MICROSTRIP LINES ON INHOMOGENEOUS SUBSTRATE

Jean-Fu Kiang
Department of Electrical Engineering
National Chung Hsing University
Taichung, Taiwan, ROC

1 Introduction

For microstrip lines deposited on an inhomogeneous substrate, the expressions of potentials (quasi-TEM analyses) or fields (full-wave approaches) in the substrate become more complicated than the expressions in a homogeneous substrate. Quasi-TEM approximation has been used to study the proximity effect of a substrate edge on the capacitance of a microstrip line [1], the capacitance of two microstrip lines separated by a notch in the middle of the substrate [2], and the capacitance of a stripline embedded in a layered medium with rectangular shape subdomains [3].

Method of lines has been developed to analyze the propagation properties of microstrip lines on a substrate of finite extent, on a substrate with notches [4], and coplanar transmission lines on a semiconductor substrate with continuous permittivity profiles [5]. Deriving the Green's function in a substrate with a piecewise constant function using conventional mode matching methods is very laborious.

In this work, an efficient mode matching technique is combined with an integral equation method to study the propagation properties of a microstrip line on an inhomogeneous substrate of which the permittivity profile can be a piecewise continuous function of the lateral coordinate. First, the eigenmodes in each layer are obtained by solving a symmetric eigenvalue matrix equation. Reflection matrices are defined across the interfaces between two contiguous layers. An integral equation is then formulated to express the tangential electric field in terms of the current on the strip surface. Galerkin's method is finally applied to solve the integral equation for the propagation constant.

2 Formulation

In Figure 1, we show the configuration of a microstrip line embedded in layer (l) of a stratified medium. The whole structure is uniform in the y direction. The dielectric constant in each layer is a piecewise continuous function of x, and is independent of y and z. Two perfect electric conductor walls are put at $x = 0$ and $x = a$ to simplify the analysis.

Assume that the waves propagate in the y direction with a propagation constant k_y, then D_x and B_x components in an inhomogeneous medium extending to infinity in the $\pm z$ direction can be expressed as a linear combination of a set of eigenmodes. These eigenmodes with their eigenvalues can be derived in terms of a set of basis functions.

Next, solve the fields generated by line currents $\bar{J}(\bar{r}) = \hat{y}I\delta(x - x_o)\delta(z - z_o)e^{ik_y y}$ and $\bar{J}(\bar{r}) = \hat{x}I\delta(x - x_o)\delta(z - z_o)e^{ik_y y}$ in layer (l) of the stratified medium to obtain the Green's function. The tangential fields generated by a microstrip line embedded in layer (l) can thus be expressed as an integral of the Green's function and the current density distribution.

Next, impose the boundary conditions that the tangential electric field vanishes on the strip surface to obtain an integral equation with the surface current as the unknown variable. To apply the Galerkin's method, first choose a set of basis functions to represent $J_x(x)$ and $J_y(x)$ on the strip surface. Substitute the current distribution in terms of these basis functions into the integral equation, then take the inner product of another set of weighting functions with the resulting equation to form a determinantal equation. The dispersion relation is obtained by solving the determinantal equation.

3 Numerical Results

In Figure 2, we show the phase constant of a microstrip line on a segmented substrate. The results with a homogeneous substrate [7] match reasonably well with our results in the high frequency range. The deviation in the low frequency range is because we model a laterally closed structure while the structure in [7] is laterally open.

In Figure 3, we show the phase constant of a microstrip line on a substrate with a parabolic permittivity profile. The maximum or minimum dielectric constant, ϵ_m, occurs at the middle of the substrate. The curve with $\epsilon_m = 10$ is that of a homogeneous substrate.

In Figure 4, we show the phase constant of a microstrip line on a substrate with a homogeneous dielectric constant and a parabolic conductivity profile. The maximum or minimum conductivity, σ_m, occurs at the middle of the substrate. The phase constant at the low frequency range increases as the conductivity increases. The slow wave phenomenon is obvious for $\sigma = 10\mho/\text{m}$. The associated attenuation constant in Figure 5 indicates that the loss is roughly proportional to the conductivity at least in the range of $0.1 < \sigma_m < 10\mho/\text{m}$, and decreases at low frequencies.

4 Conclusions

We have applied a mode matching technique and an integral equation method to study the propagation properties of a microstrip line embedded in a stratified medium where the permittivity and conductivity profiles in each layer can be continuous functions of the lateral coordinate. The phase constant and the attenuation constant with various inhomogeneous profiles have been obtained by this method. Slow wave phenomenon is also observed for structures with a lossy substrate.

References

[1] E. Yamashita, H. Ohashi, and K. Atsuki, "Characterization of microstrip lines near a substrate edge and design formulas of edge-compensated microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol.37, pp.890-896, May 1989.

[2] S. He, A. Z. Elsherbeni, and C. E. Smith, "Decoupling between two conductor microstrip transmission line," *IEEE Trans. Microwave Theory Tech.*, vol.41, pp.53-61, January 1993.

[3] A. Nyshadham and K. V. S. Rao, "Analysis of stripline filled with multiple dielectric regions," *IEEE Trans. Microwave Theory Tech.*, vol.39, pp.151-155, January 1991.

[4] M. Thorburn, A. Agoston, and V. K. Tripathi, "Computation of frequency-dependent propagation characteristics of microstriplike propagation structures with discontinuous layers," *IEEE Trans. Microwave Theory Tech.*, vol.38, pp.148-153, February 1990.

[5] K. Wu and R. Vahldieck. "Hybrid-mode analysis of homogeneously and inhomogeneously doped low-loss slow-wave coplanar transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol.39, pp.1348-1360, August 1991.

[6] J. B. Knorr and A. Tufekcioglu, "Spectral-domain calculation of microstrip characteristic impedance," *IEEE Trans. Microwave Theory Tech.*, vol.23, pp.725-728, September 1975.

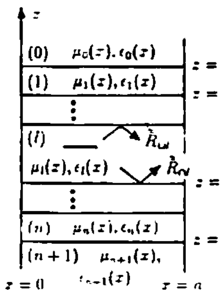

Figure 1: Geometrical configuration of a microstrip line embedded in a stratified medium consisting of inhomogeneous layers.

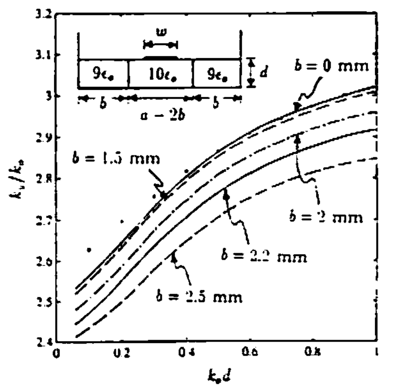

Figure 2: Normalized phase constant of a microstrip line on a segmented substrate, $a = 5$ mm, $d = 1$ mm, $w = 1$ mm, * : results with a homogeneous layer ($b = 0$) in [6]

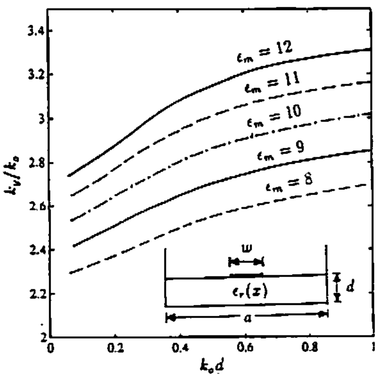

Figure 3: Normalized phase constant of a microstrip line on a substrate with a parabolic permittivity profile $\epsilon_r(x)/\epsilon_o = 10 + 4(\epsilon_m - 10)x(a - x)/a^2$, $a = 5$ mm, $d = 1$ mm, $w = 1$ mm.

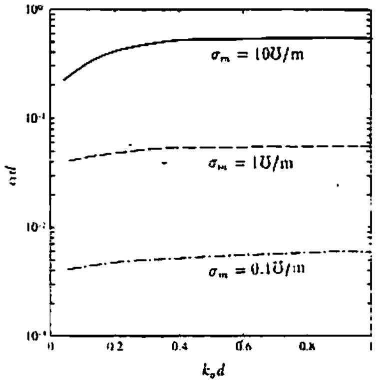

Figure 5: Attenuation constant of a microstrip line on a substrate with a parabolic conductivity profile, all the parameters are the same as in Figure 4.

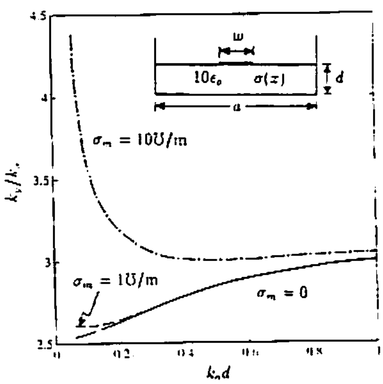

Figure 4: Normalized phase constant of a microstrip line on a substrate with a parabolic conductivity profile $\sigma(x) = 4\sigma_m x(a - x)/a^2$, $a = 5$ mm, $d = 1$ mm, $w = 1$ mm.