Analysis of Crosstalk between Multiconductor Transmission Lines in Arbitrary Directions Using a Circuit-Concept Approach

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Abstract: In this paper, a circuit-concept approach under weak-coupling assumption is expanded for the analysis of crosstalk between multiconductor transmission lines (MTLs) in arbitrary directions on a gigabit digital printed circuit board (PCB). Under the weak-coupling assumption, the calculation of crosstalk requires only self-terms of per-unit-length parameters such as capacitance and inductance. The effects of the mutual-terms of the parameters can be estimated by considering external electromagnetic fields coupling between MTLs. It is shown that the crosstalk calculated by the expanded circuit-concept approach fairly agrees well with the result obtained by solving MTL equations in which the calculation of per-unit-length parameter matrices is essentially required.

Key words: circuit-concept approach, crosstalk, multiconductor transmission lines, PCB

1. Introduction

Recently, higher data transmission and circuit density are the trends in an electronic circuit design of a gigabit digital printed circuit board (PCB). It has been frequently observed, however, an unexpected EM coupling problem such as crosstalk that can cause timing violation, false clocking, and intermittent data fault. The problem is highly associated with the effects of multiconductor transmission lines (MTLs) because PCB traces are the largest/longest component in most of all devices on a PCB. For decades, therefore, it has been actively studied by several researchers to analyze and reduce the unwanted electromagnetic (EM) coupling.

The transmission line theory in which the transverse electromagnetic (TEM) propagation for the fields surrounding conductors is assumed is suitable to estimate and explain the EM coupling phenomenon in a gigabit digital PCB. For the multiconductor transmission lines, (n+1) conductors where n>2, parallel to the line axis, i.e., uniform cross section, the crosstalk is well studied in both frequency and time domain [1]. In this analysis, the per-unit-length parameters of inductance, capacitance, conductance, resistance matrices are first determined from the cross-sectional information of given MTLs. Then, line voltages and currents can be calculated by solving the MTL equations with the incorporation of terminal conditions. In the case of nonuniform lines whose cross-sectional dimensions vary along the line axis, the per-unit-length parameter matrices become the functions of line axis. Therefore, MTL equations become complex and difficult to solve.

For the nonuniform transmission line structure of different direction and length, crosstalk between two conductors over a ground plane, i.e., (2+1) lines, is analyzed in frequency domain by using a circuit-concept approach under the weak-coupling assumption [2][3]. In this approach, the calculation of the only self-terms of per-unit-length parameters is required. The coupling between two lines shows that one line over a ground plane driven by a lumped source within a termination acts as a transmitting antenna. The other passive line plays the role of a receiving antenna.

In this paper, the circuit-concept approach for crosstalk analysis of (2+1) lines is studied and expanded for the analysis of (n+1) lines where n>2. The crosstalk calculated by the expanded circuit-concept approach is compared with the results obtained by solving MTL equations in which the calculation of per-unit-length parameter matrices is essentially required.

2. Weak-coupling assumption

A circuit of (2+1) conductors for illustrating crosstalk mechanism under weak-coupling assumption is shown in Fig. 1. One conductor line having source resistance R_S and load resistance R_L is driven by a lumped voltage source, $V_S(t)$. This circuit is referred to as a generator circuit. The other conductor terminated at the near end with a resistive load, R_{NE}, and at the far end with a resistive load, R_{FE} is referred to as the receptor circuit. From the generator circuit, the voltages and currents due to the lumped source voltage produce electromagnetic fields that excite the receptor circuit and induce the crosstalk voltage V_{NE} and V_{FE} at the terminations.

These induced voltages and currents in the receptor circuit also produce electromagnetic fields that interact with the generator circuit again. This back-interaction or second-order effect by inducing voltages and currents in the generator circuit is negligible in the weak-coupling assumption.

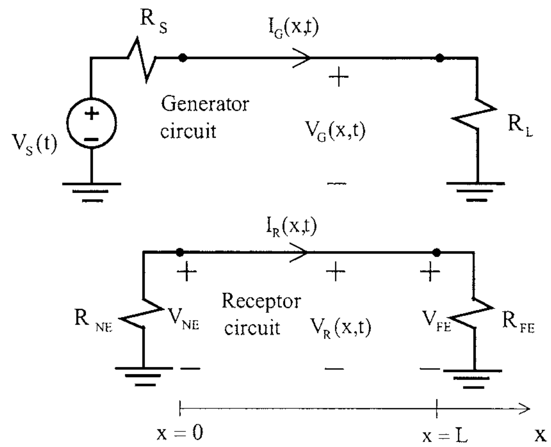


Fig. 1. The circuit of (2+1) conductors illustrating crosstalk.

The coupling coefficient is known as

$$k = l_m / \sqrt{l_G l_R} \qquad (1)$$

where l_m is the per-unit-length mutual inductance between two circuits. l_G and l_R are the per-unit-length inductances of the respective circuits. The weak-coupling between two lines can be reasonably assumed if $\sqrt{1-k^2} \cong 1$ [4].

For MTL analysis, we assume that one circuit is driven by a voltage source, i.e., a generator circuit. The other lines are passive and also weakly coupled to the generator circuit and each other. It can be intuitionally presumed that the longer the separations between the generator and receptor lines, the better the accuracy of the weak-coupling assumption.

3. 2n-port network for (n+1) conductor lines

Fig. 2 shows the structure of (n+1) finite-length microstrip lines over a ground plane. Each microstrip line is arbitrary directed and has angle theta between the lines. The thickness of the substrate having relative permittivity ϵ_r is h. In the case of (n+1) finite-length lines, effect of the electromagnetic coupling on a receptor line is caused not only by very neighbor generator line but also by the other generator lines which are in the vicinity of the receptor line. Let us consider line #1 as a receptor line. Then, the others except for the line #1 are referred to as generator lines.

The modified telegrapher equations for the voltage $V_1(x)$ and line current $I_1(x)$ on line #1 can be expressed as

$$-\frac{d}{dx}\begin{bmatrix}V_1(x_1)\\ I_1(x_1)\end{bmatrix}=\begin{bmatrix}0 & j\omega C_1\\ j\omega L_1 & 0\end{bmatrix}\begin{bmatrix}V_1(x_1)\\ I_1(x_1)\end{bmatrix}+\sum_{i=2}^{n}\begin{bmatrix}V_{fi}(x_1)\\ I_{fi}(x_1)\end{bmatrix} \qquad (2)$$

where C_1 and L_1 are per-unit length self capacitance and self inductance, respectively. ω is angular frequency. The forcing terms $V_{fi}(x_1)$ and $I_{fi}(x_1)$ are the inducement of voltages and currents in the receptor line #1. Here, the summation of forcing terms of the right-hand side in the equation (2) are voltages and currents on line #2, line #3, …, line #n.

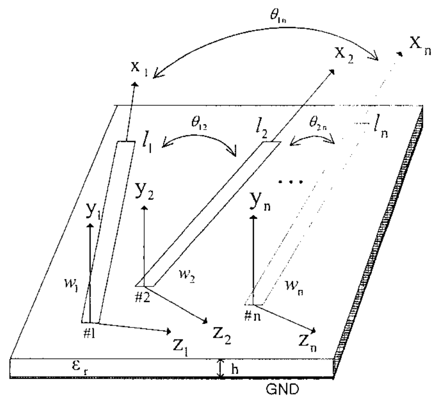


Fig. 2. (n+1) finite-length microstrip lines in the same substrate.

The above equation signifies that the characteristic impedance of the line is determined independently as an isolated line, although the effect of neighboring line has been taken into account because of weak-coupling assumption.

The solution of the equation (2) can be obtained as

$$\begin{bmatrix}V_1(0)\\ I_1(0)\end{bmatrix}=\Phi_1^{-1}(l_1)\begin{bmatrix}V_1(l_1)\\ I_1(l_1)\end{bmatrix}-\sum_{i=2}^{n}\int_0^{l_1}\Phi_1^{-1}(x_1')\begin{bmatrix}V_{fi}(x_1')\\ I_{fi}(x_1')\end{bmatrix} \qquad (3)$$

where Φ_1^{-1} is an inverse chain matrix.

$$\Phi_1^{-1}(x_1')=\begin{bmatrix}\cos\beta_1 x_1' & jZ_{01}\sin\beta_1 x_1'\\ j\sin\beta_1 x_1'/Z_{01} & \cos\beta_1 x_1'\end{bmatrix} \qquad (4)$$

where Z_{01} and β_1 are a characteristic impedance and propagation constant in line #1, respectively.

The forcing terms, V_{fi} and I_{fi}, on the i-th line can be expressed as vector potential A_i caused by the currents on i-th line as [2]

$$\begin{bmatrix} V_{fi}(x_i) \\ I_{fi}(x_i) \end{bmatrix} = \begin{bmatrix} j\omega (A_{xi})\Big|_{y'=0}^{y'=h} \\ \dfrac{C_1}{\mu_0 \varepsilon_0 \varepsilon_r} \dfrac{\partial}{\partial x} (A_{xi})\Big|_{y'=0}^{y'=h} \end{bmatrix}. \tag{5}$$

The vector potential can be expressed as

$$A_{xi} = \frac{\mu_0}{4\pi} \{ \int_0^{l_i} I_i(x_i') \frac{\exp(-jkR_{i1})}{R_{i1}} dx_i' - \int_0^{l_i} I_i(x_i') \frac{\exp(-jkR_{i2})}{R_{i2}} dx_i' \} \tag{6}$$

where

$$R_{i1} = \sqrt{(x_i - x_i')^2 + (y_i - h)^2 + z_i^2}\,. \quad R_{i2} = \sqrt{(x_i - x_i')^2 + (y_i + h)^2 + z_i^2} \tag{7}$$

The second term of right side in the equation (6) is caused by image currents. The wavenumber k is not k_0 in free space but approximately $k_0\sqrt{\varepsilon_{eff}}$ using effective permittivity. A line current $I_i(x_i')$ can be expressed by a line voltage and a line current at point $x = l_i$ on i-th line as follows:

$$I_i(x_i') = j\frac{V_i(l_i)}{Z_{0i}} \sin\beta_i(l_i - x_i') + I_i(l_i)\cos\beta_i(l_i - x_i') \tag{8}$$

Similarly, $V_i(x_i)$ is also expressed as

$$V_i(x_i') = jI_i(l_i)Z_{0i}\sin\beta_i(l_i - x_i') + V_i(l_i)\cos\beta_i(l_i - x_i')\,. \tag{9}$$

The relationship of vector potentials between $x_1 - y_1 - z_1$ and $x_i - y_i - z_i$ coordinate systems are as follows:

$$\begin{bmatrix} A_{x1} \\ A_{y1} \\ A_{z1} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} A_{xi} \\ A_{yi} \\ A_{zi} \end{bmatrix}. \tag{10}$$

Substituting (5) into (3) and using (8) and (9), we have result as

$$\begin{bmatrix} V_1(0) \\ I_1(0) \end{bmatrix} = \begin{bmatrix} a_{11} & b_{11} \\ c_{11} & d_{11} \end{bmatrix} \begin{bmatrix} V_1(l_1) \\ I_1(l_1) \end{bmatrix} + \sum_{i=2}^{n} \begin{bmatrix} a_{1i} & b_{1i} \\ c_{1i} & d_{1i} \end{bmatrix} \begin{bmatrix} V_i(l_i) \\ I_i(l_i) \end{bmatrix} \tag{11}$$

Similarly, we choose i-th line as receptor line, then the other lines are referred as the generator lines. If all cases are taken into account from line #1 to line #n, we have n similarly equation forms. Consequently, 2n-port network for the n finite-length microstrip lines is obtained as (12). We are ready to calculate crosstalk between MTLs by incorporating terminal conditions with (12). The matrix elements in (12) can be derived from [2].

$$\begin{bmatrix} V_1(0) \\ V_2(0) \\ \vdots \\ V_n(0) \\ I_1(0) \\ I_2(0) \\ \vdots \\ I_n(0) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} & b_{11} & b_{12} & \cdots & b_{1n} \\ a_{21} & a_{22} & & & b_{21} & b_{22} & & \\ \vdots & & \ddots & \vdots & \vdots & & \ddots & \vdots \\ a_{n1} & & \cdots & a_{nn} & b_{n1} & & \cdots & b_{nn} \\ c_{11} & c_{12} & \cdots & c_{1n} & d_{11} & d_{12} & \cdots & d_{1n} \\ c_{21} & c_{22} & & & d_{21} & d_{22} & & \\ \vdots & & \ddots & \vdots & \vdots & & \ddots & \vdots \\ c_{n1} & & \cdots & c_{nn} & d_{n1} & & \cdots & d_{nn} \end{bmatrix} \begin{bmatrix} V_1(l_1) \\ V_2(l_2) \\ \vdots \\ V_n(l_n) \\ I_1(l_1) \\ I_2(l_2) \\ \vdots \\ I_n(l_n) \end{bmatrix} \tag{12}$$

4. Simulation results

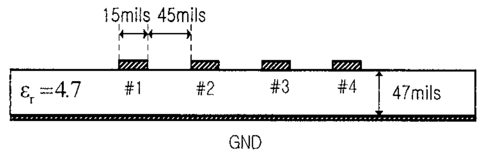


Fig. 3. Cross-sectional dimension of test PCB.

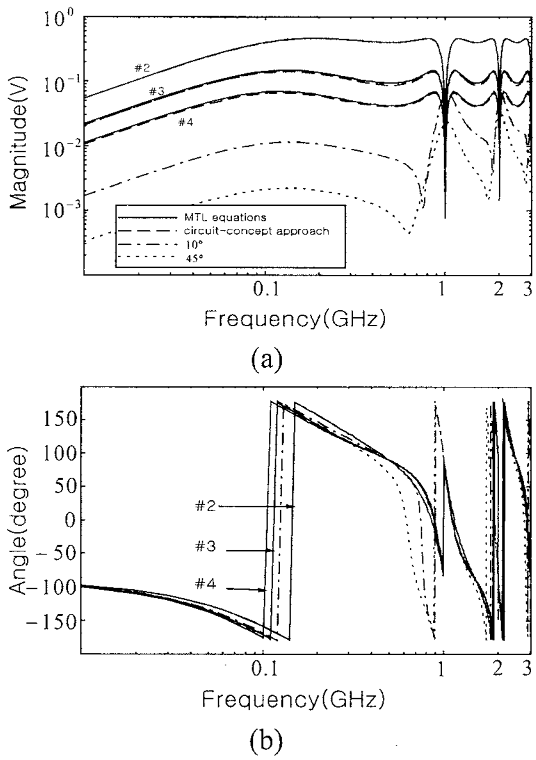
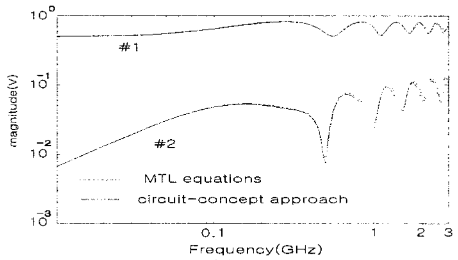
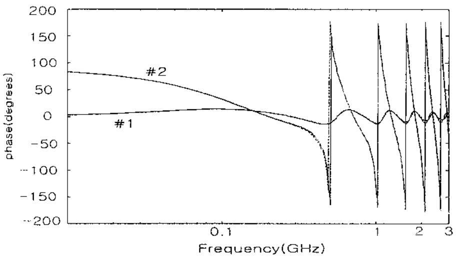


Fig. 4. Far end crosstalk voltages.
(a) magnitude, (b) phase.

The cross-sectional dimension of test PCB is shown in Fig. 3. All conductor lines are terminated with 50 Ω. The far end crosstalk voltages are shown in Fig. 4 for homogeneous case, $\epsilon_r = 1$. The arbitrary direction angles, i.e., θ_{34} in Fig. 2, of line #4 are 0°, 10°, and 45°. The other lines are parallel to each other. The effects of the mutual-terms of the per-unit-length line parameters can be easily calculated by considering external electromagnetic fields coupling in the expanded circuit-concept approach.

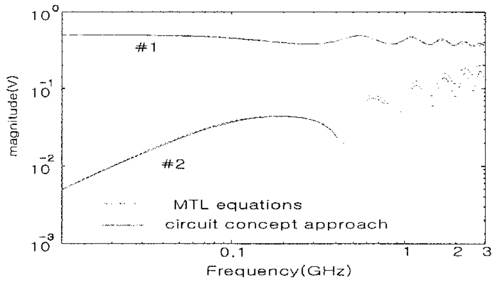


(a)

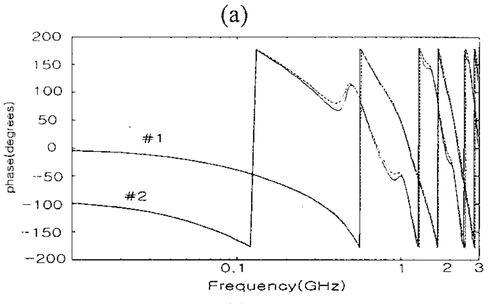


(b)

Fig. 5. Near end crosstalk voltage on line #2. (a) magnitude, (b) phase.



(a)



(b)

Fig. 6. Far end crosstalk voltage on line #2. (a) magnitude, (b) phase.

The coupling factor of the structure in Fig. 3 can be calculated as $k = 0.194$, and $\sqrt{1-k^2} = 0.98$. Thus, the weak-coupling assumption can be valid for this separation of the PCB lands. Figs. 5 and 6 show the near end and far end crosstalk voltages, respectively. The circuit concept approach agrees also well with MTL equations.

5. Conclusion

The circuit-concept approach was expanded for the crosstalk analysis of (n+1) lines where n>2. The crosstalk calculated by the expanded circuit-concept approach developed by this study was compared with the results obtained by solving the MTL equations in which the calculationof per-unit-length parameter matrices was essentially required. It was shown that the crosstalk calculated by the expanded circuit-concept approach fairly agreed well with the results of the MTL equations.

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