR, and propagation channel as components having an impact on spatial correlation coefficient (Fig. 4). The applicable conditions are different according to correlation formulas. In Rayleigh fading environments, ρ_{3Dex} properly evaluates the spatial correlation, however, in the environments with strong direct waves, ρ_{matrix} should be used for the assessment.

4. Conclusions

This paper clarified the difference in the spatial correlations derived from Bessel function, radiation patterns, S-parameters, and the channel matrix. From the results, this paper revealed that under

Rayleigh fading channels, the spatial correlation was properly assessed by the equation based on the three-dimensional radiation patterns (Eq. 3). Then, we demonstrated that for evaluating the correlation under free space propagation and Nakagami-Rice fading channels in addition to Rayleigh fading channels, the equation derived from the channel matrix (Eq. 5) should be used.

References

[1] R. G. Vaughan et al., IEEE Trans. Veh. Technol., vol. VT-36, no. 4, pp. 149-172, Nov. 1987.
[2] S. L. Loyka, Electron. Lett., vol. 35, no. 17, pp. 1421-1422, Aug. 1999.
[3] R. H. Clark, Bell Labs System Technical Journal, vol. 47, pp. 957-1000, July-August 1968.
[4] T. Takeuchi, et al., IEICE Trans. Commun. (Japanese Edition), vol. J67-B, no. 5, pp. 570-571, May 1984.
[5] T. Taga, IEICE Trans. Commun. (Japanese Edition), vol. J73-B-II, no. 12, pp. 883-895, Dec. 1990.
[6] S. Blanch, et al., IEE Electronics Letters, vol. 39, pp. 705-707, May 2003.
[7] K. Sakaguchi, et al., IEICE Trans. Commun., vol. E88-B, no. 7, pp. 3010-3019, July 2005.
[8] Y. Karasawa, Radio Propagation Fundamentals for Digital Mobile Communications, Coronasha, Tokyo, 2003 (in Japanese).
[9] K. Ogawa et al., IEICE Trans. Commun. (Japanese Edition), vol. J91-B, no.9, pp. 948-959, Sep. 2008.

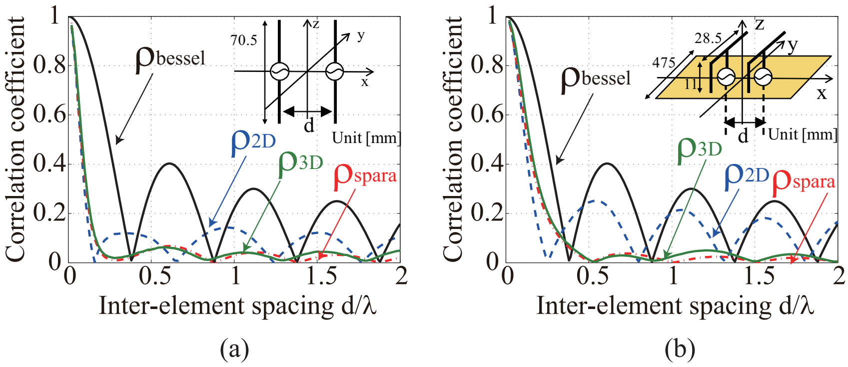


Figure 1: Spatial correlation coefficients calculated from Bessel function, (1), (2) and (4) for a two-element (a) dipole array and (b) inverted-F array, respectively.

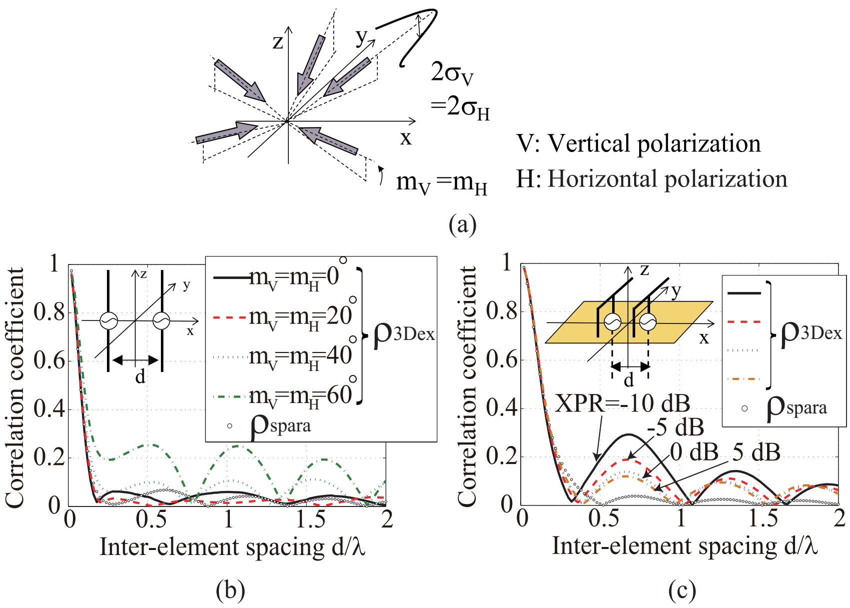
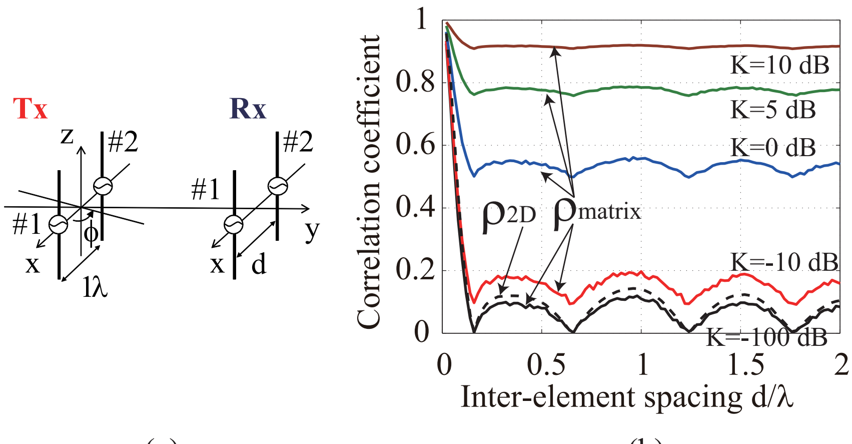


Figure 2: Spatial correlation coefficients for incoming waves with uniform distribution in the horizontal plane and Gaussian distribution in the vertical plane. (a) Conceptual diagram of the target scenario, and the correlations for incoming waves with various (b) elevation angles and (c) XPRs.



(a) (b)

Figure 3: Spatial correlation coefficients derived from channel matrix. (a) Target scenario and (b) the correlation for each Rice factor.

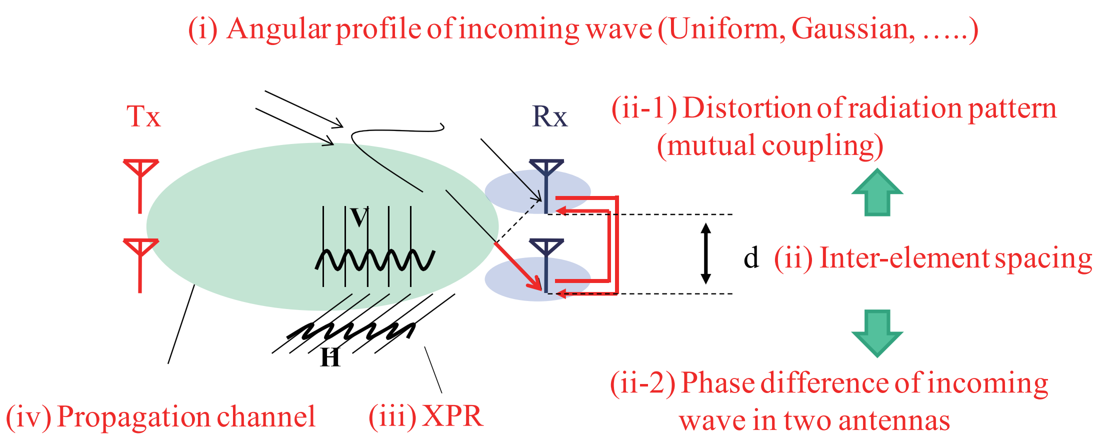


Figure 4: Components having an impact on spatial correlation coefficient.

Table 1: Applicable condition for each correlation coefficient. Here, ∘ and × mean that the components (ii-1), (ii-2), and (iii) are considered and not considered in the equations, respectively.

Correlation	Components having an impact on correlation in Fig. 4				
	(i)	(ii-1)	(ii-2)	(iii)	(iv)
ρ_{bessel}	Uniform (horizontal plane) Incoming waves are concentrated in the horizontal plane.	×	∘	×	Rayleigh fading
ρ_{2D}	Uniform (horizontal plane) Incoming waves are concentrated in the horizontal plane.	∘	∘	×	Rayleigh fading
ρ_{3D}	Uniform	∘	∘	×	Rayleigh fading
ρ_{spara}	Uniform	∘	∘	×	Rayleigh fading
ρ_{3Dex}	Applicable in any case	∘	∘	∘	Rayleigh fading
ρ_{matrix}	Applicable in any case	∘	∘	∘	Applicable in any case