MUTUAL COUPLING EFFECT ON
A SPATIALLY SMOOTHING ADAPTIVE ARRAY
APPLIED TO SPECTRUM ESTIMATION

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#### I. INTRODUCTION

Multipath propagation is a very serious problem in considering the spectrum estimation performance of an adaptive array. The multipath propagation may cause the erroneous indication of the signal direction. To improve the array performance in the presence of coherent signals, the spatial smoothing technique is one of the useful means [1,2]. By using this method, the correlation between signal sources can be decreased. By properly selecting the coefficients weighted to the array channels, the correlation among the signal sources can be completely eliminated [3].

In a spatial smoothing adaptive array, it is required that the phase difference between the received signals of two adjacent pair of elements be the same. An equally spaced linear array with identical antenna elements can be used, and it operates well when the effect of mutual coupling is ignored. However, when the effect of mutual coupling is not small, the phase relation changes and the array performance may degrade.

In this paper, the spectrum estimation by a spatially smoothing adaptive array is investigated with or without mutual coupling effects. Briefly the theoretical analysis of the performance is given in the next section, in which the method of moments is applied [4,5]. Some numerical results for linear arrays will be given in Section III. Although the effect of mutual coupling on an adaptive array has been discussed in literatures, mutual coupling on a spatial smoothing array has different effects.

#### II. ANALYSIS

Consider an N-element adaptive array with the MUSIC algorithm [6]. The waveforms X received at the N array elements are linear combinations of the D incident wavefronts and noise as in

$$X = AF + W \tag{1}$$

where A is the matrix spanned by the propagation vectors determined by the signal direction-of-arrival (DOA)  $\theta$  and the array configuration, and F is the vector consisting of complex signal sources. The noise appears as the complex vector W. From eqn.(1), the N x N covariance matrix of the vector X is

$$S = \overline{XX^*} = \overline{AFF^*A^*} + \overline{WW^*}$$
 (2)

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under the assumption that the incident signals and the noise are uncorrelated, where superscript \* denotes complex conjugate transpose.

By considering the othogonal property between the eigenvectors associated with the minimum (N-D) eigenvalues and the space spanned by the columns of the incident signal mode vector, the estimated spectrum density is expressed as

$$P(\theta) = \frac{1}{a^*(\theta)E_N E_N^* a(\theta)}.$$
 (3)

where E<sub>N</sub> is defined to be the N x (N-D) matrix spanned by the (N-D) noise eigenvectors, and a( $\theta$ ) is the propagation vector of a signal from DOA  $\theta$ .

In the presence of correlated signals, the spatial smoothing method can be used to eliminate the correlation among the arrival signals. This method devides an adaptive array into a set of subarrays and the signal covariance matrix is evaluated based on the weighted average of those of the subarrays. Therefore, the resulted M x M covariance matrix containes less or no correlation effect between the signals. In a spatially smoothing array, the estimated spectrum density has the same form as eqn. (2), while  $a(\theta)$  and  $E_N$  are reduced to an M dimensional vector and an M x (M-D) matrix, respectively.

We turn to consider the effect of mutual coupling. For simplicity, the vertically oriented half-wavelength dipoles are used as the antenna elements and all arrival signals are assumed to be vertically polarized. By using the method of moments, the incident waveforms in the presence of mutual coupling is given by [4]

$$X = -Z_{L}YAF + W \tag{4}$$

where  $Z_t$  is a diagonal load impedance matrix with the load impedances as its elements and Y is the equivalent admittance matrix between the antenna elements. Using eqn.(4), instead of eqn.(1), in evaluating the parameters appeared in eqn.(3), the spectrum can be estimated with the consideration of the effect of mutual coupling.

# III. NUMERICAL RESULTS

As the example, 3- and 7-element linear arrays are considered with the vertical resonant halfwave dipole of radius 0.005 wavelength and a resistance located at the output matches the antenna input impedance in the absence of mutual coupling.

We assume that each of the signals arrives from the horizontal direction (X-Y plane) and has a constant amplitude. Since the emphasis of this paper is placed to the effect of mutual coupling, it is also assumed that the correlation effect of the incoming signals is completely eliminated. Thus, the results obtained in this paper can be considered as the upper limit of the obtainable array performance. Furthermore, each element of the noise vector W is assumed to be a Gaussian process  $G(0,\sigma^2)$ . For simplicity,  $\sigma^2$  is set to unity hereafter. In the computations, the number of samples is chosen as 100. Each subarray is equally weighted.

First, we consider the case in the presence of single signal together with the noise. Fig.1 compares the eigenvalues of the smoothed covariance matrix S, where M is 3, N is 3 and 7,

and the interelement spacing d varies between 0.1 and 1 wavelength. The strength of the signal is defined as  $E' = (E \cdot wavelength/\sigma) = 40dB$  which arrives in DOA = 50°. In the absence of mutual coupling, the maximum eigenvalue associated with the arrival signal is almost constant with respect to d. Compared with the 3-element case, the two eigenvalues associated with the noise becomes closer in the 7-element case. This means that the covariance between diffirent noise elements is reduced after spatial smoothing.

On the other hand, in the presence of mutual coupling, the eigenvalues vary with respect to N and d. In Fig.1, as the increase of N and the decrease of d, the maximum eigenvalue associated with the arrival signal decreases, while the eigenvalues associated with the noise increase. This implies that, in a spatial smoothing array, a part of the signal power cannot be correctly recognized by the array and contributes to the increase of the noise power.

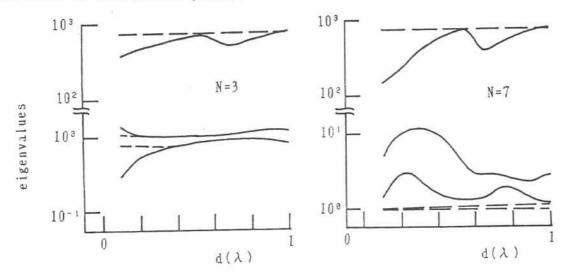


Fig. 1 Eigenvalues vs. d (—: with mutual coupling, -- -: no mutual coupling).

The estimated spectrum density is shown in Fig.2 where N is 3 and 7, M is 3, and d is chosen 0.2, 0.5 and 1 wavelength. The same parameters of the arrival signal are assumed as in Fig. 1. It is seen that, in the absence of the spatial smoothing effect (N=3), the difference in the performance with and without mutual coupling is not so great compared with that in the case that the spatial smoothing is performed. When N is 7, the performance, including the beam width, the peak level and the peak direction, degrades greatly by the effect of mutual coupling, particularly when d is small.

Fig.3 shows the estimated spectrum density when two signals arrive with the same signal strength as that in Fig.2. The DOA's of the two signals are  $\theta$  =40° and 50° with the separation of 10°. It is seen in Fig.3 that, in the absence of mutual coupling, except the 3 element array with d=0.2 wavelength (an error of about 2° is observed in the DOA of each signal), all the plots show good resolution performance. The performance is generally improved when either N or d increases. In the presence of mutual coupling, no resolved spectrum can be observed

in the 7-element array where spatial smoothing is performed.

#### IV. CONCLUSION

In the presence of mutual coupling, the spatially smoothing method will generally degrade the array performance, particularly when the interelement spacing is small. This implies that, before applying the spatial smoothing method, one should pay attention to the array configuration and make sure that the effect of mutual coupling is negligible.

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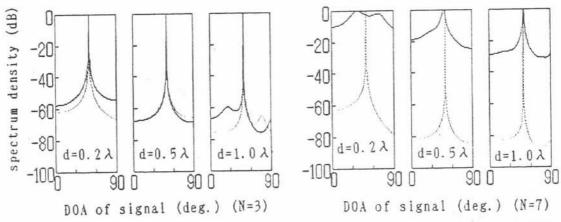


Fig. 2 Estimated spectrum density in the presence of single signal  $(D0A=50^{\circ})$  ( — :with mutual coupling, - - -:no mutual coupling).

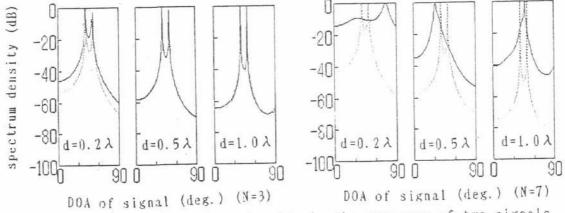


Fig. 3 Estimated spectrum density in the presence of two signals (DOA= $40^{\circ},50^{\circ}$ ) ( — :with mutual coupling, - - -:no mutual coupling).