

B-4-1

NUMERICAL INVESTIGATION OF CORRUGATED  
FEEDS BY USING IMPEDANCE BOUNDARY CONDITIONS

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The electromagnetic problem of corrugated structures can normally be solved numerically by an application of the boundary conditions on the conducting surface of the corrugation. However, due to the large size of the conducting surface and numerous sharp corners the dimension of the resulting matrix becomes too large, which requires excessive computer time and storage. These difficulties can be overcome by representing the corrugations by an appropriate impedance surface.<sup>1,2</sup> The large conducting surface of the corrugations is therefore reduced to a small impedance one, just over the corrugations and results in smaller matrix size and consequently more efficient numerical solution of the problem.

In general, the representation of a corrugated surface by impedance boundary conditions is not exact and its validity needs careful examination. Furthermore, the available expressions for the surface impedance of the corrugation is reported to be valid only for high density corrugations.<sup>3</sup> These properties are investigated by considering a two-dimensional corrugated geometry and computing its surface field distribution and the scattered field. The computed results are then compared with available data.

Next, the problem of a corrugated rectangular horn is considered. The problem is simplified by considering a two-dimensional case. A finite section of a conducting parallel plate waveguide is assumed to be connected to corrugated flanges at an arbitrary flare angle. The geometry is excited by a magnetic line source inside the waveguide region, fig. 1. It can be shown that for such a geometry the surface current  $K_s$  is related to the incident magnetic field through the integral equation

$$H_z^{\text{inc}}(\bar{t}) = \frac{1}{2} K_s(\bar{t}) - \frac{j}{4} \oint_c K_s(\bar{t}') H_1^{(2)}(k|\bar{t}-\bar{t}'|) \cos(\bar{n}', \bar{t}-\bar{t}') dk \ell'$$

$$+ \frac{1}{4} \oint_c \frac{Z_s K_s}{\eta} H_0^{(2)}(k|\bar{t}-\bar{t}'|) dk \ell'$$

where  $k$  and  $\eta$  are the propagation constant and intrinsic impedance of free space and  $Z_s$  is the surface impedance of the corrugations. For rectangular corrugations this surface impedance is represented by

$$Z_s = -j \frac{g}{g+\delta} \eta \tan kd$$

where  $g$  is the width of the corrugation slot and  $\delta$  and  $d$  are the

thickness and the depth of the corrugation teeth, respectively. Note that, in the above integral equation the effect of corrugation is to introduce an additional term, which is due to the surface impedance  $Z_s$ . The integral equation can therefore be solved by a moment method similar to the case of conducting structures. A set of computed radiation patterns are shown in figures 2-a and 2-b for conducting and corrugated horns with different corrugation depths. Fig. 2-a indicates that the radiation patterns of a conducting horn is reasonably good, but that of a corrugated horn with a corrugation depth  $d/\lambda = 0.25$  (resonant slots) has large side lobes. Increasing the corrugation depth  $d/\lambda$  in fig. 2-b reduces the side lobe levels and increases the gain. For the case of  $d/\lambda = 0.41$  the main lobe has a smooth shape and the side and back lobes are relatively small.

From these sample calculations we conclude that using the impedance boundary conditions the problem of corrugated horns can be solved numerically. Its main advantage is in its usefulness for investigation of corrugated geometries, where the corrugated surface has an arbitrary surface curvature. In such cases the available methods based on the modal analysis of the field are either too complex to be used or are totally impractical to apply.

#### References

1. Hansen, J.E. and Shafai, L., "Application of Impedance Boundary Conditions to Corrugated Horns", Electromagnetics Institute, Tech. U. of Denmark, R 187, May 1977.
2. Shafai, L. and Hansen, J.E., "Matrix Formulation of Corrugated Feeds by Using Impedance Boundary Conditions", Elec. letters, Vol. 13, No. 11, pp. 310-311, 1977.
3. Lawrie, R.E. and Peters, L., "Modification of Horn Antennas for Low Side Lobe Levels", IEEE Trans. Ant. & Propag. Vol. Ap. 14, No. 5, pp. 605-610, 1966.

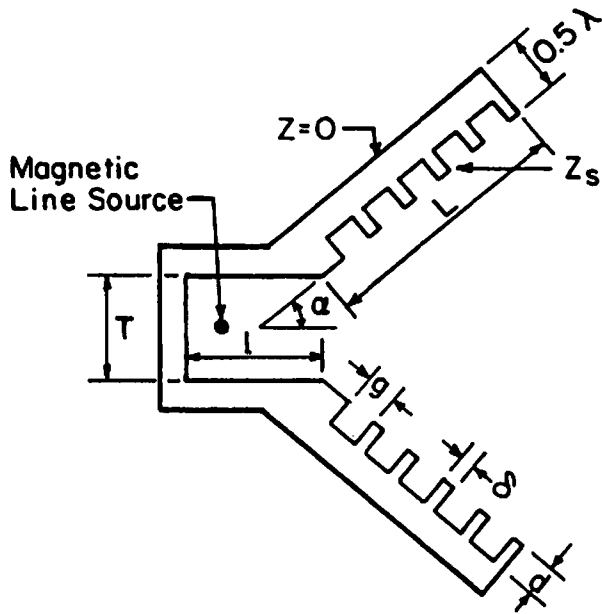


Fig. 1 Geometry of the E-plane corrugated horn

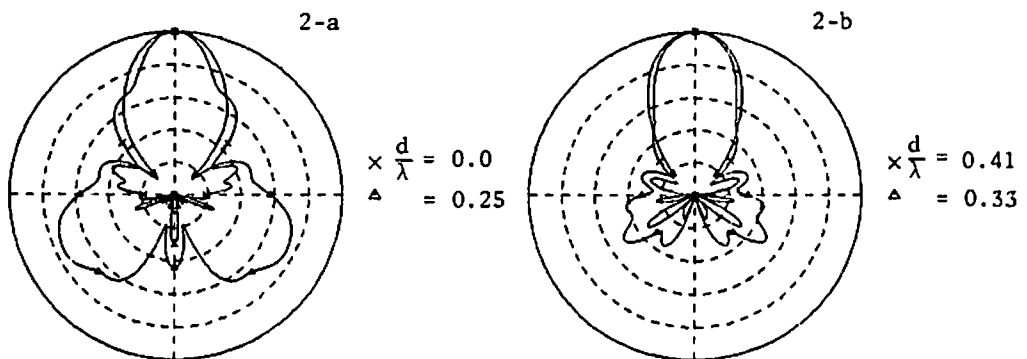


Fig. 2 Radiation patterns of the E-plane corrugated horn,  
 $\alpha=30^\circ$ ,  $L=2.15\lambda$ ,  $\ell=1.89\lambda$ ,  $T=0.32\lambda$ , scale 5dB/div.