Pattern Synthesis of Spherical Arrays

G. Seehausen
Institut für Technische Elektronik der
RWTH Aachen, Fed. Republik of Germany

Introduction

Because of their flexibility conformal phased arrays become of increasing interest in modern radar systems. These antennas allow a full space beam covering and beam forming by pattern synthesis. For planar and circular arrays several synthesis methods /1/,/2/ have already been proposed, but pattern synthesis for spherical arrays, especially for those with a large number of directive elements, does not seem to be available in the literature.

This paper describes a theory of synthesis of spherical array patterns using the two dimensional Fourier-Legendre-series. The influence of element spacing, element directivity and the radius of the sphere compared to the wavelength will be discussed. Some theoretical results for a spherical array of circular polarized elements are shown.

Pattern Synthesis for Continuous Spherical Arrays of Isotropic Elements

Consider a spherical array of radius a in the origin of a spherical coordinate system (r,Θ,Φ). The number of elements of the array tends to infinity and the elements are assumed to be isotropic point sources radiating perfect circular polarization. The current at each antenna element is chosen as a series of terms

$$i(\Theta,\Phi) = \sum_{n=0}^{N} \sum_{m=-n}^{n} I_n^m \cdot P_n^{|m|}(\cos\Theta) \cdot e^{jm\Phi} \qquad (1)$$

This is a representation of the element currents in a Fourier-Legendre-series of sequence currents, which are current components, that have a magnitude corresponding to the associated Legendre function $P_n^{|m|}(\cos\Theta)$ of the 1. kind /3/ and a progressive phase increase in Φ. The far field radiation pattern of the array is /4/

$$f(\Theta,\Phi) = \sum_{n=0}^{N} \sum_{m=-n}^{n} I_n^m \cdot 2\pi j^n \cdot J_n(k_0 a) \cdot P_n^{|m|}(\cos\Theta) \cdot e^{jm\Phi} \qquad (2)$$

with $J_n(k_0 a)$ = spherical Besselfunction of the 1. kind and $k_0 = 2\pi/\lambda^0$ (λ = wavelength). If $f_D(\Theta,\Phi)$ is the desired pattern, which is assumed as piecewise continuous, it can be represented as a Fourier-Legendre-series /4/

$$f_D(\Theta,\Phi) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} A_n^m \cdot P_n^{|m|}(\cos\Theta) \cdot e^{jm\Phi} \qquad (3)$$

where

$$A_n^m = \frac{2n+1}{4\pi} \cdot \frac{(n-|m|)!}{(n+|m|)!} \int_0^{2\pi} \int_0^{\pi} f_D(\Theta,\Phi) \cdot P_n^{|m|}(\cos\Theta) \cdot e^{-jm\Phi} \cdot \sin\Theta \, d\Theta \, d\Phi .$$

Thus a pattern can be synthesized that achieves a mean square match to the desired pattern by choosing

$$I_n^m = A_n^m/(2\pi j^n \cdot J_n(k_0a)). \tag{4}$$

As $J_n(k_0a)$ converges rapidly to zero for $n > k_0a$, the maximum number of sequence current terms is $N = k_0a$.

Synthesis for Continuous Spherical Arrays of Directive Elements

Now consider a continuous spherical array of circular polarized directive elements. Again we choose the current distribution corresponding to equ.(1). All the elements are assumed to have the same rotationally-symmetric radiation pattern about their axis, radially oriented. The pattern of the element located at the pole (Θ=0) is represented by the Legendre-series

$$h(\Theta) = \sum_{p=0}^{P} A_p \cdot P_p(\cos\Theta) \tag{5}$$

The Legendre-series result from equ.(3) by substituting m=0. For directive elements the sequence currents I_n^m are modified to

$$I_n^m = A_n^m/(\pi(2n+1)\cdot\frac{(n-|m|)!}{(n+|m|)!}\cdot\sum_{p=0}^{P} A_p\cdot\sum_{q=-p}^{p}\frac{(p-|q|)!}{(p+|q|)!}\cdot\sum_{u=|n-p|}^{n+p} j^u\cdot J_u(k_0a)\cdot B(u)) \tag{6}$$

with

$$B(u) = (2u+1)\cdot\frac{(u-|m-q|)!}{(u+|m-q|)!}\cdot\left[\int_{-1}^{+1} P_n^{|m|}(x)\cdot P_p^{|q|}(x)\cdot P_u^{|m-q|}(x)\cdot dx\right]^2,$$

where the Ap's are determined by the desired element pattern. Comparing to isotropic elements the maximum number N of current terms for directive elements is reduced by the degree P of element directivity:

$$N = k_0a - P$$

Synthesis for Discrete Spherical Arrays with Directive Elements

Now consider a discrete spherical array of directive circular polarized elements, whose pattern is described by equ.(5). The elements are arranged on the spherical surface in such a way that the array consists of K circular arrays with L_k equidistant elements. The circular arrays, whose axes coincide with the polar axis of the sphere, are located at the zeros $\Theta_k(K)$ of the Legendre function $P_K(\cos\Theta)$/4/. Thus the element coordinates are $(\Theta_k(K), \Phi_{kl} = 2\pi l/L_k)$. This special choice of element distribution has the advantage that we can make use of the following identity for $K > n+p \geq u$:

$$\sum_{k=1}^{K} c_k\cdot P_n^{|m|}(x_k)\cdot P_p^{|q|}(x_k)\cdot P_u^{|m-q|}(x_k) = \int_{-1}^{+1} P_n^{|m|}(x)\cdot P_p^{|q|}(x)\cdot P_u^{|m-q|}(x)\cdot dx,\quad x_k=\cos\Theta_k(K) \tag{7}$$

The c_k are defined in /5/. With respect to the continuous array the element currents are chosen as

$$i(\Theta_k(K),\Phi_{kl}) = \sum_{n=0}^{N}\sum_{m=-n}^{n} I_n^m(k)\cdot P_n^{|m|}(\cos\Theta_k(K))\cdot e^{j\Phi_{kl}}\ . \tag{8}$$

It can be shown, that the array factor of a discrete spherical array is expressed by the array factor for a corresponding continuous spherical array plus some correction terms. The influence of these correction terms can be kept negligibly small, if

$$K > N + P, \quad L_k > 2(P + N). \tag{9}$$

This implies an average element spacing of 0.5 λ, when the maximum number $N = 2\pi a/\lambda - P$ of current terms for best mean square match is used. The sequence currents for discrete spherical arrays, which are designed under consideration of condition (9), are given as

$$I_n^m(k) = c_k \cdot I_n^m / L_k, \tag{10}$$

where the I_n^m's result from equ.(6). For convenience the B(u)'s in equ.(6) are calculated using equ.(7).

Results

From the above investigations, we may conclude as follows:

(i) the element spacing depends on the number N of current terms and for best match to the desired pattern it should be less than 0.5 λ

(ii) the maximum number N of current terms, which can be applied to a spherical array, depends on the radius compared to the wavelength and on the degree P of element directivity

Some typical results on synthesized patterns of a spherical array are plotted in Fig. 1 (a) and 1 (b). Both patterns are rotationally-symmetric. The considered array with a radius of $a=12/k_0$ consists of 180 circular polarized directive (P = 2) elements with an average element spacing of 0.5 λ. For the synthesis N=10 current terms were used.

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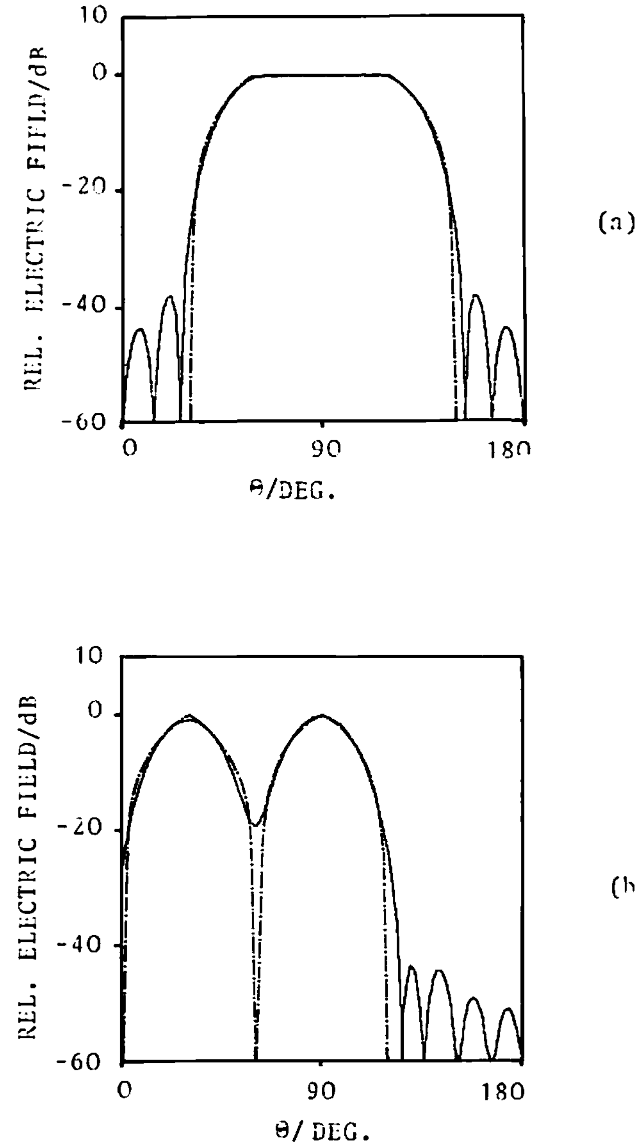

Fig.1 Far field patterns of a spherical array of 180 directive elements

—·—·— desired

——— synthesized