# Comparison of BER Performances with Receiving Antenna Selection Techniques in MIMO Systems

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Abstract – This paper compares bit error rate (BER) performances when receiving antenna selection (RAS) techniques are applied to MIMO (Multi-Input Multi-Output) systems from the viewpoint of receiver structure. We assume two receiver structures. One is a structure for controlling gain in received signals centrally for all receiving antennas. The other is a structure for controlling gain in received signals individually in each receiving antenna. We show that a receiver which controls gain in received signals centrally can obtain good BER performances when utilizing RAS techniques using the channel matrix eigenvalue (RAS-E). Additionally, we consider RAS techniques using the phase of the channel components (RAS-PC) and received power (RAS-RP) for a receiver which controls received signal gain individually in each receiving antenna. We then compute and compare BER performances when employing RAS-PC and RAS-RP under Rayleigh fading channels, when the receiver uses inverse channel detection based on inverse matrix (ICD) or maximum likelihood detection (MLD).

# I. INTRODUCTION

The rapidly increasing number of users and services in mobile communications requires efficient usage of the available frequency band. MIMO systems are one possible candidate to meet these demands [1]-[10]. Consequently, much investigation has been carried out on techniques for obtaining good BER performances in such environments [1]-[4][6]-[10]. Space-time block code [3], space-time trellis code [4] and detection techniques [10] in MIMO systems have been investigated. Recently, techniques for antenna diversity techniques in MIMO systems have been investigated [6]-[9]. It has also been shown that the minimum eigenvalue power of the channel matrix eigenvalue influences BER performances in MIMO systems [5]-[7]. From this, antenna selection techniques based on the minimum power of the channel matrix eigenvalue in MIMO systems were investigated [6][7][9] and RAS techniques using the minimum power of the channel matrix eigenvalue have been proposed [9].

This paper compares BER performances employing RAS techniques in MIMO from the viewpoint of receiver structure. We assume two receiver structures. One is a structure for controlling gain in received signals centrally for all receiving antennas. The other is a structure for controlling gain in received signals individually for each receiving antenna. We show that a receiver which controls signal gain centrally can

obtain good BER performances by employing RAS-E. Additionally, BER performances when employing RAS-PC are better than those when employing RAS-RP when the receiver controls received signal gain individually and uses ICD. On the other hand, BER performances when employing RAS-RP are better than those when employing RAS-PC when the receiver uses MLD. Thus, when the receiver has a structure for controlling received signal gain individually in each receiving antenna, the most effective RAS technique differs according to the detection method used by the receiver.

#### II. SYSTEM MODEL

The system model in this investigation is shown in Figure 1. The transmitter then has 2 antennas. The modulation signals of channels A and B are transmitted in transmitting antennas 1 and 2 respectively and the modulation is QPSK. The receiver has 3 antennas. The received signals  $Rx_k$  in the receiving antenna *k* are expressed as

$$R_{X_{k}} = \begin{pmatrix} h_{k1} & h_{k2} \end{pmatrix} \begin{pmatrix} T_{X_{a}} \\ T_{X_{b}} \end{pmatrix} + n_{k} \qquad (k=1,2,3)$$
(1)

where the transmitted signals of channel A and B are  $Tx_a$ and  $Tx_b$ .  $h_{k1}$  and  $h_{k2}$  are the channel component and  $n_k$  is white Gaussian noise. The channel model then is treated as Rayleigh fading channels, and the detection method is treated as ICD or MLD [10].

## **III. RECEIVING ANTENNA SELECTION TECHNIQUE**

In this section, first the relationship between the minimum power of the channel matrix eigenvalue and received power is described. In consideration of received power, we examine the RAS-E, RAS-PC and RAS-RP techniques.

# A. Relationship between Channel Matrix Eigenvalue and Received power

From (1), equation (2) is established in receiving antennas i and j.

$$\begin{pmatrix} \mathbf{R}\mathbf{x}_{i} \\ \mathbf{R}\mathbf{x}_{j} \end{pmatrix} = \begin{pmatrix} \mathbf{h}_{i1} & \mathbf{h}_{i2} \\ \mathbf{h}_{j1} & \mathbf{h}_{j2} \end{pmatrix} \begin{pmatrix} \mathbf{T}\mathbf{x}_{a} \\ \mathbf{T}\mathbf{x}_{b} \end{pmatrix} + \begin{pmatrix} \mathbf{n}_{i} \\ \mathbf{n}_{j} \end{pmatrix}$$
(2)

In (2), the phase difference  $\varphi_{ij}$  between the channel matrix components in receiving antennas *i* and *j* is defined by:

$$\varphi_{ij} = \theta_j - \theta_i \qquad (-\pi \leq \varphi_{ij} \leq \pi \text{ radians}) \qquad (3)$$

where the phase difference of the channel matrix components  $h_{i1}$  and  $h_{i2}$  is  $\theta_i (-\pi \le \theta_i \le \pi$  radians) and the phase difference of the channel matrix components  $h_{j1}$  and  $h_{j2}$  is  $\theta_j (-\pi \le \theta_j \le \pi$  radians). Generality is established, even if the equation of (2) is substituted by equation (4).

$$\begin{pmatrix} \mathbf{R}\mathbf{x}_{i} \\ \mathbf{R}\mathbf{x}_{j} \end{pmatrix} = \begin{pmatrix} \mathbf{r}_{i1}/\sqrt{2} & \mathbf{r}_{i2}e^{j\theta_{i}}/\sqrt{2} \\ \mathbf{r}_{j1}e^{-j(\theta_{i}+\phi_{ij})}/\sqrt{2} & \mathbf{r}_{j2}/\sqrt{2} \end{pmatrix} \begin{pmatrix} \mathbf{T}\mathbf{x}_{a} \\ \mathbf{T}\mathbf{x}_{b} \end{pmatrix} + \begin{pmatrix} \mathbf{n}_{i} \\ \mathbf{n}_{j} \end{pmatrix}$$
(4)

The eigenvalue and eigenvector of the matrix in (4) are defined by  $\lambda_{ij,1}(r_{i1},r_{i2},r_{j1},r_{j2},\phi_{ij})$ ,  $\lambda_{ij,2}(r_{i1},r_{i2},r_{j1},r_{j2},\phi_{ij})$  and  $\mathbf{v}_{ij,1}$ ,  $\mathbf{v}_{ij,2}$ , respectively. Then, the eigenvalue is expressed as:

$$\lambda_{ij,1} = \frac{1}{2\sqrt{2}} \left( r_{i1} + r_{j2} + \sqrt{\left(r_{i1} - r_{j2}\right)^2 + 4r_{i2}r_{j1}e^{-j\phi_{ij}}} \right),$$
(5)

$$\lambda_{ij,2} = \frac{1}{2\sqrt{2}} \left( r_{i1} + r_{j2} - \sqrt{\left(r_{i1} - r_{j2}\right)^2 + 4r_{i2}r_{j1}e^{-j\phi_{ij}}} \right)$$
(6)

where  $|\lambda_{ij,1}|^2 \ge |\lambda_{ij,2}|^2$ . The vector  $\mathbf{T}\mathbf{x}_{ij} = (Tx_a, Tx_b)^T$ ,  $\mathbf{R}\mathbf{x}_{ij} = (Rx_i, Rx_j)^T$ ,  $\mathbf{n}_{ij} = (n_i, n_j)^T$  is expressed as

$$\mathbf{T}\mathbf{x}_{ij} = \alpha \mathbf{v}_{ij,1} + \beta \mathbf{v}_{ij,2},\tag{7}$$

$$\mathbf{R}\mathbf{x}_{ij} = \alpha \lambda_{ij,1} \mathbf{v}_{ij,1} + \beta \lambda_{ij,2} \mathbf{v}_{ij,2} + \mathbf{n}_{ij}$$
(8)

where  $\alpha$  and  $\beta$  are coefficients given in the vector  $\mathbf{T}\mathbf{x}_{ij}$ . As shown in (8), the power of the vector  $\alpha\lambda_{ij,1}\mathbf{v}_{ij,1}$ ,  $\beta\lambda_{ij,2}\mathbf{v}_{ij,2}$ depends on the power of  $|\lambda_{ij,1}|^2$ ,  $|\lambda_{ij,2}|^2$ , respectively. Next the receiving power is considered. In the receiver, the inverse matrix operation is carried out in (8), the estimation value  $\alpha'$ ,  $\beta'$  of coefficient  $\alpha$ ,  $\beta$  is obtained and the estimation vector  $\mathbf{T}\mathbf{x}_{ij}$  of the vector  $\mathbf{T}\mathbf{x}_{ij}$  is obtained. Then the receiving power of  $\beta\mathbf{v}_{ij,2}$  in  $\mathbf{T}\mathbf{x}_{ij}$  depends on  $|\beta\lambda_{ij,2}|^2$  and the receiving power  $\alpha\mathbf{v}_1$  in  $\mathbf{T}\mathbf{x}_{ij}$  depends on  $|\alpha\lambda_{ij,1}|^2$ . Therefore the estimation accuracy of  $\alpha'$  improves but the estimation accuracy of  $\beta'$ becomes deteriorates. However, the receiving power of  $\alpha\mathbf{v}_{ij,1}$ depends on  $|\alpha\lambda_{ij,2}|^2$  minimally because of  $|\lambda_{ij,1}|^2 \ge |\lambda_{ij,2}|^2$ . Accordingly, the vector  $\mathbf{R}\mathbf{x}_{ij}$  is approximated as:

$$\mathbf{R}\mathbf{x}_{ij} \approx (\alpha | \lambda_{ij,2}| \frac{\lambda_{ij,1}}{|\lambda_{ij,1}|} \mathbf{v}_{ij,1} + \beta |\lambda_{ij,2}| \frac{\lambda_{ij,2}}{|\lambda_{ij,2}|} \mathbf{v}_{ij,2}) + \mathbf{n}_{ij}.$$
(9)

Considering the received power from (9), the minimum effective power  $C'_{ij}$  in (8) is expressed as:

$$C'_{ij}(r_{i1}, r_{i2}, r_{j1}, r_{j2}, \varphi_{ij}) = |\lambda_{ij,2}(r_{i1}, r_{i2}, r_{j1}, r_{j2}, \varphi_{ij})|^2$$
(10)

[5]. BER performances are therefore improved as the receiver obtains a high minimum effective power when using the receiving antenna selection techniques.

#### B. RAS-E

We propose a receiving antenna selection technique using the channel matrix eigenvalue in the following based on section III.A.

The receiver has *n* antennas  $(n \ge 3)$ . Therefore the receiver has  ${}_{n}C_{2}$  combinations to select 2 receiving antennas from *n* receiving antennas. The minimum power of the eigenvalue of the channel matrix in selection pattern *p*  $(p=1,2,...,nC_{2})$  is

Transmitting antenna 1



Transmitting antenna 2





Fig.2(b). Receiver structure

expressed as  $|\lambda_{\min,p}|^2$ . Then, the receiver selects the 2 receiving antennas which give the largest power  $|\lambda_{\min,p}|^2$  at  $p=1, 2, ..., {}_{n}C_{2}$ .

The receiver structure is shown in Figure 2. We assume two receiver structures. One is a structure for controlling gain in the received signal centrally for all receiving antennas as in Figure 2(a). The other is a structure for controlling received signal gain individually in each receiving antenna as in Figure 2(b). Although the receiver requires a structure like Figure 2(a) for obtaining the minimum effective power  $C'_{ij}(\mathbf{r}_{11},\mathbf{r}_{12},\mathbf{r}_{j1},\mathbf{r}_{j2},\phi_{ij})$  from (10) in all selection patterns (*p*=1, 2,..., nC<sub>2</sub>), a receiver which has a structure like Figure 2(b) cannot obtain the minimum power  $C'_{ij}(\mathbf{r}_{11},\mathbf{r}_{12},\mathbf{r}_{j1},\mathbf{r}_{j2},\phi_{ij})$ .

Receiving antenna selection techniques for a receiver which has a structure like Figure 2(b) are described in sections III.C and III.D according to the two parameters which the receiver can easily obtain when the receiver has a structure like Figure 2(b): the phase of the channel matrix component, and the received power. Section III.C and III.Ddescribes the RAS-PC and RAS-RP technique, respectively.

## C. RAS-PC

The approximation from (10) is considered for a receiver which has a structure for controlling received signal gain individually in each receiving antenna as in Figure 2(b). It can be considered that the minimum effective power  $C_{ij}$  is a function of  $\varphi_{ij}$  alone, because the receiver controls received signal gain individually in each receiving antenna. So, if  $|h_{i1}|=|h_{j2}|$  or  $|h_{i2}|=|h_{j1}|$  is established and the phase differences  $\varphi_{\rm S}$  and  $\varphi_{\rm L}$  from (3) satisfies  $0 \le |\varphi_{\rm S}| \le |\varphi_{\rm L}| \le \pi$  radians, equation (11) is established.

$$|\lambda_{ij,2}(\varphi_S)|^2 \leq |\lambda_{ij,2}(\varphi_L)|^2 \tag{11}$$

Based on this, we propose a receiving antenna selection technique using the phase of the channel components in the following.

The receiver has *n* antennas  $(n \ge 3)$ . The phase difference from (3) in selection pattern *p* (*p*=1, 2, ..., <sub>n</sub>C<sub>2</sub>) is expressed as  $\varphi_p$ . The receiver selects the 2 receiving antennas which give the largest absolute value of the phase difference  $|\varphi_p|$  at  $p=1, 2, ..., _nC_2$ .

#### D. RAS-RP

The antenna selection pattern is expressed as p when the receiver selects receiving antennas i and j. The received power  $C_p$  is expressed as:

$$C_{p} = |h_{i1}|^{2} + |h_{i2}|^{2} + |h_{j1}|^{2} + |h_{j2}|^{2}$$
(12)

When noise is considered, the affect of the noise can be reduced by increasing the received power in (12). In the following, we consider a receiving antenna selection technique using the received power.

The receiver has *n* antennas  $(n\geq 3)$ . The received power from (12) in selection pattern *p* (*p*=1, 2, ..., <sub>n</sub>C<sub>2</sub>) is expressed as  $C_p$ . The receiver selects the 2 receiving antennas which give the largest  $C_p$  at *p*=1, 2, ..., <sub>n</sub>C<sub>2</sub>.

# IV. COMPARISON OF BER PERFORMANCES

#### A. BER Performances Using Inverse Channel Detection

In this section, BER performances employing RAS techniques are shown using ICD.

The relationship between BER and C/N (carrier-to-noise power ratio) with 3 receiving antennas employing RAS-E, RAS-PC and RAS-RP techniques is shown in Figure 3.  $G_n$  is defined as the antenna gain at a diversity with *n* antennas and can be expressed as:

$$G_n = 10\log(n/2) (dB)$$
 (13)

where the antenna gain with 2 antennas is set to be 0dB. Antenna gain increases according to the increase in the number of antenna and the carrier power also increases. The relationship between BER and  $E_b$ /No (energy per bit-to-noise spectral density ratio) with 3 receiving antenna employing RAS-E, RAS-PC and RAS-RP techniques is shown in Figure 4. As a comparison, BER performances with 2 receiving antennas are shown in Figures 3 and 4.

Here, we consider the performance of the receiver in Figure 3. The receiver with 3 antennas employing RAS-RP, RAS-PC and RAS-E techniques gives a margin of 4, 12 and



Fig.3. BER versus C/N per receiving antenna employing inverse channel detection.



Fig.4. BER versus E<sub>b</sub>/No employing inverse channel detection.

16dB at BER= $1.0 \times 10^{-4}$  compared to the receiver with 2 antennas. Thus, the effectiveness for improving BER performances is the greatest when the receiver use the RAS-E technique compared with the other techniques.

On the other hand, the RAS-PC technique is effective in improving BER performances when the receiver has a structure for controlling received signal gain individually in each receiving antenna although the RAS-RP technique is almost ineffective.

Furthermore, as can be seen in Figure 4, the bit energy efficiency is improved when applying the RAS techniques proposed here compared with that with 2 receiving antennas.

#### B. BER Performances Using MLD

In this section, BER performances employing RAS techniques are shown using MLD.

The relationship between BER and C/N with 3 receiving antennas employing RAS-E, RAS-PC and RAS-RP techniques is shown in Figure 5. In addition, the relationship between BER and  $E_b$ /No with 3 receiving antennas employing RAS-E, RAS-PC and RAS-RP techniques is shown in Figure 6. As a comparison, BER performances with 2 receiving antennas are shown in Figures 5 and 6. Here, we consider the performance of the receiver from Figure 5. The receiver with 3 antennas employing RAS-RP and RAS-E techniques gives a margin of 2 and 5dB at  $BER=1.0\times10^{-4}$  compared to the receiver with 2 antennas. However, the receiver with 3 antennas employing the RAS-PC technique cannot yield a margin. Therefore, the effectiveness for improving BER performances is the greatest when the receiver uses the RAS-E technique compared with the other techniques.

On the other hand, the RAS-RP technique is effective in improving BER performances when the receiver has a structure for controlling received signal gain individually in each receiving antenna, although the RAS-PC technique is not effective,.

Furthermore, as can be seen from Figure 6, although bit energy efficiency is improved by applying the RAS-E technique compared to that with 2 receiving antennas, it is not effective to apply the RAS-PC and RAS-RP techniques for the improvement of bit energy efficiency.

#### C. Results

As a result of the analysis of BER performances in section IV.*A*, IV.*B*, the following three results are particularly noteworthy:

• The effectiveness in improving BER performances is greatest when the receiver uses the RAS-E technique compared to the RAS-PC and RAS-RP techniques when employing either ICD or MLD.

• When the receiver has a structure for controlling received signal gain individually in each receiving antenna, the most effective RAS technique differs according to the detection method used by the receiver.

• The effectiveness for improvement of BER performances when applying RAS techniques with ICD is greater than that of MLD.

# V. CONCLUSIONS

This paper has made a comparison of BER performances when employing RAS-E, RAS-PC and RAS-RP techniques. We have shown that the effectiveness for improving BER performances when employing either ICD or MLD is greatest when the receiver uses the RAS-E technique compared to the RAS-PC and RAS-RP techniques. Furthermore, we demonstrated that the most effective RAS technique when the receiver has a structure for controlling received signal gain individually in each receiving antenna differs according to the detection method used by the receiver.

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Fig.5. BER versus C/N per receiving antenna employing LD.



Fig.6. BER versus E<sub>b</sub>/No employing LD.

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