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RADIATION OF SPHERICAL SECTOR HORNS WITH IMPEDANCE WALLS

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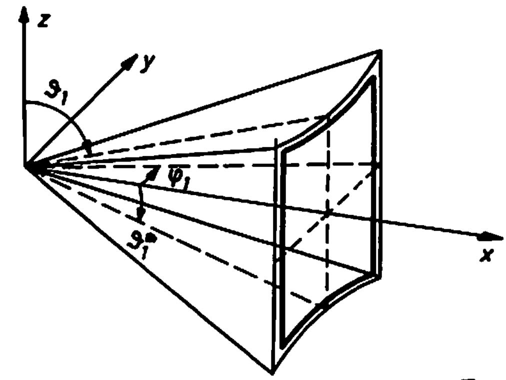
Introduction

Horn feeds with modified walls for example such with corrugations or dielectric layers are widely used in antenna practice. While there exist many investigations dealing with the conical horn, only few topics concerning horns having a rectangular shape of the aperture have been theoretically studied until now. As there is no orthogonal coordinate system for describing in a simple way the walls of a pyramidal horn, in this paper the spherical sector horn (Fig. 1) is investigated, which is especially for not too large flare angles a good approximation for the interesting object. In order to get results which may be used for different feed problems, the considerations are made for walls with general surface impedances.



$\vartheta^* = \vartheta - \frac{\pi}{2}$

Fig. 1: Spherical sector horn

Mathematical methods

The horn may be conveniently analysed by use of the Hertzian potential formulation $\vec{\Pi}_{E,H} = \vec{u}_r \, \Pi_{E,H}$. With

$$\Pi_{E,H} = f_{E,H}(\varphi,\vartheta) \, g(r) \tag{1}$$

the differential equation for $f_{E,H}(\varphi,\vartheta)$ is found to be

$$\frac{1}{\sin^2 \vartheta} \frac{\partial^2 f}{\partial \varphi^2} + \cot \vartheta \frac{\partial f}{\partial \vartheta} + \frac{\partial^2 f}{\partial \vartheta^2} + \nu(\nu+1) f = 0 \tag{2}$$

with $\nu(\nu+1)$ as separation constant. The solution for $g(r)$ is $g(r) = H^{(1),(2)}_{\nu+1/2}(kr)\sqrt{kr}$. The surface impedances are defined as usual: At the walls $\vartheta^* = \pm \vartheta_1^*$ we have $Z_{\varphi r}(\varphi,r) = \pm E_\varphi/H_r$ and $Z_{r\varphi}(\varphi,r) = \mp E_r/H_\varphi$, analog at the walls $\varphi = \pm \varphi_1$. By means of Maxwells' equations the boundary conditions for f_E and f_H are to be formulated as

$$- j \frac{C_g\ kr}{\sin\vartheta} Z_{r\varphi} \frac{\partial f_H}{\partial\varphi} + jkr \frac{Z_{r\varphi}}{Z} \frac{\partial f_E}{\partial\vartheta} - \nu(\nu+1)\, f_E = 0,$$

$$\vartheta^* = \vartheta_1^*$$

$$j \frac{C_g\ kr}{\sin\vartheta} \frac{1}{Z_{\varphi r}} \frac{\partial f_E}{\partial\varphi} + jkr \frac{Z}{Z_{\varphi r}} \frac{\partial f_H}{\partial\vartheta} - \nu(\nu+1)\, f_H = 0, \qquad (3)$$

$$j \frac{kr}{\sin\vartheta} \frac{Z_{r\vartheta}}{Z} \frac{\partial f_E}{\partial\varphi} + jC_g\ kr\ Z_{r\vartheta} \frac{\partial f_H}{\partial\vartheta} - \nu(\nu+1)\, f_E = 0,$$

$$\varphi = \varphi_1$$

$$j \frac{kr}{\sin\vartheta} \frac{Z}{Z_{\vartheta r}} \frac{\partial f_H}{\partial\varphi} - jC_g\ kr \frac{1}{Z_{\vartheta r}} \frac{\partial f_E}{\partial\vartheta} - \nu(\nu+1)\, f_H = 0.$$

with $Z = \sqrt{\mu_o\mu_r/\epsilon_o\epsilon_r}$, $k = 2\pi/\lambda$ and C_g defined by $\partial g(r)/\partial r = - jk\, C_g\, g(r)$. In order to have no discrepancy between these equations and the starting point (1), for the impedances must be $Z_{\varphi r}(\varphi,r) = Z^*_{\varphi r}(\varphi)\, r$ and $Z_{r\varphi}(\varphi,r) = Z^*_{r\varphi}(\varphi)/r$, with analog equations for $Z_{\vartheta r}$ and $Z_{r\vartheta}$. Further it is evident, that for arbitrary surface impedances the modes inside the horn are combinations of a TM- and a TE-mode, but, as can be shown, in some cases pure TE or TM modes are possible. These cases are of special interest for feed problems.

For the differential equation (2) a solution in product form exists, which may be written as

$$f(\varphi,\vartheta) = f_1(\varphi) \cdot f_2(\vartheta)\ . \qquad (4)$$

It is pointed out, that the fields of spherical sector horns with impedance walls can be analysed by (4) only when the impedances satisfy certain conditions. These depend on the existence of single or coupled TE- and TM-modes. If for a horn $kr \gg \nu(\nu+1)$ is valid (which is true for many feed problems), then $g(r) \sim e^{-jkr}$ and we get the very simple condition

$$Z_{r\vartheta}\, Z_{\varphi r} + Z_{\vartheta r}\, Z_{r\varphi} = 0.$$

By this formula it may be understood, why the analysis of rectangular feeds with corrugations on all four walls as done for example in [1] gives results which agree well with experiments, although the starting points are not correct [2].

If the separation of variables cannot be used, the solution may be obtained by the finite-difference method. A computer program was developed using difference formulas of high order, so that a very satisfying accuracy can be obtained for many problems without dealing with large matrices. The errors arising in the numerical treatment have been investigated in detail.

Application

As an application the wall impedances of a spherical sector horn were specified for high aperture efficiency and equal shape of the main beam of E- and H-plane for both horizontal and vertical polarisation. Taking into account some physical considerations it can be shown, that this is realizable in the case of a symmetrical horn ($\varphi_1 = \vartheta_1^*$) having surface impedances as

$$Z_{r\varphi} = Z_{r\vartheta} = 0$$

and

$$Z^*_{\varphi r}(\varphi) = Z^*_{\vartheta r}(\vartheta^*) = jZx_{\varphi r}(\varphi) \quad \text{for} \quad \varphi = \vartheta^*$$

when the real function $x_{\varphi r}(\varphi)$ has a suitable form. To find this the E- and H-plane of a horn was calculated for

$$x_{\varphi r} = x_{\varphi ro}\left[(1-t)\cos^m\left(\frac{\varphi}{\varphi_1}\frac{\pi}{2}\right) + t\right]$$

changing the parameters $x_{\varphi ro}$, t and m in the ranges

$$0 < x_{\varphi ro} < \infty, \quad 0 \le t \le 1 \text{ and } m = 1,2,3,4.$$

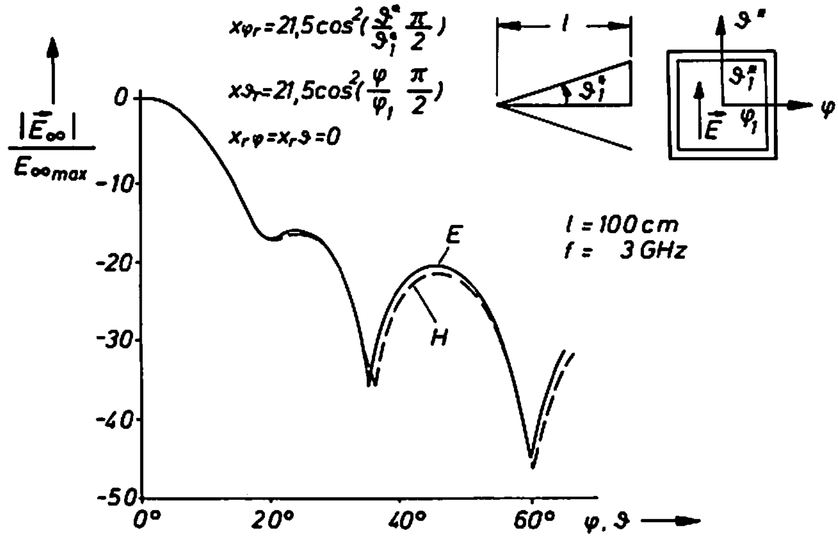


Fig. 2: E- and H-plane pattern of the specified horn

Fig. (2) shows for a horn with $\vartheta_1^* = \varphi_1 = 10^o$, $l = 100$ cm and $f = 3$ GHz the radiation field in the best case. The calculated aperture efficiency is about 0,90. The required impedances may be realized by corrugations parallel to the direction of wave propagation with diminishing depth from the centre of each wall to the edge.

References

[1] Bryant, G.H., Propagation in Corrugated Waveguides. Proc. Inst. Elec. Eng., Vol, 116, pp. 203 - 213, 1969

[2] Dybdal, R.B., Peters, L. and Peake, W.H., Rectangular Waveguides with Impedance Walls. IEEE Trans. Microwave Theory Tech., Vol. MTT-19, pp. 2 - 9, 1971